A New Link-level Urban Traffic Modeling Paradigm based on Interrupted Flow Fundamental Diagrams

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Abstract

An efficient and accurate dynamic traffic model which can reproduce the congestion propagation within an urban road network is required to assess the effect of different traffic management strategies. This paper proposes a link-level traffic flow modeling paradigm based on interrupted flow fundamental diagrams, which describes the relationship between flow, mean speed, and density on an urban link for each path. With the link traffic dynamics depicted by FDs, signal timing information does not need to be modeled explicitly. The adopted path-based approach and event-based resolution scheme ensures first-in-first-out at intersections for multi-commodity flow in congested situations with spillback. We test the model in scenarios with different network layouts and path compositions. The performance of the model is also compared with the store-and-forward models. The outcomes show a high level of compliance with the results computed from SUMO. It is anticipated that the model can be integrated into model-based optimization or control problems in an urban network.

Keywords

Congestion propagation; Dynamic urban traffic modeling; First-in-first-out; Interrupted traffic flow; Macroscopic fundamental diagram; Queue spillback; Path-based approach

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1 Introduction

Many studies related to macroscopic urban traffic modeling adapted models which were originally proposed for uninterrupted flow without the influence from traffic signals. For instance, there were studies which extended the cell-transmission model (CTM) and link-transmission model (LTM) for urban traffic (Adacher and Tiriolo, 2018; Hao *et al.*, 2018; van de Weg *et al.*, 2019, 2020). The delay due to red lights and the queue discharge rate during the switch of signal phases are represented in these models. These studies often utilized the fundamental diagram (FD), the flow-speed-density relationship for uninterrupted flow, to describe the traffic flow dynamics. However, the dynamics of interrupted flow in the urban context are essentially different from uninterrupted flow on motorways. Deliberately capturing the progression of uninterrupted vehicle flow between small cells or urban links has little positive contribution to the accuracy (Aboudolas *et al.*, 2009).

For the modeling of interrupted traffic flow in the urban context, Gazis and Potts (1963) proposed the store-and-forward (SaF) model, which considers each road link to be a queue of vehicles. The signal timing plan is explicitly included as parameters or even variables in the discrete-time model formulation. Therefore, the model were often utilized for traffic signal timing optimization purposes (Aboudolas *et al.*, 2009; Kouvelas *et al.*, 2014). To consider the link travel time and available storage space in the downstream links, Lin *et al.* (2012) proposed the S model by separating the moving and queuing vehicles. The transfer of vehicle flows within a link, hence, also needs to be modelled at every time step. By keeping track of the location of the tail of the queue, they aimed to capture the stop-and-go movement in detail. There were also other studies which applied link-level macroscopic urban traffic models to other problems. To determine the dedicated bus lane allocation plan, Tsitsokas *et al.* (2021) developed an extended SaF model for the optimization framework. Different from the original SaF model, the model also incorporates the computation of delay within the link using the same logic in both the BLX model and the S model in Lin *et al.* (2012). However, no study has ever tested and discussed the accuracy of these modeling techniques while it is known that time-discretization can only yield an approximation of the exact solution to the traffic model (Raadsen and Bliemer, 2019).

On the other hand, many macroscopic traffic models simply simulated single-commodity flow by designating turning vehicles following the pre-determined split ratios at intersections without considering the detailed paths of each origin-destination (OD) pair, which may lead to unrealistic scenarios and disallow the incorporation of dynamic traffic assignment (DTA). To overcome this limitation, there were many studies which aimed to make the CTM or LTM formulation suitable for multi-commodity flow modeling (Szeto and Lo, 2004; Ukkusuri *et al.*, 2012; Yperman *et al.*, 2006; Raadsen and Bliemer, 2019). However, these DTA studies are only able to ensure first-in-first-out (FIFO) condition to a certain extent. The FIFO violation becomes significant when there is spillback on downstream links. In light of this problem, Blumberg and Bar-Gera (2009) introduced the concept of anticipated arrival order into the iterations between the route model and traffic flow model so that the route choice outcome and actual travel time can be consistent. This approach can be regarded as a reverse engineering method to approximate FIFO. To achieve exact FIFO, Bar-Gera and Carey (2022) explicitly extended the CTM by defining "traffic cohorts" and fully tracking their movements from the origin cell to the destination cell. The cohorts can be split when necessary due to the limited exit capacity. Nevertheless, the method can hardly be applied to large-scale network due to its relatively intensive computation requirement and intricate model formulation.

All the aforementioned DTA models applied uninterrupted flow FDs to describe the traffic state on each cell or link, which is only suitable for the dynamic traffic loading of a motorway network. For urban traffic modeling in particular, De Souza *et al.* (2022) proposed the multi-commodity SaF by differentiating the routes in the network. The flow of each commodity is governed by the proportion of green period it receives. However, as acknowledged in the article, the model could only prevent the violation of FIFO by avoiding long queues through the signal timing optimization framework coupled with the model. For pure modeling purposes, the accuracy cannot be guaranteed in such an approach.

There has been little endeavor which applied multi-commodity traffic flow modeling to an urban network considering the effect of signal control and different OD paths. This complexity degrades the accuracy of the macroscopic modeling outcome. A technique which can overcome such a problem and still possess smaller computation complexity and better adaptiveness to different conditions for model-based decision-making compared to microscopic traffic simulation is required. In this study, we propose a link-level urban traffic modeling paradigm by first exploiting the merit of the macroscopic fundamental diagram (MFD) in depicting urban traffic dynamics. The path-based approach and event-based resolution scheme enable space- and time-continuous modeling and also ensure exact FIFO for multi-commodity flow. This also allows the incorporation of route choice behavior for different paths in different departure time periods easily.

2 Model Description

This section first introduces the method to derive the interrupted flow fundamental diagram and then describes the proposed link-level urban traffic model step-by-step.

2.1 Interrupted flow fundamental diagram

In this study, continuous-time modeling is considered one of the key aspects to improve the accuracy of traffic models. However, the switch of phases in the traffic signal timing plan is an obstacle for the development of a continuous-time urban traffic flow model. To get rid of such a complexity, the interrupted traffic flow dynamics on urban road links needs to be accurately represented.

The FD has been a well-known tool to describe traffic dynamics by capturing the density-flow-speed relationship. There were a few studies which seek to estimate the FD for urban links based on empirical data (Wu *et al.*, 2011; Dakic and Stevanovic, 2018; Yin *et al.*, 2022). However, these studies omitted the heterogeneity of link traffic dynamics experienced by vehicles with different paths caused by traffic signal control. For instance, a turning vehicle at a four-leg intersection has a different travel time on the downstream link compared to the travel time experienced by a vehicle which comes from an upstream link with better signal coordination design even under the same density. Hence, the resulting empirical FD is not fixed but influenced by the path heterogeneity. For traffic flow modeling purposes, it is important to differentiate the FDs of different paths. In addition, to adapt to different signal timing plans, a theoretical method to derive the FDs for different paths on an urban link without relying on empirical data is required. Although Helbing (2009) aimed to derive the urban FD through a utilization-based approach, the assumption of uniform arrival rate also indicates the ignorance of different paths from the upstream of the link. There is still no proper method to analytically derive the FD of a path on an urban link.

The macroscopic fundamental diagram (MFD) has been used as a convenient tool for urban traffic modeling at the corridor- and network-level. Here, we believed that the average traffic dynamics experienced by a path on an urban link should be similar to that of a corridor consisting of multiple identical links with the same length and downstream signal timing plan. The variety caused by the arrival patterns at different time points during a green period can be accounted for with the signal coordination effect between consecutive links. Hence, it is speculated that the shape of an interrupted flow FD may be approximated with the MFD of such a homogeneous corridor. The accuracy of such an approximation will be discussed in section 4.

In this study, the shape of an analytical corridor MFD is derived with the variational theory (VT) method proposed in Leclercq and Geroliminis (2013). In the VT method, an MFD is composed of a set of practical cuts in the density-flow (k, q) plane. The cuts are derived from the mean speeds and mean costs of several shortest path moving observers in the VT graph, which is a time-space (x, t) plane containing the signal timing plans of intersections along the corridor. For detailed information regarding the VT method, the reader is referred to the original papers.

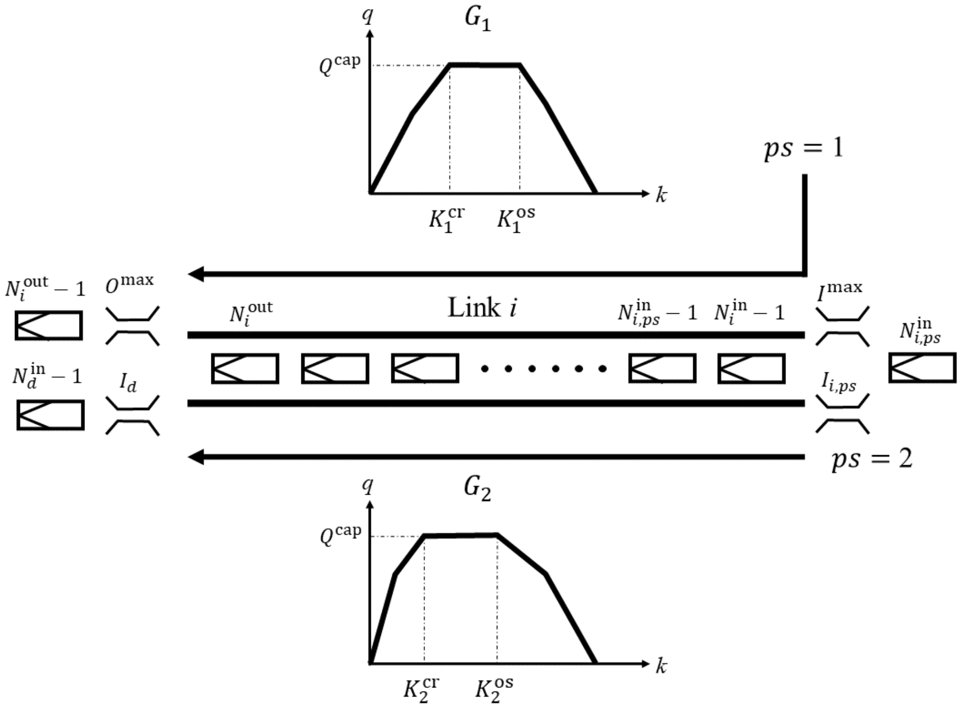
2.2 Fundamental diagram-based model

As discussed in the previous subsection, this study aims to exploit the theoretical knowledge developed for aggregated-level urban traffic flow to the modeling of link-level traffic dynamics by using the approximated interrupted flow FDs. This means that the effect of signal control, including the green-to-cycle ratio and the offset, is already considered. Hence, the signal timing information would not need to be explicitly considered anymore in the model.

This study adopts the event-based resolution scheme coupled with the path-based approach, which is similar to the trip-based MFD modeling approach proposed in Mariotte *et al.* (2017). Different from the network-level modeling, this approach models the transmission of vehicles between links. The remaining travel distance of each vehicle on each link is updated at every event step based on its assigned speed upon entry of the link. Under such formulation, each link is considered a queue system as in the SaF. FIFO can be fully ensured as the order of vehicles in the queue is tracked. Figure 1 explains the modeling principle on a single link with two path sets.

Each path set contains paths that enter the link from the same upstream link as they experience the same dynamics (signal timing setup) on this link. G_{ps} denotes the flow FD which belongs to the path set ps on the link. For each path set, the traffic state is considered saturated between the critical density K^{cr} and the over-saturated density K^{os}. The transmission of a vehicle between links is governed by its remaining travel distance

Figure 1: Representation of the link-level traffic flow model based on the path FDs



on the current link and its mean speed on the link which can be derived from the speed FD $V(k) = G_{ps}(k)/k$ according to the link density k (number of vehicles ahead) upon entry.

N_i^{in} and N_i^{out} are the order of the vehicles which are about to enter and exit the link, respectively. In particular, $N_{i,ps}^{\text{in}}$ denotes the order of the entering vehicle of path set ps, which will also be used for the entry supply function. The entry flow into the link for each path is limited by an entry supply function $I_{i,ps}$, while the exit flow is constrained by both the maximum outflow $O^{\max}$, which should be equal to the maximum flow Q^{cap} shown in the FDs, and the entry supply of the downstream link d of the next exiting vehicle N_i^{out}.

To consider the spillback effect at the entry of the link, the concept of entry supply limitation described in Mariotte and Leclercq (2019) is included. Eq. 1 formulates the entry supply function for each path set $I_{i,ps}$, which is a function of the link density k_i. When the link is over-saturated, i.e., the link density exceeds K^{os}, only the remaining production $G_{ps}(k_i)$ can be utilized by the inflow demand. The inflow would also be limited by the inflow capacity $I^{\max}$, which depends on the total green time of the upstream path sets. The earliest possible (supplied) entry time of vehicle N_i^{in} can then be calculated by Eq. 2.

$$I_{i,ps}(k_i) = \begin{cases} Q^{\text{cap}}, & \text{if } k_i \leq K^{\text{os}} \\ G_{ps}(k_i), & \text{otherwise} \end{cases} \tag{1}$$

$$t_{\text{entry supply}}^{N_{i,ps}^{\text{in}}} = \max(t_{\text{entry}}^{N_{i,ps}^{\text{in}}-1} + \frac{1}{I_{i,ps}(k_i)}, t_{\text{entry}}^{N_i^{\text{in}}-1} + \frac{1}{I^{\max}}) \tag{2}$$

In under-saturated conditions, the time of the next exit demand is determined by the remaining distance of vehicle N_i^{out} on the link and the link speed computed from the FD. In Eq. 3, L_i denotes the total length of link i, while $l^{N_i^{\text{out}}}$ is the remaining distance for vehicle N_i^{out} to travel on link i. For saturated or over-saturated conditions, the extension to account for the maximum exit demand during saturation is also applied. When the link density exceeds K^{cr}, the last vehicle on the link N_i^{out} would be pushed to exit the link so that the exit flow can fulfill the maximum outflow $O^{\max}$. On the other hand, besides the capacity constrain, the exit time is also limited by the entry supply of the downstream link of vehicle N_i^{out}, which is denoted by d here. The supplied exit time of vehicle N_i^{out} can hence be expressed as Eq. 4.

$$t_{\text{exit demand}}^{N_i^{\text{out}}} = \begin{cases} t + (L_i - l^{N_i^{\text{out}}})/v(k_i), & \text{if } k_i \leq K^{\text{cr}} \\ \max(t, t_{\text{exit}}^{N_i^{\text{out}}-1} + \frac{1}{O^{\max}}), & \text{otherwise} \end{cases} \tag{3}$$

$$t_{\text{exit supply}}^{N_i^{\text{out}}} = \max(t_{\text{exit}}^{N_i^{\text{out}}-1} + \frac{1}{O^{\max}}, t_{\text{entry supply}}^{N_d^{\text{in}}}) \tag{4}$$

The numerical procedure of the event-based modeling scheme for the proposed link-level FD-based urban traffic model is summarized below:

1. Initialize all the vehicle entry requests at every inflow node by assigning the desired

entry time, path, the first link length (remaining link travel distance), and link speed.

2. Determine the next time step and time step size by finding the event with the minimum entry or exit time on all links following the demand and the supply limitations described in Eq. 2, Eq. 3, and Eq. 4. Note that it is possible to have multiple events taking place at the same time step.
3. For all existing vehicles, update their remaining link travel distances based on the assigned travel speeds and the time step size determined in step 2.
4. For each link, insert and/or remove a vehicle according to the events identified in step 2.
5. For each link, compute the travel speed based on the updated density for the vehicle that will enter in the next time step.
6. Return to step 2.

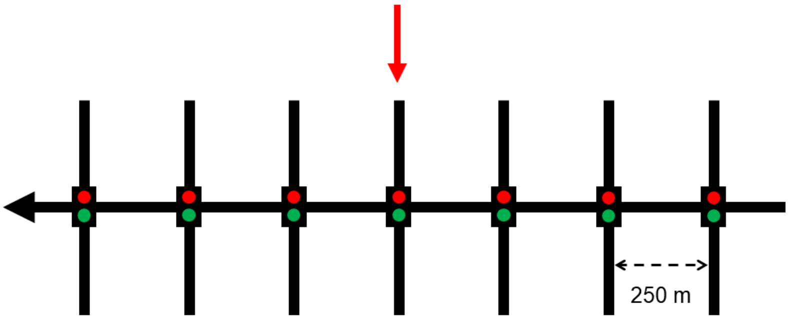
3 Case Study

The model performance is tested in a case study to evaluate its accuracy. The open-source microscopic traffic simulation tool, SUMO (Lopez *et al.*, 2018), is used to compute the groundtruth simulation outcome. The desired maximum speed in the car-following model is set to $v_\mathrm{f} = 12.5$ m/s. There is no stochasticity and heterogeneity in the driving behavior. This also influences the parameters used in the VT method. The resulting intersection capacity q_m becomes 0.6 veh/s, while the jam wave speed w is therefore 7.5 m/s according to the assumed triangular FD in the VT method. It is worth noting that they are larger than the values typically used in practice since a homogeneous driving behavior is considered to align with the setting in the microsimulation tool for validation purposes.

A one-way single-lane corridor with a homogeneous link length of 250 m is implemented in the first scenario, as shown in Figure 2. It is assumed that the signals along the corridor have the same green length of 25 s and cycle length of 60 s. There is no endogenous bottleneck since the lane configuration and signal timing plan are both uniform. The offsets are designed in a way that approximately half of the vehicle platoon which starts from the upstream intersection can pass through the downstream intersection if there is no queuing vehicle, i.e., link length / free flow speed / 2.

A three-hour scenario with varying inflow demand is designed to mimic a typical peak period. Besides the main path (path 1) which passes through every intersection along the corridor, one turning path (path 2) is included as a source of disturbance to the steady traffic state on the corridor. Turning vehicles enter the corridor from the fourth intersection and exit via the seventh intersection. The demand profile of the turning path is set to one-fifth of the main path.

Figure 2: Layout of the homogeneous corridor



In this case study, it is possible to compare our model with the SaF models as this scenario can be simulated in the single-commodity setup. Therefore, we also simulate the scenario using the original SaF and the extended SaF, which are presented in the next section.

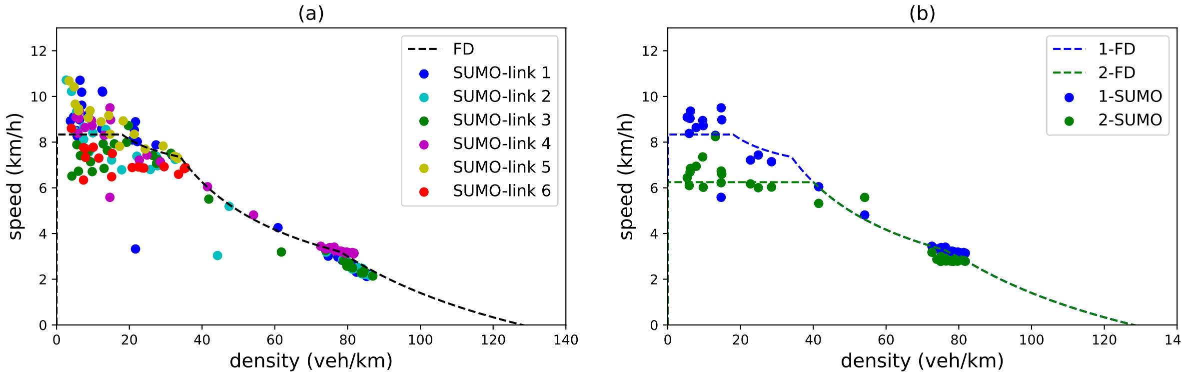
4 Results

This section first inspects the ability of the approximated FDs to describe the link traffic dynamics of different paths. The evolution of link density and path travel time in the simulation period are then discussed.

Figure 3 shows the mean link travel speed for each path at every five-minute interval. As can be seen, the mean link travel speed computed from the SUMO outcomes comply with the approximated theoretical FDs, which proves the accuracy of the FDs derived with the VT method in describing the link traffic performance under different density conditions.

The link density evolution simulated using SUMO, the SaF models, and the proposed FD-based model is then shown in Figure 4. On links 1-4, the onset of congestion is delayed when using the original SaF due to its inability to consider link travel time. Vehicles are pushed out of the link faster than in the reality. On the other hand, the extended SaF

Figure 3: Mean link travel speed of (a) path 1 on all links and (b) both paths on link 4



also fails to precisely simulate the development of congestion. The link density starts to increase earlier and drops slightly slower than the ground-truth. This shows the problem of inaccurate update of the number of vehicles joining and exiting the waiting queue in the discrete-time formulation. The same problem can also clearly be found on links 5 and 6, where the traffic state is supposed to maintain steady at the critical density when the upstream is congested. Comparatively, the proposed model shows consistent results with the SUMO ground-truth. The onset and recovery of congestion on every link can be captured quite well.

In addition, to validate the suitability of the model for multi-commodity modeling, it is important to examine the path travel speed evolution. As shown in Figure 5, the mean travel speed on both paths analyzed from the results of the FD-based model also be accurately predicted. This indicates the potential to incorporate path choice behavior in the future.

5 Conclusions

This study proposes a new modeling paradigm to simulate urban traffic dynamics at the link-level. By using the path-based approach and event-based resolution scheme, we allow the model to incorporate path choice behavior considering travel time affected by congestion in a DTA framework with fully ensured FIFO. They also enable continuous-time modeling, which significantly improves the accuracy. However, the drawback is longer computation time required particularly in a large network with a great amount of links and

Figure 4: Density evolution on all links

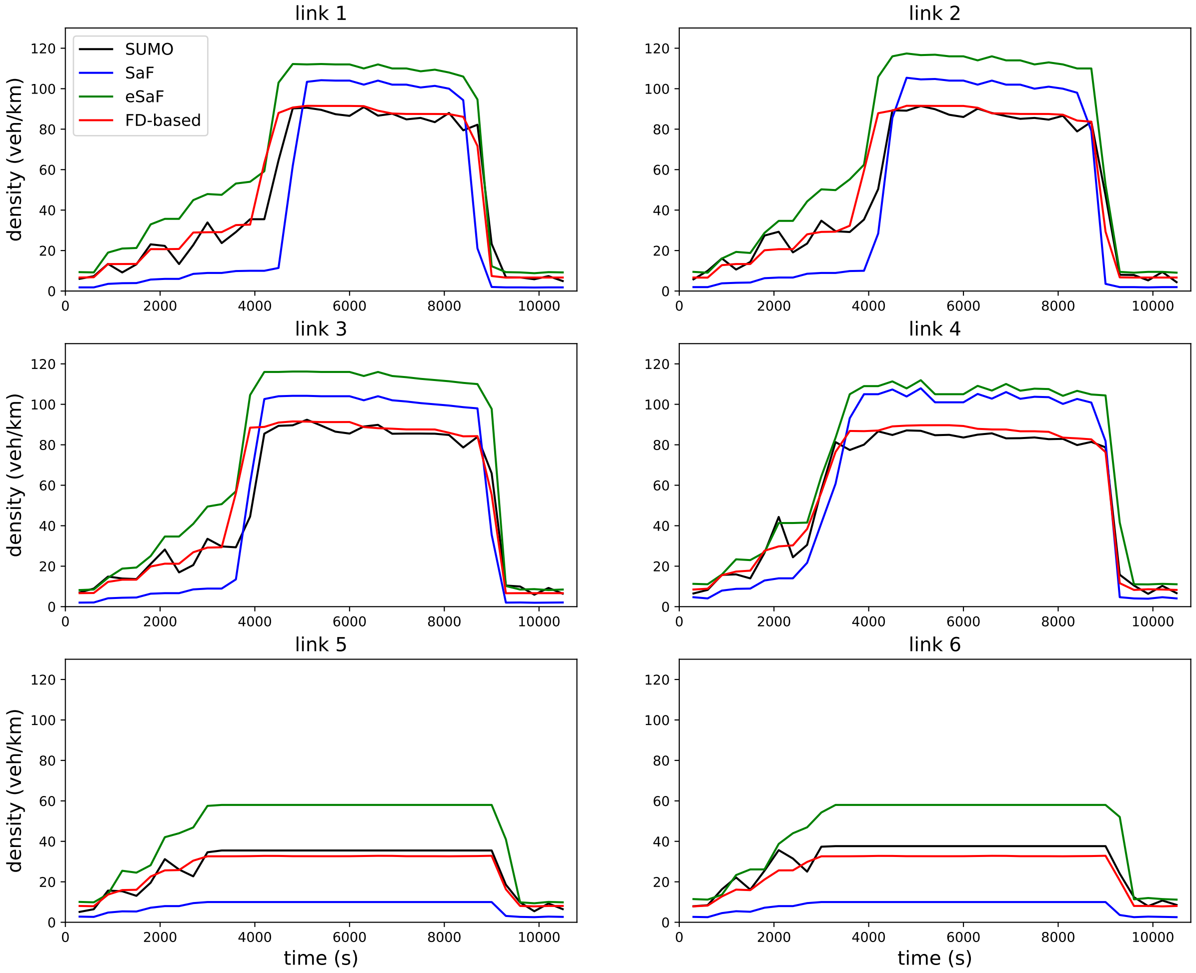
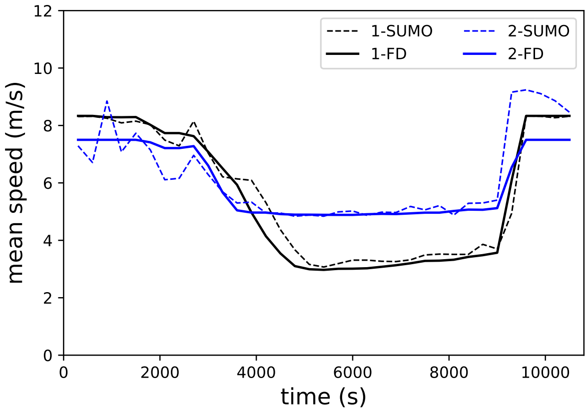


Figure 5: Path mean travel speed evolution



circulating vehicles compared to the other discrete-time models. In terms of computation efficiency, the model is considered an alternative which lies between microscopic traffic simulation tools and typical macroscopic link-level traffic modeling methods.

Most importantly, the use of FD to describe link traffic dynamics allows convenient adaptation for changes in lane configuration on the road space of the corridor. Hence, the model can be easily applied to urban traffic management problems, such as multi-modal urban road space allocation strategies.

Future work will extend the case study to the network-scale to investigate the ability of the model to reproduce the congestion propagation on vertical links due to spillback and identify other potential problems that are not yet discovered. Heterogeneity in other aspects, such as link length and signal timing plan, will also be considered.

6 References

Aboudolas, K., M. Papageorgiou and E. Kosmatopoulos (2009) Store-and-forward based methods for the signal control problem in large-scale congested urban road networks, *Transportation Research Part C: Emerging Technologies*, **17** (2) 163–174, 4 2009, ISSN 0968090X.

Adacher, L. and M. Tiriolo (2018) A macroscopic model with the advantages of microscopic model: A review of Cell Transmission Model's extensions for urban traffic networks, *Simulation Modelling Practice and Theory*, **86**, 102–119, 8 2018, ISSN 1569190X.

Bar-Gera, H. and M. Carey (2022) Constructing a cell transmission model solution adhering fully to first-in-first-out conditions, *Transportation Research Part B: Methodological*, **161**, 247–267, 7 2022, ISSN 01912615.

Blumberg, M. and H. Bar-Gera (2009) Consistent node arrival order in dynamic network loading models, *Transportation Research Part B: Methodological*, **43** (3) 285–300, 3 2009, ISSN 01912615.

Dakic, I. and A. Stevanovic (2018) On development of arterial fundamental diagrams based on surrogate density measures from adaptive traffic control systems utilizing stop-line detection, *Transportation Research Part C: Emerging Technologies*, **94**, 133–150, 9 2018, ISSN 0968090X.

De Souza, F., R. C. Carlson, E. R. Muller and K. Ampountolas (2022) Multi-Commodity

Traffic Signal Control and Routing With Connected Vehicles, *IEEE Transactions on Intelligent Transportation Systems*, **23** (5) 4111–4121, 5 2022, ISSN 1524-9050.

Gazis, D. and R. Potts (1963) The oversaturated intersection, paper presented at the *Proceedings of the Second International Symposium on Traffic Theory*, 221–237, London, UK.

Hao, Z., R. Boel and Z. Li (2018) Model based urban traffic control, part I: Local model and local model predictive controllers, *Transportation Research Part C: Emerging Technologies*, **97**, 61–81, 12 2018, ISSN 0968090X.

Helbing, D. (2009) Derivation of a fundamental diagram for urban traffic flow, *The European Physical Journal B*, **70** (2) 229–241, 7 2009, ISSN 1434-6028.

Kouvelas, A., J. Lioris, S. A. Fayazi and P. Varaiya (2014) Maximum Pressure Controller for Stabilizing Queues in Signalized Arterial Networks, *Transportation Research Record: Journal of the Transportation Research Board*, **2421** (1) 133–141, 1 2014, ISSN 0361-1981.

Leclercq, L. and N. Geroliminis (2013) Estimating MFDs in simple networks with route choice, *Transportation Research Part B: Methodological*, **57**, 468–484, 11 2013, ISSN 01912615.

Lin, S., B. De Schutter, Y. Xi and H. Hellendoorn (2012) Efficient network-wide model-based predictive control for urban traffic networks, *Transportation Research Part C: Emerging Technologies*, **24**, 122–140, 10 2012, ISSN 0968090X.

Lopez, P. A., E. Wiessner, M. Behrisch, L. Bieker-Walz, J. Erdmann, Y.-P. Flotterod, R. Hilbrich, L. Lucken, J. Rummel and P. Wagner (2018) Microscopic Traffic Simulation using SUMO, paper presented at the *2018 21st International Conference on Intelligent Transportation Systems (ITSC)*, 2575–2582, 11 2018, ISBN 978-1-7281-0321-1.

Mariotte, G. and L. Leclercq (2019) Flow exchanges in multi-reservoir systems with spillbacks, *Transportation Research Part B: Methodological*, **122**, 327–349, 4 2019, ISSN 01912615.

Mariotte, G., L. Leclercq and J. A. Laval (2017) Macroscopic urban dynamics: Analytical and numerical comparisons of existing models, *Transportation Research Part B: Methodological*, **101**, 245–267, 7 2017, ISSN 01912615.

Raadsen, M. P. and M. C. Bliemer (2019) Continuous-time general link transmission model with simplified fanning, Part II: Event-based algorithm for networks, *Transportation Research Part B: Methodological*, **126**, 471–501, 8 2019, ISSN 01912615.

Szeto, W. and H. K. Lo (2004) A cell-based simultaneous route and departure time choice model with elastic demand, *Transportation Research Part B: Methodological*, **38** (7) 593–612, 8 2004, ISSN 01912615.

Tsitsokas, D., A. Kouvelas and N. Geroliminis (2021) Modeling and optimization of dedicated bus lanes space allocation in large networks with dynamic congestion, *Transportation Research Part C: Emerging Technologies*, **127**, 103082, 6 2021, ISSN 0968090X.

Ukkusuri, S. V., L. Han and K. Doan (2012) Dynamic user equilibrium with a path based cell transmission model for general traffic networks, *Transportation Research Part B: Methodological*, **46** (10) 1657–1684, 12 2012, ISSN 01912615.

van de Weg, G. S., M. Keyvan-Ekbatani, A. Hegyi and S. P. Hoogendoorn (2020) Linear MPC-Based Urban Traffic Control Using the Link Transmission Model, *IEEE Transactions on Intelligent Transportation Systems*, **21** (10) 4133–4148, 10 2020, ISSN 1524-9050.

van de Weg, G. S., H. L. Vu, A. Hegyi and S. P. Hoogendoorn (2019) A Hierarchical Control Framework for Coordination of Intersection Signal Timings in All Traffic Regimes, *IEEE Transactions on Intelligent Transportation Systems*, **20** (5) 1815–1827, 5 2019, ISSN 1524-9050.

Wu, X., H. X. Liu and N. Geroliminis (2011) An empirical analysis on the arterial fundamental diagram, *Transportation Research Part B: Methodological*, **45** (1) 255–266, 1 2011, ISSN 01912615.

Yin, R., N. Zheng and Z. Liu (2022) Estimating fundamental diagram for multi-modal signalized urban links with limited probe data, *Physica A: Statistical Mechanics and its Applications*, **606**, 128091, 11 2022, ISSN 03784371.

Yperman, I., S. Logghe, C. Tampere and B. Immers (2006) The multi-commodity link transmission model for dynamic network loading, paper presented at the *The 85th Annual Meeting of the Transportation Research Board*, Washington DC, USA, 1 2006.