Concept-NN:
substituting complex functions in decision models with constrained representation learning

STRC 2021

Brian Sifringer

Summary

- Research Goals
- Theoretical
  - Advances in DCM and DL in Transport
  - Proposed Contribution
  - Concept-NN definition
- Experimental
  - Literature in Human Trajectory Forecasting
  - Replacing complex DCM functions with Concept-NN
  - Results
- Pending Improvements
- Conclusion, Questions & Suggestions

Research Goals

Representation Learning

Predictive Strength

Discrete Choice Modeling

High Interpretability

Image: https://dumielauxepices.net/sites/default/files/yinyang-tattoos-clipart-koi-fish-829882-434406.jpg

EPFL

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- Better fit to data
- Avoid misspecification
- New input types
- …

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Research Goals

- Explainable A.I.
- Regularizer
- …

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Related Literature in DL and DCM in Transport

Deep Learning

- Extended applications of machine learning methods [1]

Classical

- Improving Specification [2]
- Improving Architecture [3]

[1] T. Hillel, M. Bierlaire, M. Z. Elshafie, and Y. Jin. A systematic review of machine learn- ing classification methodologies for modelling passenger mode choice. *Journal of choice modelling*, 2021.

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Hybrid

- DL as replacing architecture [4]
- Reading econometric information from DL methods [5]
- DL in utility specification [6]

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[4] Bentz, Y., Merunka, D. Neural networks and the multinomial logit for brand choice modelling: a hybrid approach. J. Forecast. 19 (3), 2000

[5] S. Wang, Q. Wang, and J. Zhao. Deep neural networks for choice analysis: Extracting complete economic information for interpretation, 2018b.

[6] B. Sifringer, V. Lurkin, and A. Alahi. Enhancing discrete choice models with neural networks. In *hEART 2018–7th Symposium of the European Association for Research in Transporta- tion conference*,, 2018.

Previous works including DL directly in parts of the utility function

- Finding missing utility specification [7]:

$$\boldsymbol{U}_n = \boldsymbol{f}(\mathcal{X}_n; \boldsymbol{\beta}) + \boldsymbol{r}(\mathcal{Q}_n; \boldsymbol{w}) + \boldsymbol{\varepsilon}_n.$$

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- Finding better taste-functions for parameter heterogeneity [8]:

$$V_{in} = \boldsymbol{\beta_i}^{TN}(\boldsymbol{z_n}; \boldsymbol{w})' \boldsymbol{x_{in}}^{TN} + \boldsymbol{\beta_i}^{MNL'} f(\boldsymbol{x_{in}}^{MNL}, \boldsymbol{z_n})$$

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- Using ‘embedded representation’ attribute for new input type [9,10]

$$V_n = f(\boldsymbol{x}_n, \boldsymbol{\beta}) + r(\mathcal{Z}_n)$$

$\mathcal{Z}_n$ is an image

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[9] Van Cranenburgh, S. (2020). Blending computer vision into discrete choice models. *preprint*.

[10] B.Sifringer, A. Alahi, Add Image to my Choice, STRC 2020 - Presentation only

Proposed contribution

- Replace complex hand-designed components of the utility with a Neural Network (NN)
- Use expert modeling to constrain input and output of the NN
- Interpretability:
  - Follow interpretability conditions
  - The output of NN is a new “representation” of its input
- In short:

$$V_n = f(\boldsymbol{x}_{1n}, \boldsymbol{\beta}_1) + g(\boldsymbol{x}_{2n}, \boldsymbol{\beta}_2) + h(\boldsymbol{x}_{3n}, \boldsymbol{\beta}_3)$$

Concept-NN

- Select a meaningful subset: $Z_n \subset X$
- Output defined as:

$$c_{*i} = c_*(\boldsymbol{z_i}) = a_*\left((L_K \circ S_+ \circ L_h \circ S_+ \circ L_h)(\boldsymbol{z_i})\right)$$

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S: Softplus activation
h: hidden dimension
K: output dimension
a: final activation
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- The final activation defines the output constraint:

positive	negative	both
$a_+ = R_+$	$a_- = -(R_+)$	$a_0 = I$

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$c_*(z_i)$

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Human Trajectory Forecasting: Prior Work

- Knowledge-Based[1, 2]
  - Hand-crafted functions designed by experts based on social concepts.

- Deep Learning[3, 4]
  - Neural network to approximate the prediction model directly from data.

Brian Sifringer (slide: Parth Kothari)

Concept-NN: substitute complex DCM functions

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Human Trajectory Forecasting: Prior Work

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- Knowledge-Based
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✓ Simple and Generalizable

✓ **Model decisions can be explained**

× Fail to model long-term dependencies

× Cannot model complex interactions

- Deep Learning
  - Neural network to approximate the prediction model directly from data.

✓ **Model long-term dependencies**

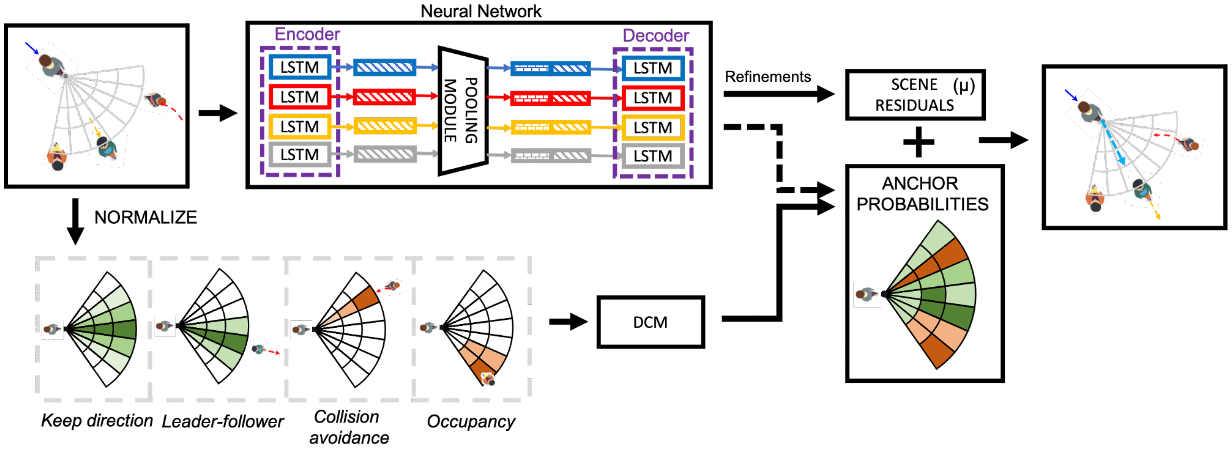
✓ **High accuracy on existing datasets**

× Model decisions cannot be explained

Combining the best of the 2 worlds!

- Concept-NN: substitute complex DCM functions

Prior Work: Social Anchors [1]



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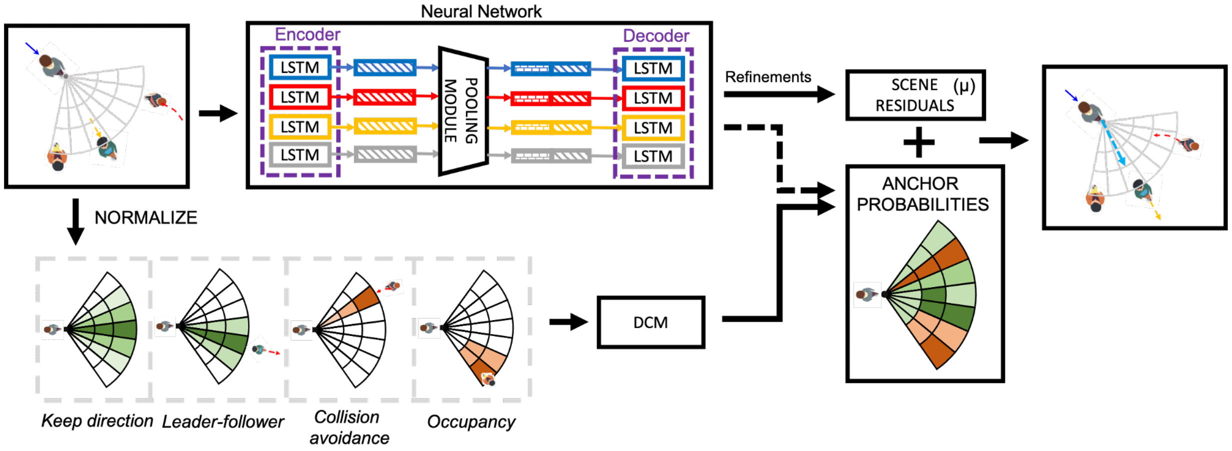

- Discretization and DCM functions for interpretability [2,3]

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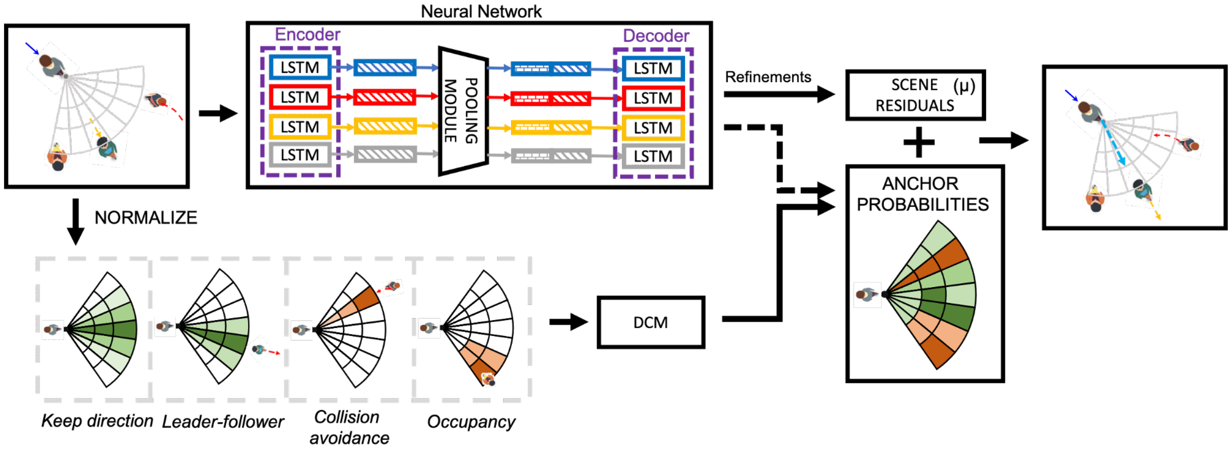

- Discretization and DCM functions for interpretability [2,3]
- NN to encode:
  - "Past trajectory" + "complex social interactions" in utility function
  - Predict residuals to project : Discrete => Continuous

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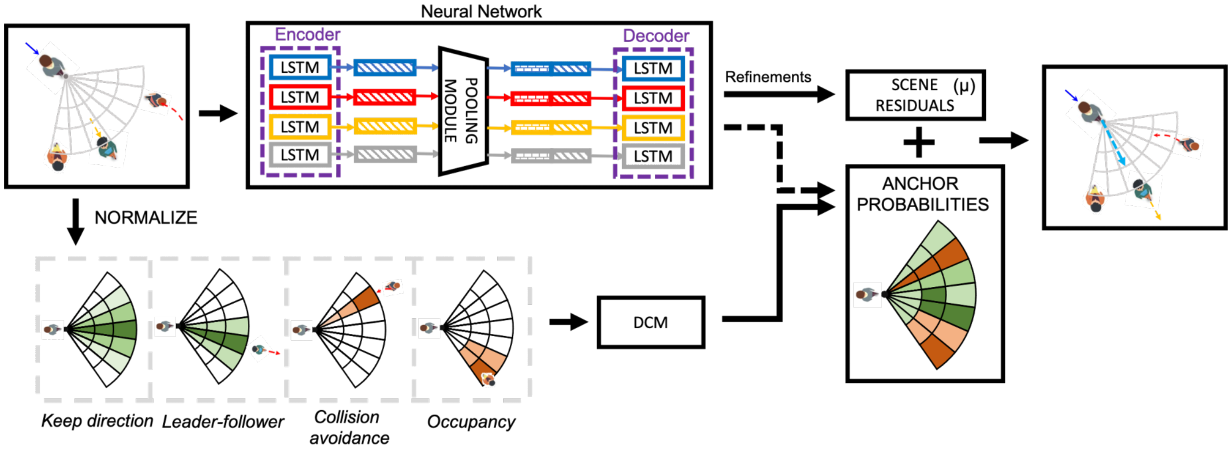
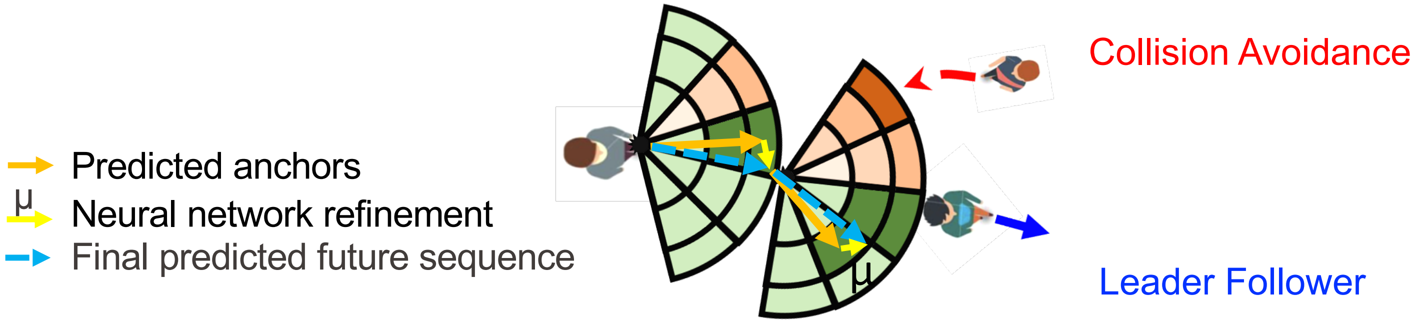


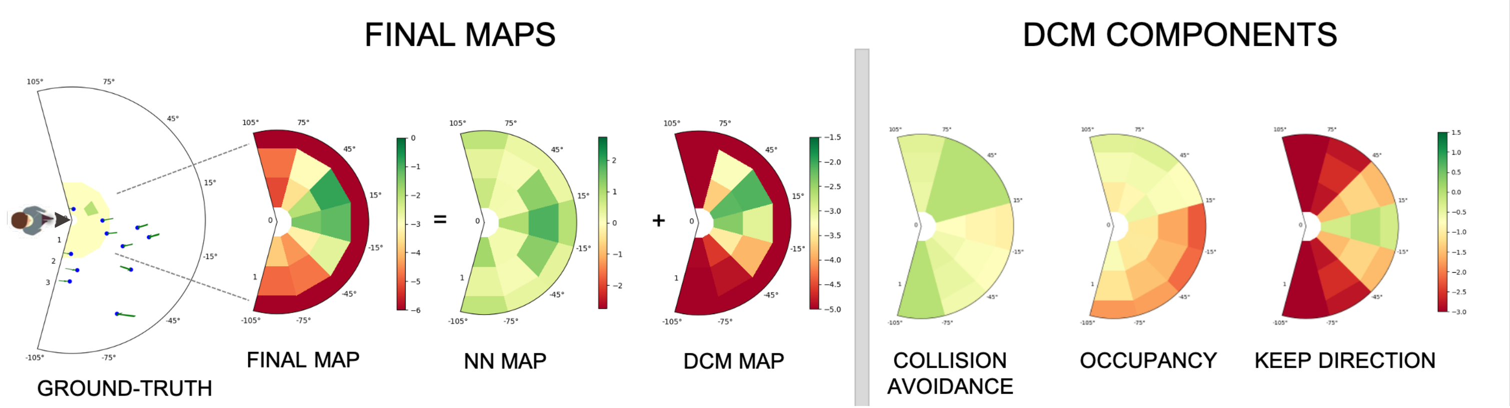


Favourable regions in green, unfavourable in red

Concept-NN: substitute complex DCM functions

Prior Work: Social Anchors - Results

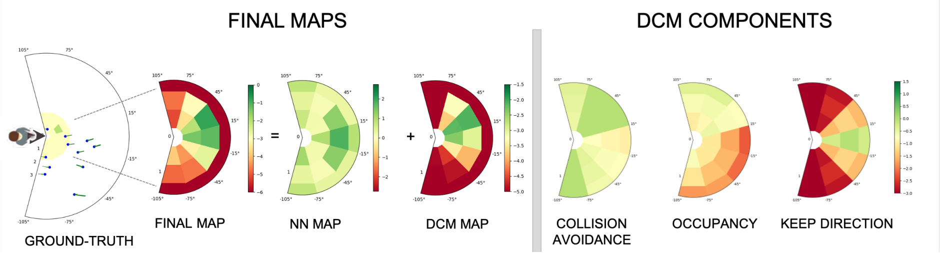
✓



✓

- Trajectory prediction on-par with state-of-the-art
- Added interpretability

Prior Work: Social Anchors - Results

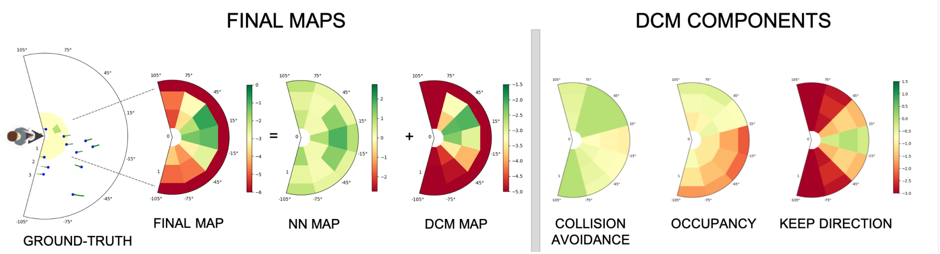


✓

✓ - Trajectory prediction on-par with state-of-the-art
- Added interpretability

However …

Prior Work: Social Anchors - Results



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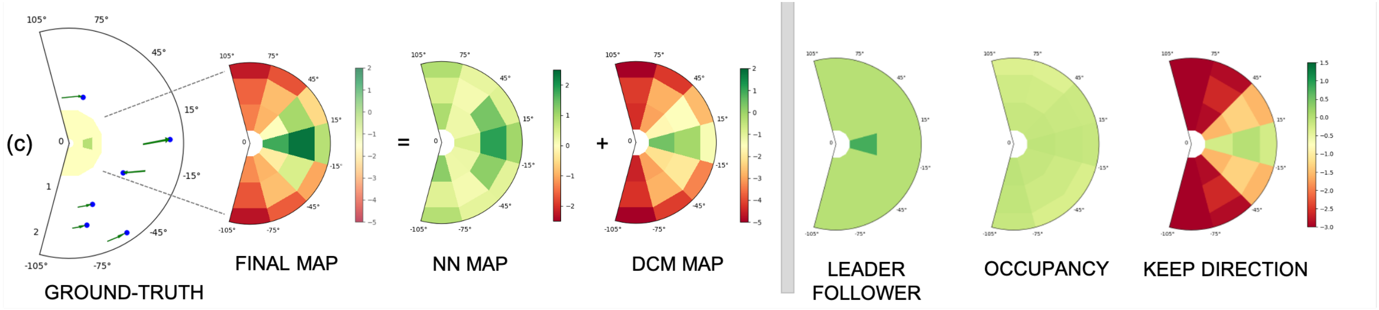
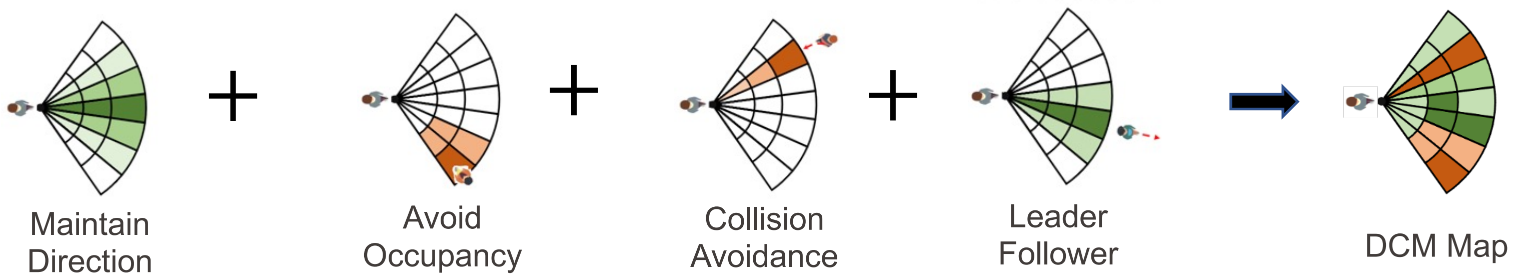


Too often, dominating result is predicted by the NN map

Concept-NN: substitute complex DCM functions

Replacing complex functions with Concept-NNs



$$u(\mathbf{X}) = \underbrace{\beta_{dir} dir}_{keep\ direction} + \underbrace{\beta_{occ} occ}_{avoid\ occupancy} + \underbrace{\beta_{C} col}_{collision\ avoidance} + \underbrace{\beta_{acc} L_{acc} + \beta_{dec} L_{dec}}_{leader\text{-}follower}$$

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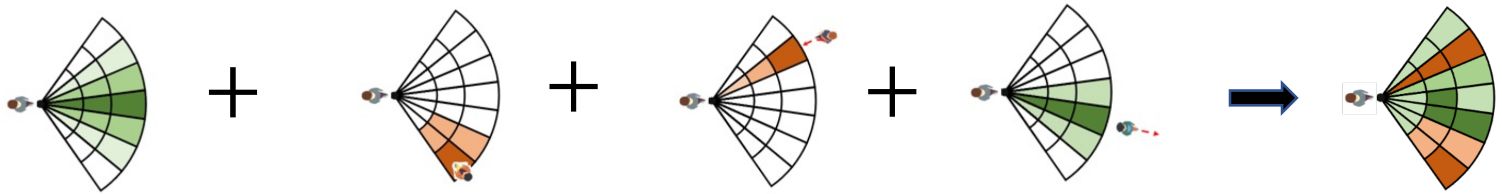
Replacing complex functions with Concept-NNs



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Details:

$$\begin{aligned} V_{vdn} = {} & \beta_{\text{dir\_central}} \text{dir}_{dn} I_{d,\text{central}} \\ & + \beta_{\text{dir\_side}} \text{dir}_{dn} I_{d,\text{side}} \\ & + \beta_{\text{dir\_extreme}} \text{dir}_{dn} I_{d,\text{extreme}} \\ & + \beta_{\text{dec}} I_{v,\text{dec}} (v_n / v_{\max})^{\lambda_{\text{dec}}} \\ & + \beta_{\text{accLS}} I_{n,\text{LS}} I_{v,\text{acc}} (v_n / v_{\text{maxLS}})^{\lambda_{\text{accLS}}} \\ & + \beta_{\text{accHS}} I_{n,\text{HS}} I_{v,\text{acc}} (v_n / v_{\max})^{\lambda_{\text{accHS}}} \end{aligned}$$

$$+ \beta_{\text{occ}} \sum_{k=1}^{N} I_{kd} e^{-\gamma_1 \| p_k - c_{vdn} \|}$$

$$+ I_{d,\text{C}} \alpha_{\text{C}} e^{\rho_{\text{C}} D_{\text{C}}} \Delta v_{\text{C}}^{\gamma_{\text{C}}} \Delta \theta_{\text{C}}^{\delta_{\text{C}}}$$

$$\begin{aligned} & + I_{v,\text{acc}} I^{\text{L}}_{d,\text{acc}} \alpha^{\text{L}}_{\text{acc}} D_{\text{L}}^{\rho^{\text{L}}_{\text{acc}}} \Delta v_{\text{L}}^{\gamma^{\text{L}}_{\text{acc}}} \Delta \theta_{\text{L}}^{\delta^{\text{L}}_{\text{acc}}} \\ & + I_{v,\text{dec}} I^{\text{L}}_{d,\text{dec}} \alpha^{\text{L}}_{\text{dec}} D_{\text{L}}^{\rho^{\text{L}}_{\text{dec}}} \Delta v_{\text{L}}^{\gamma^{\text{L}}_{\text{dec}}} \Delta \theta_{\text{L}}^{\delta^{\text{L}}_{\text{dec}}} \end{aligned}$$

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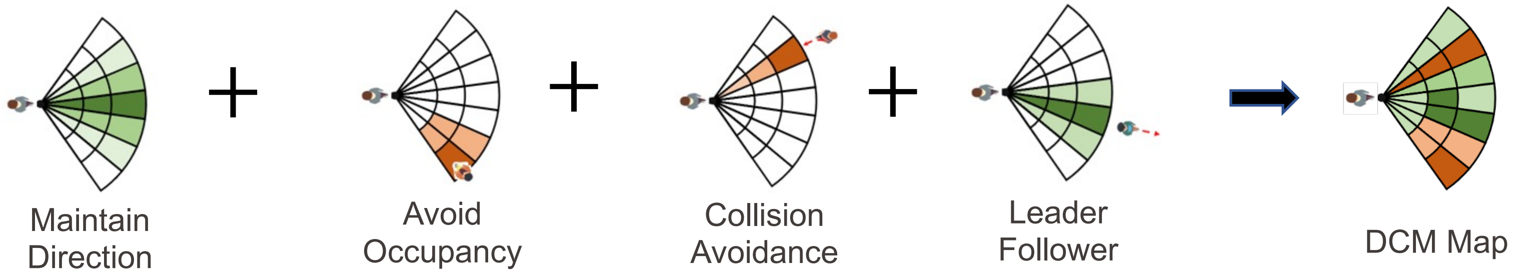

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Replace:

$$\rightarrow \quad u(\mathbf{X}) = c_0(\boldsymbol{z}^t_{dir}) + c_-(\boldsymbol{z}^t_{occ}) + c_-(\boldsymbol{z}^t_{col}) + c_+(\boldsymbol{z}^t_{lf})$$

$$\boldsymbol{z}^t_{dir} = \boldsymbol{V}^t_0 \qquad \boldsymbol{z}^t_{occ} = \mathcal{I}_{occ} \cdot \boldsymbol{X}^t \qquad \boldsymbol{z}^t_{col} = \mathcal{I}_{col} \cdot (\boldsymbol{X}^t, \boldsymbol{V}^t) \qquad \boldsymbol{z}^t_{lf} = \mathcal{I}_{lf} \cdot (\boldsymbol{X}^t, \boldsymbol{V}^t)$$

Replacing complex functions with Concept-NNs

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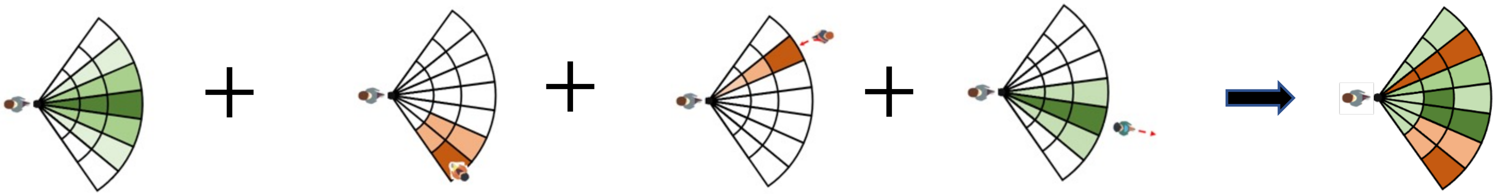

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Results – Rank Loss

Rank Loss:

Average position of the ground truth anchor in the anchor selection's ranking:

Best = 1 Worst = # anchors

Method	Rank Loss
dcm	1.95
$dcm + nn$	1.45
nn	1.45
$c_{dir} + c_{lf} + c_{col}$	1.85
$c_{dir} + c_{lf} + c_{col} + c_{occ}$	1.60
c_{full}	1.40
$c_{dir} + c_{lf} + c_{col} + c_{occ} + nn$	**1.35**

Previous

Current

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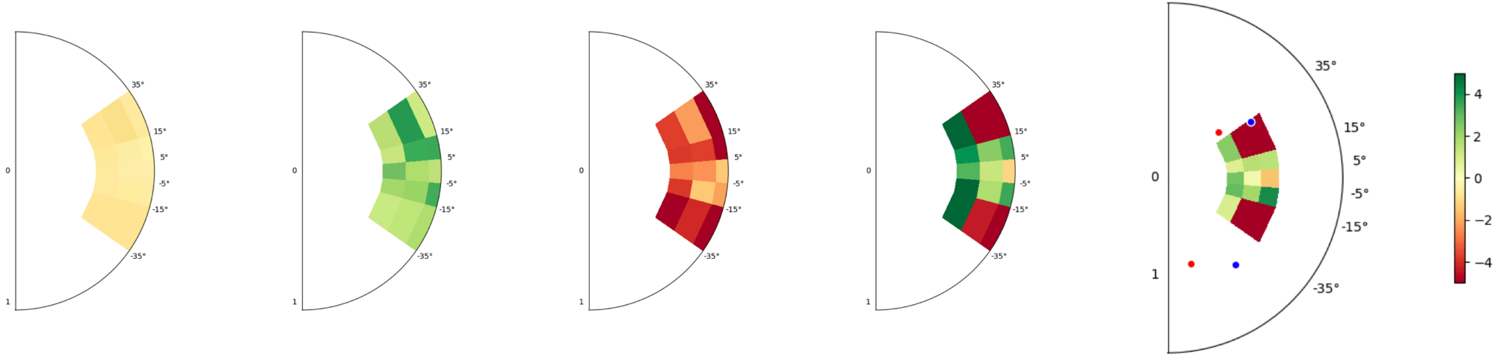
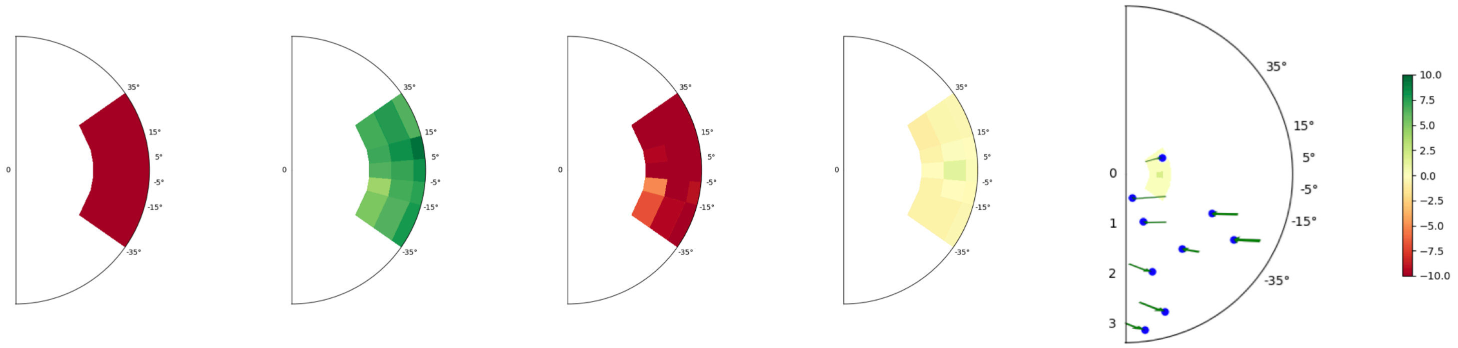


Collision Leader-Follower Occupancy Direction Final Map

- Discovering direction function is more complex
  => High speeds has effect on turns
- Leader-Follower and Occupancy are coherent

EPFL

Results



Collision | Leader-Follower | Occupancy | Direction | Ground Truth Map

- Still work to do due to design:
  - High number of pedestrians saturates some outputs
  - Disequilibrium with non-scaling functions (direction, past observations, etc…)

Pending Improvements

- Currently 2 - 4% improvement on trajectory metrics
- Re-scale based on # of neighbors
- Further constrain output (e.g. area of neighbor effect)
- Train a few epochs with DCM functions as Teacher/Regularizer
- Ablation study (replace 1 concept at a time)

Conclusion

- Theoretically:
  - A data driven term creates a “representation” of its inputs
  - Expert modeling can make them explainable

- Experimentally:
  - There **are** more complex functions to be found
  - Work in progress, but encouraging results
    - More constraints
    - Help first steps training with DCM

Thank you for your time!

Questions?