Traffic estimation by fusing static and moving observations in highway networks

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Traffic estimation by fusing static and moving observations in highway networks

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Abstract

Traffic monitoring and control constitute necessary steps in order to ensure the efficient function of transport networks. To that end, traffic state estimation and prediction are crucial tasks, normally relying on the (limited) deployment of sensor infrastructure. In recent years, apart from the traditional stationary sensors (i.e. loop detectors), new data collection alternatives have emerged at different levels of spatial andtemporal aggregation (e.g. Global Navigation Satellite System (GNSS) data, Bluetooth tracking, Car-floating data, etc.). The abundance of new data sources provides a unique opportunity to improve existing traffic monitoring strategies or develop new ones. A key role within this process is data fusion, the process of integrating multiple data sources in order to produce consistent and accurate state estimations. Furthermore, it is essential that the development of these techniques could be robust to data outages, among the various sources. In this regard, different data fusion techniques have been developed to allow an integration to take place. Transport networks are known for highly nonlinear behavior, posing a challenge and an opportunity with regard to the aspects above. This research deals with the issue of traffic state estimation for highway networks with limited, and potentially incomplete, measurement data from different sensor infrastructures. In the current framework, we suggest the use of Unscented Kalman Filter (UKF) which inherently incorporates the data fusion process in an algorithmic concept. In particular, we combine the second order traffic flow model METANET, with filtering methods (e.g. Unscented Kalman filter) properly modified to account for spatio-temporal correlations between the corresponding noise terms. Our results are compared against reference data to help us make decisive statements about the efficacy of the new methodology in tackling this problem.

Keywords

State estimation; feedback control; data-driven traffic assimilation; highway road networks; Unscented Kalman Filter; data fusion; Cell Transmission Model (CTM); METANET model.

1 Introduction

Traffic state estimation and forecasting systems have the potential to ameliorate traffic conditions and decrease travel delays by facilitating better utilization of available capacity. The need of an estimation engine is imperative, as a wide range of online traffic control loop applications face difficulties to produce confident estimates of the current state due to a variety of reasons, such as data dropouts, detector failures, availability of sensors. Traffic estimation is an important part of the online traffic feedback control loop. In (Treiber *et al.*, 2011), the authors mark the necessity of trajectory data as supplement to loop detector data. Thus, the properties of a jammed cluster can be characterized in a more robust fashion. Given a set of available real-time measurements, it is possible to estimate all traffic variables that are required by a controller as state feedback at the current time. The purpose of this research paper is to outline the existed state of the art methodologies on traffic estimation methods and to propose novel approaches that are able to improve and give insights to complement current literature.

Two well-known challenges are the lack of precision in online traffic estimation due to its high nonlinearity and lack of data sources; this paper addresses these shortcomings by exploiting the sophistication of Unscented Kalman Filter (UKF). In this work, UKF is used for fusing heterogeneous data (measurements model) coming from loop detectors (i.e. static) and vehicle trajectories (i.e. moving) on a real-world case in the ring road in Antwerp, which was calibrated based on empirically observed traffic counts, in addition to traffic dynamics stemming from macroscopic models METANET/CTM (process/transition model).

During the past years traffic state estimation drew an increasing academic attention, as it remains a challenging problem for the transportation community, not only due to the poor deployment of sensing infrastructure, but also to its daily impact on safety, environment, and economy. However, emerging data collection technologies such as Bluetooth, Global Navigate Satelite System (GNSS) data, and traditional loop detectors can contribute in that way. Therefore, data acquisition from various sensor sources and their fusion into forecasting models can have a quintessential importance for the optimal traffic state estimation. To provide traffic state estimates, there are several mathematical models, such as the Cell Transmission Model (CTM) (Daganzo, 1992) and METANET (Messner and Papageorgiou, 1990), which are the most widely used for highway traffic flow modelling. The proposed methodological framework is a combination of data fusion stemming from various sources, the utilization of the aforementioned traffic flow models coupled with estimation methods such as the Unscented Kalman Filter (UKF), the

Ensemble Kalman Filter (EnKF), Extended Kalman Filter (EKF), which are estimation methods for combining dynamic systems and traffic sensor measurements.

State estimation methods based on physical traffic models such as METANET, CTM have advantages over those based purely on statistical models according to (Wright and Horowitz, 2016). Their rigorous mathematical derivation allows them to estimate conditions at non-instrumented locations, as well as predict network's behaviour in space and time based on previously unobserved conditions. It is common to tackle the problems with filtering methods where estimates are propagated forward in time through traffic models and updated with information coming from measurements. Examples can be found in the works of Sun *et al.* (2004); Wang and Papageorgiou (2005); Work *et al.* (2010). In the study of Wright and Horowitz (2016) the authors have employed the macroscopic flow model CTM coupled with an estimation method based on Rao-Blackwellized particle filters, a sequential Monte Carlo scheme.

2 Literature Review

As early as 1956, the need for traffic state estimation had been identified as an important task by Schmidt and Campbell (1956). According to them, estimation is concerned with the determination of normal patterns in diverse places, their limits or ranges, and their correlations with the major causal factors and conditions of travel. The fundamental input of traffic state estimation is traffic sensing data. In the work of van Erp *et al.* (2018), loop detectors and data from probe vehicles and their fusion is discussed. Previous works were published on traffic state estimation employing static sensors (Wang and Papageorgiou, 2005; Hegyi *et al.*, 2006) and for moving GPS sensors (Work *et al.*, 2008; Herrera and Bayen, 2010). Furthermore, there are methods in literature for reconstructing traffic states by fusing heterogeneous data (Treiber *et al.*, 2011; Bachmann *et al.*, 2013).

In Treiber *et al.* (2011) the method used for fusing heterogeneous data is generalized adaptive smoothing (GASM). GASM can interpolate locally inconsistent data. The authors have concluded that GASM is effective, and capable of removing the noise and retaining the structure. In the work of Bachmann *et al.* (2013) a rigorous analysis about seven multi-sensor data fusion-based estimation techniques is provided and investigated. According to their results, there is an improvement over single sensor approaches, and also, they state that this improvement depends on the technique, number of probe vehicles, and traffic conditions. Moreover, the fusion of loop detectors generally outperforms the midpoint method independently from the fusion method used; however, loop detectors are found to perform poorly in congested conditions. Finally, the paper highlights that by adding data such as flow and density and using either solely neural networks, Kalman Filter, or the combination of two, has the potential to improve the overall accuracy. In Liu *et al.* (2018) the authors integrate traffic features extracted from wireless communication records and measurements from microwave sensors. To track the dynamic traffic conditions they employ a Progressive Extended Kalman Filter (PEKF).

Furthermore, in the paper of Van Lint and Hoogendoorn (2010) they employ the extended generalized Treiber-Helbing filter (EFTG) for fusing the data. They hve managed to develop a robust method for fusing heterogeneous data coming from different sources, such as induction loops or moving vehicles. The authors state that the developed method is generic and can be applied to any fixed or moving data source. According to their results, the filter produces an unbiased estimate of the ground-truth data even if 50% of the data is missing. In this work the authors assume that the filter could be used for real time state estimation even though they have not examine that case in their study.

In Trinh *et al.* (2019) UKF in combination with the first order CTM model is investigated. This work shows the potential of using UKF, however, further validation on a realistic network and dataset is missing. Hence, the combination of the fusion procedure and estimation properties of UKF with METANET traffic flow model can prove to add value to the developments of online traffic estimation research community. Therefore, in the current paper, we tackle the problem of data fusion and state estimation by employing Unscented Kalman filtering on the macroscopic traffic level. We employ bothMETANET and CTM as spate-space models of freeway traffic flow. While both models are nonlinear, CTM comprises a first-order traffic model and METANET a second-order. In the literature, there is a lot of discussion about the precision of these two approaches (but also higher order models), and thus, a comparison between these two different outputs is deemed interest. To deal with the nonlinearities of METANET we will employ the Unscented Kalman Filter (Wan and Van Der Merwe, 2000), which can address highly nonlinear dynamics and fuse data from multiple sources. Our data collection comes from a ring road motorway in Antwerp, Belgium. Trajectory data are extracted from a microsimulation scenario in Aimsun software, replicating a peak-hour demand. One challenge to overcome is, as stated in Wright and Horowitz (2016), to accurately match probe measurements to individual road segments as macroscopic models work with discrete spatial cells.

The study in Ampountolas and Kouvelas (2015) exploits Kalman filter methodology to estimate in real time the critical accumulation (or its derivative) in urban regions. Finally, recently, the authors in Kouvelas *et al.* (2017b); Saeedmanesh *et al.* (2019) have developed a systematic state estimation approach based on Extended Kalman Filter to be used for control purposes Kouvelas *et al.* (2017a, 2019) in urban multi-region traffic networks. Although the dimension of the problem (number of states) is considerably smaller than in a highway network, the theoretical framework as well as application domain are quite similar.

The remainder of the paper is organized as follows: The macroscopic freeway traffic flow models are presented in Section 3; in Section 4, we analyze UKF and data fusion and provide a workflow of the system design to be followed for traffic state estimation. Section 3 presents the case study network of Antwerp that will be used for testing our methodology. Finally, Section 5 provides a discussion on the assessment methodology to be followed, anticipated results, and future work plans.

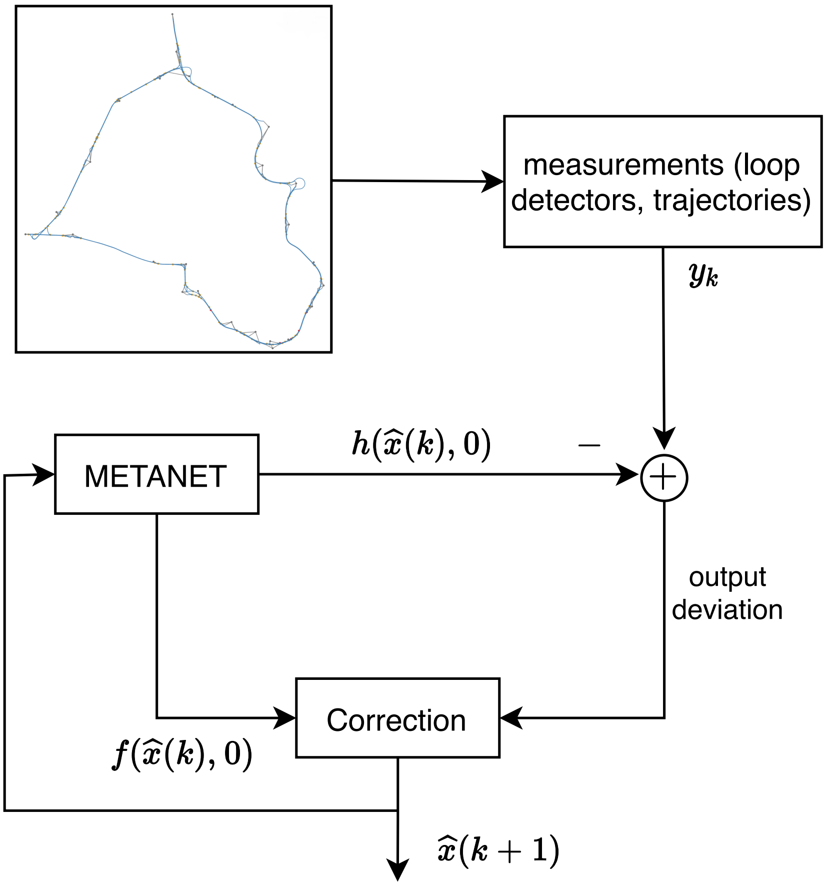
3 Methodology

The flow chart of the proposed methodological framework is illustrated in Figure 1. Two data sources are used as input for the proposed algorithm, GNSS trajectory data (moving or Langrangian observations) from microsimulation results and loop detector densities (moving or Eulerian observations) from the Antwerp network. One of the contributions of the current study is to highlight how spatial outages in the input data and noise levels impact the final state estimation. This procedure is planned to be formalized at a later state of this work. Furthermore, comparison between CTM and METANET could showcase the sensitivity and accuracy of each model to the conditions mentioned above. The remaining of this section presents the formulations of METANET model, UKF method, and describes the Aimsun case study and data sources and collection.

3.1 METANET model

METANET is a macroscopic second order model of freeway traffic flow that represents the dynamics of a freeway segment i with length Δ_i and number of lanes λ_i as follows

Figure 1: The flowchart of the proposed framework.



(Wang and Papageorgiou, 2005):

$$\rho_i(k) = \rho_i(k-1) + \frac{1}{\Delta_i \lambda_i}\Big(q_{i-1}(k-1) - q_i(k-1) + r_i(k-1) - s_i(k-1)\Big) \tag{1}$$

$$s_i(k) = \beta_i(k) q_{i-1}(k) \tag{2}$$

$$\begin{aligned} v_i(k) = v_i(k-1) + \frac{T}{\tau}\Big(V(\rho_i(k-1)) - v_i(k-1)\Big) - \frac{\delta}{\Delta_i \lambda_i} \frac{r_i(k-1) v_i(k-1)}{\rho_i(k-1) + k} \\ + \frac{T}{\Delta_i} v_i(k-1)\Big(v_{i-1}(k-1) - v_i(k-1)\Big) - \frac{v}{\tau \Delta_i} \frac{\rho_{i+1}(k-1) - \rho_i(k-1)}{\rho_i(k-1) + k} \end{aligned} \tag{3}$$

$$V(\rho_i(k)) = v_{f,i} \exp\left(- \frac{1}{a_i} \left(\frac{\rho_i(k)}{\rho_{\text{cr},i}} \right)^{a_i} \right) \tag{4}$$

$$q_i(k) = \rho_i(k) v_i(k) \lambda_i \tag{5}$$

where $k = 1, 2, \ldots, K$ is the discrete time index and $\rho_i(k)$, $q_i(k)$ and $v_i(k)$ denote the traffic density, flow, and space mean speed in segment i, respectively; τ, v, k, and δ are user-defined model parameters. Based on equation (1), i.e. conservation equation, the difference between total amount of input flows (i.e. $q_{i-1}(k)$ and on-ramp inflow $r_i(k)$) and output flows (i.e. $q_i(k)$ and off-ramp outflow $s_i(k)$), at time k in segment i, equals to the change in the amount of density $\rho_i(k)$. The exiting rate $\beta_i(k)$ demonstrates the ratio of $s_i(k)$ to $q_{i-1}(k)$. Equation (3), the dynamic speed equation, is composed by different terms. The first term is the relaxation term that demonstrates the tendency of vehicles to achieve the desired speed (stationary speed $V(\rho_i(k))$). The third and fourth terms model the impact of spatial heterogeneity. The third term is the so-called convection term, which expresses the effect of inflow, and the fourth is the so-called anticipation term that demonstrates the effect of upcoming change in density. The relationship between stationary speed and traffic flow according to the fundamental diagram is demonstrated in equation (4), where $v_{f,i}$ and $\rho_{\mathrm{cr},i}$ denote the free flow speed and critical density, respectively.

4 Unscented Kalman Filter

4.1 UKF algorithm

One of the very first techniques used for nonlinear problems, which still remains the most common technique is the Extended Kalman Filter (EKF). To handle nonlinear problems, EKF linearizes the system at the current estimate point, and then the linear Kalman filter is used to provide estimates (based on Jacobian matrices). However, in EKF, the state distribution is approximated by a Gaussian Random Variable (GRV), which can introduce large errors in the true posterior mean and covariance, and may lead to divergence of the filter. This problem is alleviated with Unscented Kalman Filter, which does not add significant computational cost and is utilized in this work. The *unscented transform* calculates the statistics of some random variables, which undergoes a nonlinear transformation. Although UKF a deterministic approach, similarly to EKF, UKF sampling approach is represented by the so-called sigma points, which are chosen carefully to keep computational complexity at the same order as EKF.

The basic principle of *unscented transform* is that we draw some samples from a Gaussian distribution and transform them through a nonlinear function $\mathbf{g}(\cdot)$. Then the mean and covariance of the transformed points are computed. In other words, UKF tries to have the best Gaussian approximation of the transformed points. In order to decrease the number of needed samples to approximate the transformed points, we employ a special approach in the sampling method of the initial Gaussian distribution.

Unscented Kalman Filter consists of two steps, predict and update. In step predict it computes the *prior estimate* using the process model $\mathbf{f}(\cdot)$. After the computation of sigma points $\mathcal{X}$ and their respective weights W^m, W^c these points are then propagated through time and the new prior, the transformed sigma points are estimated:

$$\boldsymbol{\mathcal{Y}} = \mathbf{f}(\boldsymbol{\chi}, \Delta t)$$

Using the unscented transform (UT) the mean and covariance of the prior $\mathbf{x}$ are computed, from the new transformed sigma points:

$$\bar{\mathbf{x}}, \bar{\mathbf{P}} = \mathrm{UT}(\mathcal{Y}, w_m, w_c, \mathbf{Q}) \tag{6}$$

$$\bar{\mathbf{x}} = \sum_{d=0}^{2n} w_i^m \boldsymbol{\mathcal{Y}}_d \tag{7}$$

$$\bar{\mathbf{P}} = \sum_{d=0}^{2n} w_d^c (\boldsymbol{\mathcal{Y}}_d - \bar{\mathbf{x}})(\boldsymbol{\mathcal{Y}}_d - \bar{\mathbf{x}})^\mathsf{T} + \mathbf{Q} \tag{8}$$

where n are the dimensions of the state input. To perform the update step, the sigma points of the prior must be converted to the measurement space through the function: $\boldsymbol{\mathcal{Z}} = h(\boldsymbol{\mathcal{Y}})$. Then the weighted mean and covariance of the sigma points of the measurement space is computed:

$$\boldsymbol{\mu}_z, \mathbf{P}_z = UT(\boldsymbol{\mathcal{Z}}, w_m, w_c, \mathbf{R}) \tag{9}$$

$$\boldsymbol{\mu}_z = \sum_{d=0}^{2n} w_d^m \boldsymbol{\mathcal{Z}}_d \tag{10}$$

$$\mathbf{P}_z = \sum_{d=0}^{2n} w_d^c (\boldsymbol{\mathcal{Z}}_d - \boldsymbol{\mu}_z)(\boldsymbol{\mathcal{Z}}_d - \boldsymbol{\mu}_z)^\mathsf{T} + \mathbf{R} \tag{11}$$

Residual is next to be computed:

$$\mathbf{y} = \mathbf{z} - \boldsymbol{\mu}_z \tag{12}$$

where $\mathbf{z}$ is the measurement and $\boldsymbol{\mu}_z$ is the mean of the measurements. Then, in order to compute the Kalman gain, cross covariance between measurements and the state has to be calculated:

$$\mathbf{P}_{xz} = \sum_{d=0}^{2n} w_d^c (\boldsymbol{\mathcal{Y}}_d - \bar{\mathbf{x}})(\boldsymbol{\mathcal{Z}}_d - \boldsymbol{\mu}_z)^\mathsf{T} \tag{13}$$

Lastly, the Kalman gain can be computed as follows:

$$\mathbf{K} = \mathbf{P}_{xz}\mathbf{P}_z^{-1} \tag{14}$$

where $\mathbf{P}_{xz}$ denotes the uncertainty between the state and measurements and $\mathbf{P}_z^{-1}$ the uncertainty for the measurement. The last step of the update step is to compute the new state estimate and the new covariance:

$$\mathbf{x} = \bar{\mathbf{x}} + \mathbf{K}\mathbf{y} \tag{15}$$

$$\mathbf{P} = \bar{\mathbf{P}} - \mathbf{K}\mathbf{P}_\mathbf{z}\mathbf{K}^\mathsf{T} \tag{16}$$

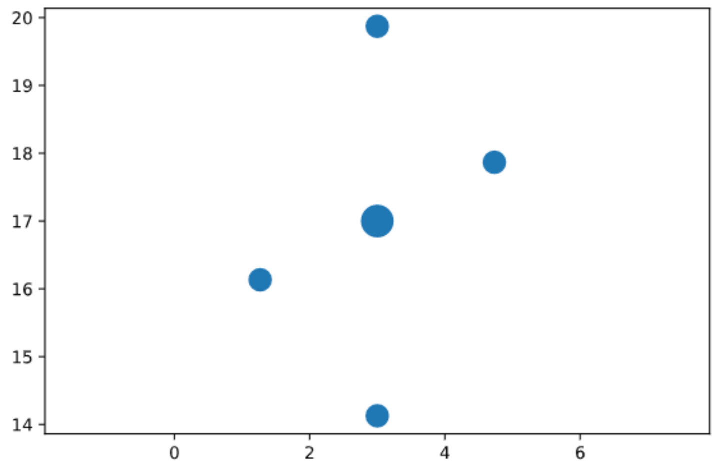
To sum up, in the predict step we calculate the weighted sigma points with which we will then calculated the transformed sigma points. After that, we will have the new mean and covariance of the process model plus the noise coming from the process model.

In the update step, we use the sigma points we calculated earlier and transform them through the measurement space. We acquire the new mean and covariance in measurement space plus the noise of the measurement function. To calculate the Kalman gain we need to calculate the cross-correlation between sigma points in state space and sigma points in the measurement space. Having calculated the Kalman gain we can predict the final state estimate and the uncertainty of the estimate.

4.2 Computation of sigma points

For computing effectively the sigma points we compute the mean and covariance of the Gaussian distribution and generate weighted sigma points. Figure 2 provides an example

Figure 2: How sigma points are distributed around $\mu = (3, 17)$ and $\Sigma = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 3 \end{bmatrix}$ according to Julier technique.



of how sigma points are distributed around the given mean μ and covariance Σ for $2D$ space. The requirements for this specific selection of sigma points are given through the following equations:

$$\sum_{d}^{2n} w_d^m = 1, \qquad \sum_{d}^{2n} w_d^c = 1 \tag{17}$$

$$\mu = \sum_{d}^{2n} w_d^m f(\mathcal{X}_d) \tag{18}$$

$$\Sigma = \sum_{d}^{2n} w_d^c (f(\mathcal{X})_d - \mu)(f(\mathcal{X})_d - \mu)^\mathsf{T} \tag{19}$$

Equation (17) provides the constraint for the weights of mean and covariance, respectively; equations (18) and (19) compute the weighted mean and covariance, respectively, for the two variables. There is not the only solution to find the weights and sigma points. The weights play an important role in UKF, as by varying them each point can incorporate some of the knowledge about the distribution, trying to minimize the effect that process model nonlinearities have to the final estimate.

The computation of sigma points can be done as described in Van Der Merwe *et al.*

(2004). To control the distribution of sigma points the formula has 3 parameters α, β and κ. Parameter α controls the spread of the points around the mean and weights them accordingly. Below are the formulas to compute the sigma points:

$$\mathcal{X}_0 = \mu \tag{20}$$

$$\boldsymbol{\mathcal{X}}_d = \begin{cases} \mu + \left[\sqrt{(n+\lambda)\Sigma}\right]_d & \text{for d=1 .. n} \\ \mu - \left[\sqrt{(n+\lambda)\Sigma}\right]_{d-n} & \text{for d=(n+1) .. 2n} \end{cases} \tag{21}$$

$\mathcal{X}_0$ is the first sigma point and is the mean of the input; the rest sigma points are computed by equation (21); $\lambda = \alpha^2(n+\kappa) - n$, where n is the dimension of input state $\mathbf{x}$.

4.2.1 Weight Computation

The formulas for the weights are as follows:

$$W_0^m = \frac{\lambda}{n+\lambda} \tag{22}$$

$$W_0^c = \frac{\lambda}{n+\lambda} + 1 - \alpha^2 + \gamma \tag{23}$$

$$W_d^m = W_d^c = \frac{1}{2(n+\lambda)} \quad d = 1..2n \tag{24}$$

where equation (22) is the weight for the mean of $\mathcal{X}_0$; equation (23) is the weight for the covariance of $\mathcal{X}_0$. The weights for the rest sigma points are the same for the mean and the covariance and are computed with (24).

5 Case study

As case study to test our framework we uilize the ring road highway around the city of Antwerp, in Belgium, a network consisting of 120km of roads. A map of the ring road is illustrated in Figure 3. The microsimulation model of this network has been designed and acquired by Mattas *et al.* (2018). To adjust appropriately the ring road for METANET, the network needs to be partitioned into consecutive segments/cells each of 500 $\pm$ 200 meters length, with at most one on-ramp and one off-ramp. With these restrictions in mind, we partition the network of Antwerp accordingly. The freeway network has been

Figure 3: The Antwerp ring road.

loaded and calibrated in Aimsun microsimulation software.The base scenario demand utilizes traffic counts from morning peak hours. The Origin-Destination (OD) matrix has been derived within Aimsun software from static OD adjustment by using Frank and Wolfe algorithm.

6 Discussion and Future Work

In this study, we study the problem of real-time traffic state estimation on freeway networks. Concluding from current bibliography, there are still reservations regarding the sensitivity of the existing traffic models on noise and insufficient data. Often than not, data gaps, missing information or data corruption is the case in most studies. Nowadays, the exposure to new sort of data (loop detectors, GPS data, BT data) offers great insights for traffic state estimation. According to previous studies, using data fusion from various sources has a positive effect on estimating traffic states. In addition to data coming from loop detectors we take advantage of UKF sophistication and fuse data coming from vehicle trajectories.

In this study, we work on data fusion from heterogeneous sources which may not necessarily have the same frequency or accuracy. In our case, Antwerp network in AIMSUN is calibrated based on real counts of light and heavy duty vehicles from an OD matrix. Microsimulation outputs are used as reference dataset. Then, the performance of our developed filter is compared to traffic conditions with varying number of vehicles and sampling sensor frequencies. We intend also to evaluate the performance of our approach using indicators such as the Root Mean Square Error (RMSE) for density ρ and speed v_i for each cell coming from UKF versus the reference data, which are generated by AIMSUN's microsimulation model. Furthermore, we will compare the predicted state from UKF with the state generated by AIMSUN every k period time. Lastly, to evaluate the overall performance of the proposed filter we will compare real counts with estimated counts per segment over peak hours.

To contemplate for the system nonlinearities the use of UKF has proven to be robust. The use of other algorithms which deal efficiently with nonlinear systems are known (e.g. EKF, PF). A comparison between these algorithms can prove the efficiency of our initial argument. For each different approach, we can compare their responses in the variable noises to be tested against them and the different percent of insufficient data to be provided as input.

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