Dynamic congestion pricing for multi-region networks: A traffic equilibria approach

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Abstract

The growing number of people living in cities results in rising mobility demand, and as a consequence, the limited capacity of traffic networks gets more stressed. Hence, congested network links are causing travel delays and negative impacts on the environment, postulating for a methodology to overcome this challenge. Considering the broad range of traffic management systems, congestion pricing is a very effective tool to tackle today's cities traffic problems. Different strategies are available in literature or even applied in real-world that show a positive effect on the traffic situation.
This paper proposes a framework design that allows the testing of pricing policies and to evaluate their performance in alleviating congestion. The study implements a multi-region urban network, where the urban regions are considered as homogeneous and replicated with a representative Macroscopic Fundamental Diagram (MFD). To assess the impact that different pricing policies may have on traffic behavior, a route choice algorithm is utilized and a concept for the computation of the dynamic user equilibrium, as well as the system optimum, are proposed. A case study is presented, where the modeling approach is applied to the heterogeneous road traffic network of the city of Zurich, Switzerland.

Keywords

Dynamic congestion pricing; multi-region urban modeling; Macroscopic Fundamental Diagram (MFD); rolling-horizon optimization

Introduction

The fact that more and more people are living in cities puts significant pressure on the mobility services of urban regions. One major challenge of today's transportation systems is the mitigation of congestion. To overcome this challenge, many traffic management approaches have been proposed in the past. One well-known system is perimeter control, allowing to reduce the user delay in a protected region significantly by controlling traffic lights at the region border (Geroliminis *et al.* (2013); Keyvan-Ekbatania *et al.* (2012)). To maintain the system at an optimal point the properties of the Macroscopic Fundamental Diagram (MFD) are used, showing that operating at the critical density, allows serving the corresponding maximal traffic volume. Nevertheless, this approach is not considering external effects that are especially present when focusing on car traffic. Air pollution, noise, accidents, congestion, and space occupation are examples of costs that the road user is not charged for. Hence, this results in negative effects on the performance of a traffic system, the environment and the economy (Hansen (2018)).
To reduce the user delay but also ensure the internalization of external effects, congestion pricing is a well-known approach, where users are charged for using the road network. This can either be implemented by HOT (High-Occupancy Toll)-lanes, where a vehicle has to be certainly occupied or the road user is willing to pay for the lane usage. Secondly, cordon-based congestion pricing approaches are available that charge the user for entering a protected region. Both methodologies are leading to a reduction in the Total Time Traveled (TTT) and recent research shows that the method is beneficial for mitigating traffic congestion. Nevertheless, it is challenging to compare different pricing policies and provide a guideline for recommended charges and potential system improvements.
In the proposed work, we focus on a multi-region-network based on the work from Sirmatel and Geroliminis (2018). The defined regions are considered as homogeneous with different characteristics (size, capacity, average trip length) in the heterogeneous traffic network of the City of Zurich. Every region is defined by a well-defined MFD with a novel method from Ambühl *et al.* (2018). By using a route guidance algorithm that considers the splitting rates of users to different route possibilities, the impact of different pricing policies on the traffic behavior, as well as on the route choice can be determined. The performance of this novel model is shown by the computation of the Dynamic User Equilibrium (DUE) and the Dynamic System Optimum (DSO). By introducing different pricing policies we can determine and compare the improvements towards the DSO. To provide a baseline scenario for the testing of congestion pricing strategies, demand patterns are found by solving an optimization problem. A case study is presented that applies the methodology to a modeling scenario of the city of Zurich.
The remainder of this paper is organized as follows: Section 1 introduces the concept of congestion pricing and the current state-of-the-art research. The methodology, i.e. the modeling

of the simulation plant, the MFDs, and the determination of the demand patterns are described in Section 2. The case study is presented in Section 3 with determined results and an outline of the determination of the DUE and DSO. The paper closes with a conclusion, as well as future research ideas, in Section 4.

1 Background and Motivation

With an increasing mobility demand over the last centuries, traffic congestion rose and with that the necessity of trying to eliminate this problem. The past has shown that traffic planners have tried to tackle this problem with the extension of the existing network. Nevertheless, it has been proven that the Vehicle Kilometers Traveled (VKT) and the lane kilometers of a network increase proportionality. Logically, such investments are not solving congestion problems. In addition, individual transportation induces several negative effects on the social, economic and environmental system that are also known as external effects. Users are not charged for these impacts and therefore the efficient consumption level of transportation will always be lower than the actual one, resulting in external effects, congestion, and consequently, time loss (Eliasson (2017)).

Congestion pricing is a traffic management approach to address the problem with the internalization of the external effects by charging users for the usage of road infrastructure. This can be either implemented with HOV-lanes or with the definition of a protected area that has gates to enter, where a tool needs to be paid. By applying this concept, a road user is charged for the time loss that is caused to others. The approaches for charging road users differ in terms of (a) the pricing infrastructure (HOV, protected area, etc.), (b) the pricing policy (time ranges of charging, exceptions for residents, etc.), and (c) the methodology used for cost determination (time-based, distance-based, joint-charging approaches, etc.). Consequently, it can be stated that the city structure is from great importance, when designing congestion pricing. This is supported by study from Börjesson (2018) that investigates in the performance of congestion pricing in Sweden. Eliasson (2017) lists several examples of systems in operation that support the variety of approaches. Since 2016, vehicles entering the city center of London (UK) need to pay a fixed price, regardless the traffic situation. Stockholm (Sweden) is varying the price, dependent on peak-hours and off-peak-hour entrances. In addition, entering the center during the night and on weekends is free of charge. Gothenburg (Sweden) is charging users during any one-hour period with a varied price that is dependent on the time of day.

The three approaches share the property, that prices are not continuously evaluated or even changing. This is different in Singapore, where users are charged per trip but the prices are revised four times a year based on the deviation of speed measurements from set targets.

In the last years, several research towards a more smarter and dynamic congestion pricing was accomplished. A pricing policy where the price is changed based on traffic density measurements and road popularity is proposed by Soylemezgiller *et al.* (2013). Nevertheless, this approach requires RFID-based toll booths at every junction in an urban region. Kachroo *et al.* (2017) shows an optimal control law with the Hamilton Jacobi bellman equation. Nevertheless, both approaches are independent from aggregated traffic measurements (e.g. an MFD) and Kachroo *et al.* (2017) is proposing the final toll calculation with three static parameters that need to be adjusted by the operator. To bridge the concept of MFDs and the derivation of the optimal toll, Zheng *et al.* (2012) shows a framework that tries to maintain a network at the optimal traffic density and also to cover route choice with an agent-based simulation model. This work was further extended with a time-dependent pricing scheme in Zheng *et al.* (2016). A comparison of distance-, time-, and delay-tolls are shown in Gu *et al.* (2018). Furthermore, the study proposes two new concepts named the Joint Distance and Time Toll (JDTT), and the Joint Distance and Delay Toll (JDDT), respectively.
Because of the high computational complexity, the presented research is mostly based on static traffic assignment methods. The concept of Dynamic Traffic Assignment (DTA) is utilized for predicting travel times more accurate than static traffic assignment methods. Ekström *et al.* (2016) is introducing a surrogate-based optimization approach for the computation of the DUE and DSO. We will also aim for the computation of these equilibria to evaluate the performance of congestion pricing (an outline is given in Section 3.4).

2 Methodology

2.1 Macroscopic Fundamental Diagram modeling

Previous shown work, such as Sirmatel and Geroliminis (2018), are using mathematical relationships for modeling an MFD that is represented as an polynomial of the degree n (e.g. in Sirmatel and Geroliminis (2018) the approximation takes the form of $G(n(t)) = (an^3 + bn^2 + cn)/\bar{L}$, where the coefficients a, b, c are derived from measurement data). Furthermore, other approximations such as an exponential or multi-regime linear function are used. However, the function parameters lack physical meaning and might introduce problems with the application of optimization procedures. Instead of assuming a functional relationship, another approach is to estimate an MFD from measurement data. Nevertheless, the quality of data or difficulties in data acquisition might not lead to reasonable approximations (Ambühl *et al.* (2018)). For the modeling of the function $G(\cdot)$ the novel procedure developed by Ambühl *et al.* (2018) is used. The work presents an approximation of the trapezoidal diagram with the properties of smoothness, concavity, and continuity defined by:

$$q(k) = -\lambda \ln\Big(\exp\Big(-\frac{ak}{\lambda}\Big) + \exp\Big(-\frac{q_{out}}{\lambda}\Big) + \exp\Big(-\frac{(\kappa - k)b}{\lambda}\Big)\Big). \tag{1}$$

The function $q(\cdot)$ is the estimated outflow (veh/s) with respect to the input density k (veh/m).q_{out} is considered as the maximal outflow (capacity) in (veh/s), κ the jam-density in (veh/m), a and b as the slope of the free-flow and congestion regime, respectively, and λ as the smoothing parameter.

2.2 Urban multi-region modeling

A multi-region city network partitioned into homogeneous regions $\mathcal{R}$ is introduced, defined by $\mathcal{R} = \{1, 2, \ldots, K\}$, where K is the number of regions. Every region R_i, where $i = \{1, 2, \ldots, R\}$ is the index of the region, is modeled with a well-defined MFD, represented by the function $G(N_I(t))$, where $N_I(t)$ is the accumulation of a region R_i at time step t. Consequently, the dynamics equations can be defined as follows:

$$\frac{dN_{II}(t)}{dt} = Q_{II}(t) - M_{II}(t) + \sum_{\mathcal{H} \in \mathcal{N}_i} M_{HII}(t), \tag{2}$$

$$\frac{dN_{IJ}(t)}{dt} = Q_{IJ}(t) - \sum_{\mathcal{H} \in \mathcal{N}_I} M_{IHJ}(t) + \sum_{\mathcal{H} \in \mathcal{N}_I; \mathcal{H} \neq \mathcal{J}} M_{HIJ}(t), \tag{3}$$

where the indices $I \in \mathcal{R}$, $H \in \mathcal{N}_{\mathcal{I}}$ and $J \in \mathcal{R}$ are representing the origin region, the stop-over region, and the destination region, respectively. $\mathcal{N}_I$ is containing all regions that are neighbors of I. The internal demand within one region is defined by $Q_{II}(t)$. Demands with the origin I and destination J are introduced by $Q_{IJ}(t)$. Flows are stated by the functions $M_{II}(t)$ and $M_{IHJ}(t)$ representing the internal flows in a region and the transfer flows from region I via H to J, respectively defined as follows:

$$M_{II}(t) = \frac{N_{II}(t)}{N_I(t)} G(N_I(t)), \tag{4}$$

$$M_{IHJ}(t) = \theta_{IHJ}(t) \frac{N_{IJ}(t)}{N_I(t)} G(N_I(t)). \tag{5}$$

The functions $N_{II}(t)$ and $N_{IJ}(t)$ are introducing the accumulation from region I to I and J, respectively. The function $\theta_{IHJ}(t)$ represents the route choice at t, where for the computation an implementation of a k-shortest path algorithm is used. The sequence of regions a user can choose in the proposed model is not arbitrary. If the indices IHJ are parametrized with $I = J$, paths are restricted (e.g. $IHJ = 131$). This assumption denies unrealistic path choices and improves the quality of the model. For regulating the traffic with pricing, $\theta_{IHJ}(t)$ is influenced by a proposed function that is dependent on the application of specified pricing policy and the given accumulation at the time step t. Please note that the transfer flows need to be restricted by (6) stating that the minimum of the incoming transfer flow or the maximal capacity of the region is considered, providing a network region from accepting incoming flows that are exceeding the capacity limit. The latter is modeled with the function $C_{IHJ}(N_H(t))$.

$$M_{\tilde{IHJ}}(t) = \min\left(C_{IHJ}(N_H(t)), \theta_{IHJ}(t) \frac{N_{IJ}(t)}{N_I(t)} G(N_I(t))\right). \tag{6}$$

2.3 Demand pattern determination

The simulation plant for the multi-region modeling is designed to receive the demand patterns as trapezoids. A trapezoid is defined as a symmetric shape by specifying the rising time $t_{q,r}$ (s), the falling time $t_{q,f}$ (s) (where $t_{q,r} = t_{q,f}$), the time the demand remains constant $t_{q,c}$ (s), and the demand magnitude Q_t in (veh/sec). Often these parameters are found by generating random numbers that satisfy the given application. In this work, an optimization procedure from Kosmatopoulos and Kouvelas (2009) is utilized to find the parameters $t_{q,r}$, $t_{q,f}$, $t_{q,c}$, and Q_t, producing a desired simulation scenario (e.g two regions with and two regions without congestion). By setting a target on the MFD per region, different scenarios for testing congestion pricing strategies can be found efficiently.

3 Case Study

This section presents a case study, where the modeling is based on an example of the city of Zurich. First, the MFD design and the corresponding parameters are introduced. In Subsection 3.2, the derived demand patterns are shown, followed by the simulation output in Subsection 3.3.

3.1 MFD Design

We base our modeling on a region design of the city of Zurich. The regions are derived from analyzing the traffic main arteries of Zurich, the geographical reference of the available Loop Detectors (LD), and with respect to providing a good baseline for congestion pricing scenarios (Figure 1).

The city center (R_1) is considered with an area of 1.5 (km^2) and holds 245 available LDs. Consequently, the parameters for the MFD design are assumed with reasonable values as follows: Jam accumulation $N_{1,jam}$ = 5000 (veh), average trip length $\bar{L_1}$ = 1000 (m), and a network length of 30 lane kilometers. $R_2 - R_4$ are designed as the border regions of the city center with an area of 5.0 (km^2), each. The number of detectors for region R_2, R_3, and R_4 are 157, 211, and 198, respectively. The MFD for the border regions is designed with $N_{2,3,4,jam}$ = 8000 (veh), $\bar{L_{2,3,4}}$ = 2000 (m) and a network length of 48 lane kilometers, respectively. Hence, the network is designed for a storage capacity of 29000 vehicles. Note that in Figure 1 the connection between region R_2 and R_4 is highlighted separately. Considering the model shown in Figure 2, we are proposing a four region network, where the region $R_{i=1}$ is representing the city center. The input parameters to determine the presented MFDs in Figure 2 with (1) are listed in Table 1. Note that the parameters a and b are the slopes of the free-flow and the congested regime of the outflow MFDs, and do not have a physical meaning.

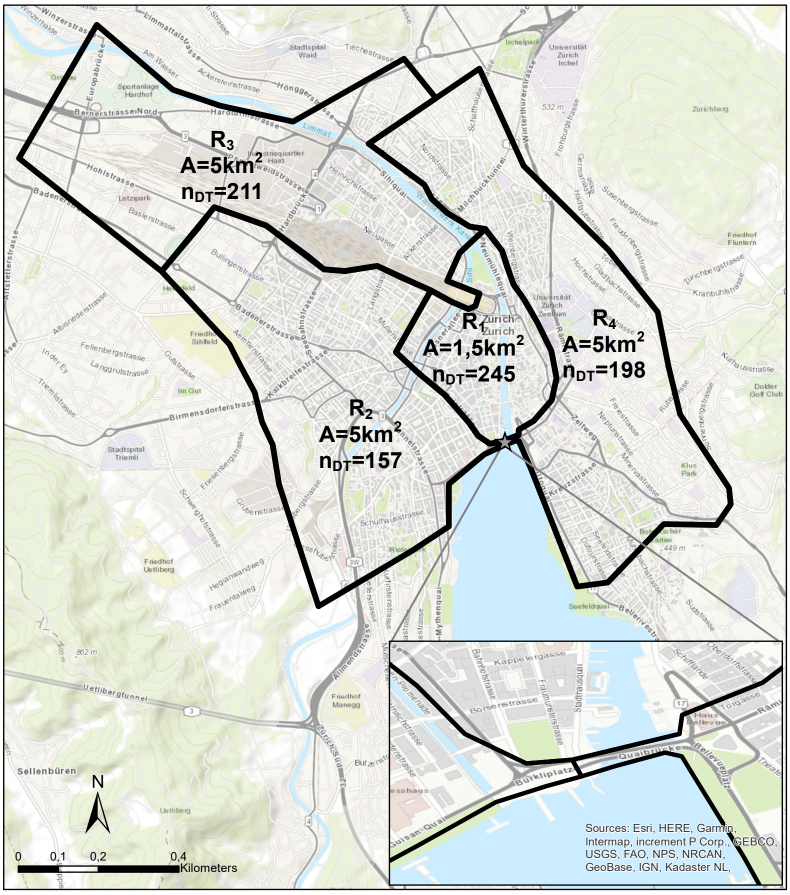
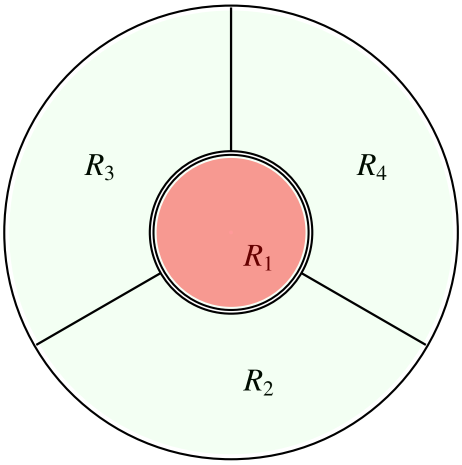


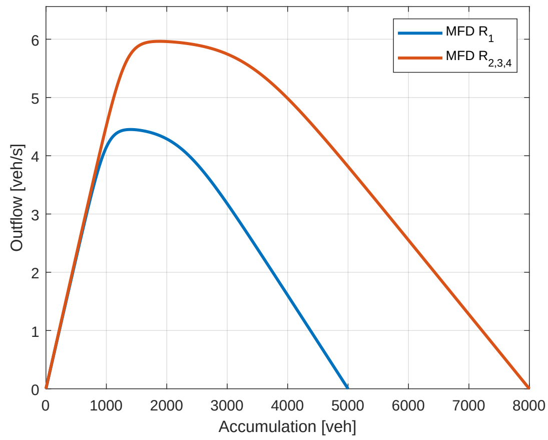
Figure 1: Region design of the city of Zurich. Every region is stated with an id R_i, the area A and the number of available LDs n_{DT}.

Table 1: Input parameters for MFD approximation.

Parameter	Unit	City Center R_1	Border Regions R_2 - R_4
a	[m/s]	135.00	219.38
q_{out}	[veh/s]	4.50	6.00
κ	[veh/m]	0.16	0.16
b	[m/s]	48.21	61.28
λ	[-]	0.50	0.60



(a) Region model



(b) MFD designs.

Figure 2: The multi-region-network model and the corresponding MFD designs. a) The region R_1 is modeled as the city center and treated as a protected region with pricing (indicated by the double lines), $R_2 - R_4$ are representing the boundaries to the center. b) The MFDs are designed according to assumptions related to the City of Zurich (region size, partitioning, etc.) and one can note that $R_2 - R_4$ are modeled as larger regions with greater capacity.

3.2 Demand determination

To provide a relevant peak-hour simulation scenario for congestion pricing, representative demand patterns need to be derived. To utilize the optimization procedure used, for every region a target value is defined (Figure 3).

R_1 and R_2 are representing a traffic situation in the congested regime, whereas R_3 is close to the optimal traffic density and R_4 operates in the non-congested regime. The derived demand patterns are depicted in Figure 4.

It is depicted that the demands from R_2 to R_1, R_3, and R_4 have the highest magnitudes, followed by the demand of the city center R_1 to the other regions. The lowest magnitudes are shown in R_3.

3.3 Simulation output

The found demand patterns are utilized as a simulation input for the baseline scenario. The configuration of the simulation plant is defined with a time step t of 20 (s) and the simulation length of 595 time steps.

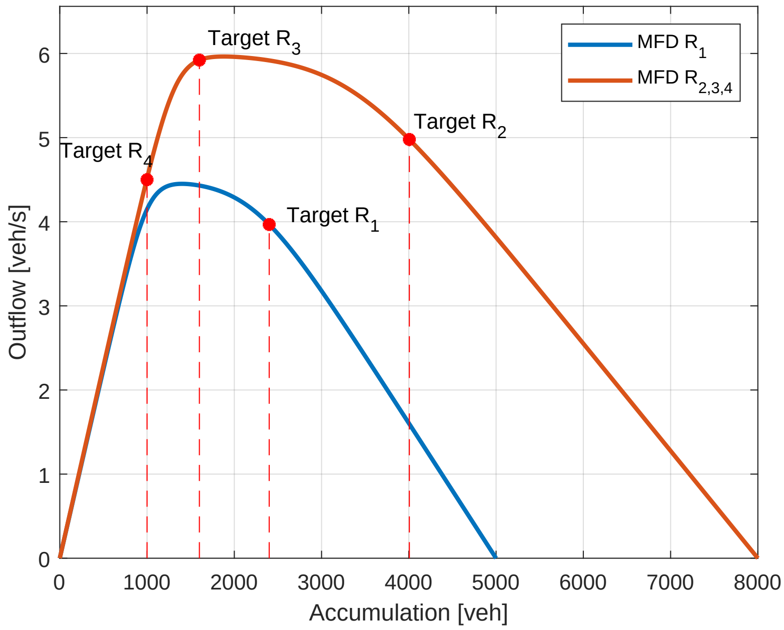


Figure 3: Target accumulation for demand determination for R_1 - R_4.

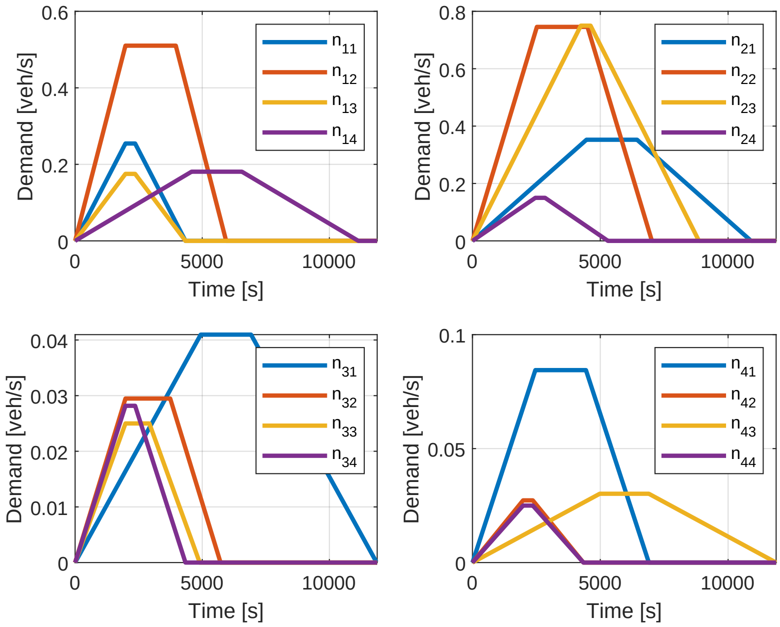


Figure 4: Traffic demand per region and pre-defined simulation horizon; configuration is for a 4X4 OD matrix, where i specifies to the origin and j the destination.

Figure 5 demonstrates the MFD functions $G(\cdot)$ and the simulation output data resulting from the demand scenario. The data points (in red) are representing the output, where every sample represent the relationship between the outflow (veh/s) and the accumulation (veh) for every t. It can be shown that the defined targets from Figure 3 are accurately representing the desired MFD regime.

Consequently, the accumulations outputs can be shown from origin i to destination j, as well as

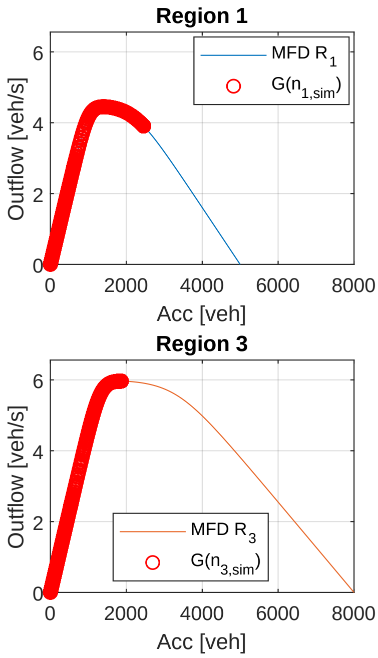


Figure 5: MFD functions with the corresponding simulation output, representing the relationship between outflow (veh/s) and the accumulation (veh) for every region.

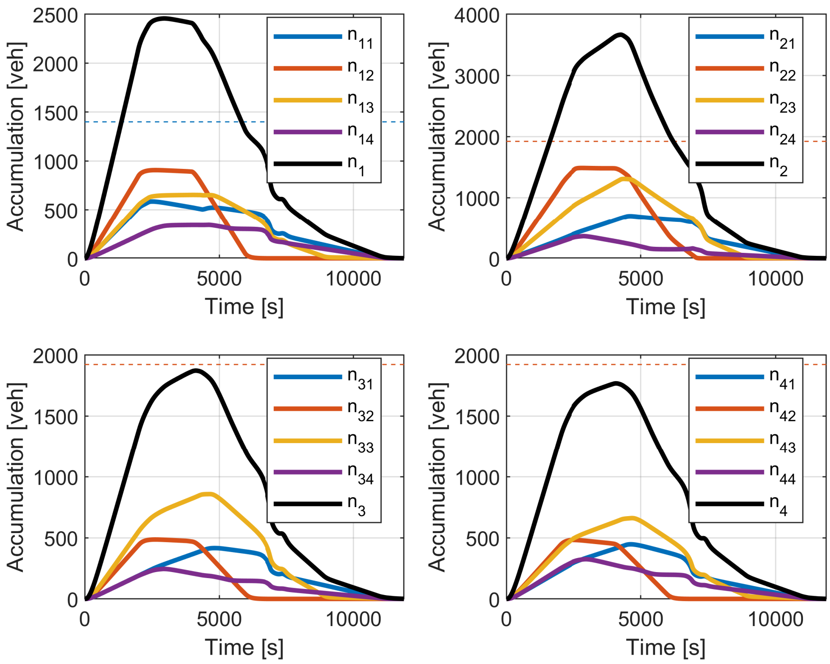


Figure 6: Accumulation (veh) over time in (s) per region, where i specifies to the origin and j the destination.n_1-n_4 are representing the accumulation summation per region

the summation per region n_1, n_2, n_3 and n_4 (Figure 6).

It is shown that R_1 and R_2 are exceeding the critical accumulation (dashed lines), reaching the congested regime. However, R_3 and R_4 are not reaching the critical density.

3.4 Dynamic equilibrium and optimum determination

Our proposed model uses route guidance and the optimal splitting rates $\theta_{IHJ}(t)$ are calculated for every time step of a performed rolling horizon optimization. This methodology implies, that the route of a user is not fixed over the simulation horizon and with that, the TTT varies compared to a pre-defined and fixed route. Hence, the static UE and SO are not sufficient for this problem and the DUE and DSO need to be determined. The DUE seeks for the Wardropian user equilibrium which is representing a state where no user can improve the experienced user travel time (UTT) by switching route. Otherwise, the DSO is the minimization of the total system travel time (STT), where every user would be better off. Nevertheless, this implies full knowledge about the route information and also full compliance of the users. The DSO problem can be formulated as an optimization problem as follows:

$$\text{minimize} \quad \sum_{l \in \mathcal{L}} \int_0^{t_c} N_I(t)dt. \tag{7}$$

where x_l is representing the equilibrium flow on a path $l \in \mathcal{L}$, where $\mathcal{L}$ holds all the possible paths for an origin I to a destination J in the proposed network. t_c can be defined as the choice time horizon of a users route decision and be set equal to the simulation time horizon. The function $N_I(t)$ represents again the accumulation in the region I. The DUE problem aims for minimization of the travel times per path for every user in the system. As this constitutes a non-linear problem and high computation costs, we are working on a methodology to reduce its relationship to a convex function (solvable with linear programming).

Both optimization problems must be solved with respect to the dynamics in (2), (3), non-negative, and capacity constraints. After the computation of the system states DUE and DSO as a reference, different pricing strategies can be applied and quantitative feedback about the system improvement can be given. The closer the model is operating at the DSO, the better the performance and with that the lower the TTT for all users in the network gets. Hence, the provided modeling allows the comparison of different pricing strategies and the evaluation of their performance in the system. In a detailed case study, several novel pricing functions J are tested and different pricing strategy combinations are tested and evaluated numerically. The MFD control in the border regions, as well as in the city center is providing feedback for dynamic pricing on corridor entrances. This approach allows us to test and evaluate current pricing strategies and to develop novel and improved methodologies. The results lead us to a traffic network with a minimized TTT and with that mitigated negative environment and social impacts.

4 Future research and conclusion

This paper presents a generic framework for the evaluation of congestion pricing policies. With the novel methodology used for MFDs design that allows a smooth, concave and continuous approximation of the trapezoidal diagram, a beneficial baseline for solving the equilibrium and optimum optimization problems is provided. To describe the dynamics between urban regions, a well-known multi-region model approach is utilized. The demand patterns, used as a simulation input, are determined by solving an optimization problem with defined targets on the corresponding region MFDs.

The presented case study shows a region design of the city of Zurich. Reasonable parameters were assumed by utilizing the traffic main arteries and the geographical reference of the installed LDs. The derived parameters from the region design, as well as from the demand optimization problem, are used in the MFD design and the simulation model, respectively. The outputs from the simulation plant represent a reasonable baseline scenario for the evaluation, improvement and potential re-design of congestion pricing policies. Furthermore, by using a route guidance algorithm that considers the splitting rates of users to different route possibilities, the impact of different pricing policies on the traffic behavior, as well as on the route choice can be determined. In future research, the performance of pricing algorithms is shown by the computation of the Dynamic User Equilibrium (DUE) and the Dynamic System Optimum (DSO). By introducing different performance indicators, we can determine and compare the improvements towards the DSO.

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