# Integrated and coordinated control for highway networks

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## Abstract

The growing level of freeway traffic congestion comprises an everyday life issue with social, economic, and environmental implications for modern metropolitan areas. Although there is evidence that Variable Speed Limits (VSL) and Ramp Metering (RM) are two effective practical approaches to ameliorate traffic congestion, their real-time field application is deemed cumbersome due to computational complexities. The positive effects that these approaches can have on traffic flow and congestion can be demonstrated with the augmented METANET model, which is one of the most widely used macroscopic models for freeway traffic. From the mathematical and systems theory viewpoint, METANET is a nonlinear, non-affine, Multiple Input Multiple Output (MIMO) system, which is affected by disturbances. Feedback linearization is a useful methodology in the literature to deal with nonlinear MIMO systems, and simplify the control design process. In the current work, by applying this method, we manage to represent the closed-loop system with a linear model that under certain conditions be an exact replication of the original system. Furthermore, the existence of a zero dynamic system, the controller stability, and the disturbance decoupling problem are investigated. Afterwards, we present and discuss the closed-loop linear representation of the METANET model by applying an appropriate feedback linearization method. Finally, a pole placement feedback loop is developed in order to regulate the closed-loop linear model.

## Keywords

# Introduction

During the past decades freeway traffic has attracted a lot of attention from academia, due to its crucial impact on safety, economy and environment. Prior studies have proposed different approaches to improve traffic condition on freeway networks. Ramp Metering (RM) and Variable Speed Limits (VSL) have been widely considered as two effective methods to regulate the traffic flow on the freeway. RM methods provide better traffic conditions on the freeway through controlling the on-ramp outflow on the freeway mainstream. RM strategies can be categorized as local and coordinated.

Local RM regulates the on-ramp outflow based on its neighbourhood traffic information, in order to ameliorate the local traffic conditions. ALINEA, presented in M. Papageorgiou and Blosseville (1991), is one of the most popular methods in this category, that employs a feedback control method to regulate the on-ramp outflow. Although local ramp metering is well-known and widely used, especially due to its simplicity, it is easy to show that it can be outperformed by coordinated ramp metering. In practice, local strategies demonstrate poor performance, and one of the main reasons is the presence of limited ramp storage space.

On the other hand, coordinated methods regulate the on-ramps outflow by utilizing system-wide traffic information, in order to enhance the overall network performance. Various methods have been presented in the literature for coordinated ramp metering; for instance C.Lu and Gong (2017) applied a reinforcement learning method to deal with equity issues (i.e. users from different on-ramps have equal access to the mainline). H. Haj-Salem and Bhouri (2018) presented a multi-objective nonlinear optimization that includes two cost functions, for traffic and safety (based on a risk index model). A model predictive hierarchical control method is developed by I. Papamichail and Papageorgiou (2010), where the structure is composed from an estimation, an optimization, a and direct control layers, with focus on optimizing the total time spent. Finally, R. L. Landman and Hoogendoorn (2015) have developed a heuristic algorithm based on the synchronization of on-ramp saturation time.

Note that various RM methods have been frequently implemented on motorways around the world. Nevertheless, the efficiency of these strategies deteriorates when they have to deal with high demands, or in cases of considering the equitable allocation of benefits among users (see A. Kotsialos and Middelham (2001) for more details). VSL is another actuator that compared to RM can provide more direct and efficient control on the mainline traffic flow. Many different methods have been proposed that apply VSL in order to improve freeway traffic conditions. For instance, A. Hegyi and Hellendoorn (2005b) and J. R. D. Frejo and Camacho

(2013) have proposed a model predictive control (MPC) approach. An optimal control strategy based on minimization of $L^2$ quadratic error to the desired outflow was developed by M. L. D. Monache and Rossi (2017). In another work, Khondaker and Kattan (2015) studied VSL for an environment with connected vehicles, and designed a multi-objective optimization function to simultaneously optimize mobility, safety and environmental sustainability.

The integration of RM and VSL methods provides the opportunity to control mainline traffic flow conditions, while, at the same time, considering the equity and on-ramp queuing issues. Zhang and Ioannou (2017) developed a feedback linearization VSL approach, coordinated with ALINEA/Q as a RM strategy, in order to maximize the flow rate and manage the on-ramp queues simultaneously. The combination of ALINEA and HERO as local and coordinated RM, respectively, together with a VSL algorithm is proposed by D. Li and Ceder (2014). A. Hegyi and Hellendoorn (2005a) developed an MPC approach to compute the optimal coordination of RM and VSL. Although the proposed control methods for the combination of RM and VSL have presented promising results, this problem requires further attention, in order to develop computationally feasible and efficient algorithms for field implementations. Note that the optimization and MPC methods applied in the literature can guarantee the optimal coordination but they provide open form solutions; nevertheless, most of the state-of-the-art approaches result in solving optimization problems repeatedly, which requires a large amount of computational power. On the other hand, heuristic algorithms can reduce the computational effort, however, they may not be able to provide optimal solutions.

In this paper, we study the problem of RM and VSL coordination on the macroscopic level. We utilize METANET as the model of freeway traffic flow, which is a nonlinear system. Feedback linearization is a useful method to deal with nonlinear systems. This method simplifies the controller design process. The application of this methodology, leads to the formulation of a linear closed-loop representation of the original system, that can be easily regulated with simple linear control strategies (e.g. pole-placement or optimization). The linear control strategies result in closed form solutions that in comparison to heuristic algorithms are more accurate and more reliable. Furthermore, solving an optimization problem for the linear representation of the nonlinear system is faster and requires less computational burden.

The reminder of the paper is organized as follows: The macroscopic model of the freeway traffic flow and the modeling impact of applying RM and VSL methods are described in Section 2. In Section 3, we outline the feedback linearization as an effective method to deal with nonlinear systems. Afterwards, we investigate the characteristics of the macroscopic model regarding the presented nonlinear system properties. Finally, we propose a closed-form control solution for the problem at hand. Section 4 presents the conclusions and future work directions.

# 1 System Dynamics

## 1.1 METANET model

The METANET is a macroscopic second order model of freeway traffic flow that represents the dynamics of each freeway segment with the length of $\Delta_i$ and the lane number of $\lambda_i$ as follows Wang and Papageorgiou (2005)

$$\dot{\rho}_i(t) = \frac{1}{\Delta_i \lambda_i}\Big(q_{i-1}(t) - q_i(t) + r_i(t) - s_i(t)\Big), \tag{1}$$

$$s_i(t) = \beta_i(t)q_{i-1}(t), \tag{2}$$

$$\dot{v}_i(t) = \frac{1}{\tau}\Big(V(\rho_i(t)) - v_i(t)\Big) + \frac{1}{\Delta_i}v_i(t)\Big(v_{i-1}(t) - v_i(t)\Big) - \frac{v}{\tau\Delta_i}\frac{\rho_{i+1}(t) - \rho_i(t)}{\rho_i(t) + k} - \frac{\delta}{\Delta_i \lambda_i}\frac{r_i(t)v_i(t)}{\rho_i(t) + k}, \tag{3}$$

$$V(\rho_i(t)) = v_{f,i}\exp\left(-\frac{1}{a_i}\left(\frac{\rho_i(t)}{\rho_{\text{cr},i}}\right)^{a_i}\right), \tag{4}$$

$$q_i(t) = \rho_i(t)v_i(t)\lambda_i, \tag{5}$$

where $\rho_i(t)$, $q_i(t)$ and $v_i(t)$ denote the traffic density, traffic flow and space mean speed in segment i, respectively. $\tau$, $v$, $k$ and $\delta$ are the model parameters. Based on (1), the conservation equation, the difference between total amount of input flows ($q_{i-1}$ (t) and the on-ramp inflow $r_i(t)$) and total amount of output flows ($q_i(t)$ and off-ramp outflow $s_i(t)$) in segment i causes change in the amount of $\rho_i$ (t). The exiting rate $\beta_i(t)$ demonstrates the ratio of $s_i(t)$ to $q_{i-1}(t)$. Equation (3), the dynamic speed equation, is composed of different terms. The first term is the relaxation term that demonstrates the tendency of vehicles to achieve the desired speed (the stationary speed $V(\rho_i(t))$). The second and third terms model the impact of special heterogeneity. The second term is convection term, which expresses the effect of inflow, and the third one is anticipation term that demonstrates the effect of upcoming change in density. The relationship between stationary speed and traffic flow according to the fundamental diagram is demonstrated in (4),

where $v_{f,i}$ and $\rho_{cr,i}$ denote the free speed and critical density, respectively.

## 1.2 Modeling impact of VSL

To model the impact of engaging VSL on the traffic condition (fundamental diagram), R. C. Carlson and Messmer (2010) represents the following equation.

$$v_{f,i}(b_i(t)) = \hat{v}_{f,i} b_i(t) \tag{6}$$

$$\rho_{cr,i}(b_i(t)) = \hat{\rho}_{cr,i}\left(1 + A_i(1 - b_i(t))\right) \tag{7}$$

$$a_i(b_i(t)) = \hat{a}_i\left(E_i - (E_i - 1)b_i(t)\right) \tag{8}$$

where $\hat{v}_{f,i}$, $\hat{\rho}_{cr,i}$ and $\hat{a}_i$ denote the parameters of (4) before applying the VSL. The VSL rate $b_i(t)$ is a control variable, confined to (0,1]. If $b_i(t) = 1$, no VSL is applied. Also, we can suppose $b_i(t)$ is roughly equal to the ratio of VSL-induced speed to non-VSL free flow speed. $A_i$ and $E_i$ are constant parameters equal to 0.67 and 1.82, respectively, that are selected through estimation from real data.

## 1.3 Modeling impact of RM

The following equations present the impact of applying RM on the on-ramp outflow $r_i(t)$ R. C. Carlson and Messmer (2010).

$$\dot{w}_i(t) = d_i(t) - r_i(t), \tag{9}$$

$$r_i(t) = c_i(t) \min\left(Q_0, Q_0 \frac{\rho_{max} - \rho_i(t)}{\rho_{max} - \rho_{cr,i}}, d_i(t) + \rho_{w_i} v_{f,i}\right). \tag{10}$$

The dynamics of the queue length $w_i(t)$ is presented in (9), where $d_i(t)$ is the on-ramp demand flow. Another control variable is presented as the metering rate $c_i(t)$ that is confined to $[c_{min}, 1]$,

where $c_{\min}$ is a minimum admissible value. According to (10) RM-induced on-ramp inflow is a portion of inflow in absence of RM, which is a minimum of three terms. The first term is on-ramp flow capacity $Q_0$, the maximum ramp inflow for under critical traffic situation of the main stream. The second term named by supply of space, expresses the effect of mainstream congestion on the ramp inflow, where $\rho_{\max}$ denotes the maximum density. The third term, demand by space, is the actual accessible demand flow by involving the vehicles waiting on the queue, where $\rho_{w_i}$ is the queueing density.

# 2 Methodology

## 2.1 Feedback Linearization

One of the well-known approaches to deal with nonlinear systems is feedback linearization. Through this approach, we achieve a linearized representation of the (whole or part of the) closed loop nonlinear system. Afterwards, we can apply the linear control method to manipulate the system. There are some important notions that are used frequently in feedback linearization approach, such as affine systems, relative degrees, internal dynamics, zero dynamics and disturbance decoupling problem. These notions are briefly defined as below Slotine and Weiping (1991).

- **Affine system:** a nonlinear system is input or disturbance affinity if it is linear with respect to the input or disturbance.
- **Relative degree:** input or disturbance relative degree is the number of times you need to take the derivative of the output before the input or disturbance appears. This concept is analogues to the difference in the number of zeros and poles in linear systems.
- **Internal dynamics:** a part of system that is unobservable from the external linearized input-output relationship. The dimension of internal dynamics is equal to the difference between the number of system states and the input relative degree.
- **Zero dynamics:** is the internal dynamics of the system when the system output is zero. The zero dynamics is functionally analogues to zero in linear system transfer function.
- **Disturbance decoupling problem:** to design a proper nonlinear feedback control that simultaneously linearize the relation between transformed input and output while completely eliminate the effects of disturbance on output signals.

The feedback linearization method is applicable for affine nonlinear systems with respect to inputs and disturbances. However, from (1-5), by considering $c_i(t)$ and $b_i(t)$ as input signals, the METANET is a non-affine system with respect to inputs. To deal with this problem, we can present an extended system that is affine. Assume the previous input signals as the new states of the extended system and the derivative of them as the input signals of the extended system Henson and Seborg (1997). The extended system of the METANET model is presented in the state space form as follows.

$$\dot{X}(t) = f(X) + g(X)U(t) + p(X)D(t) \tag{11}$$

$$Y(t) = h(X) \tag{12}$$

$$1 \leq i \leq N \tag{13}$$

$$X(t) = \left[\rho_i(t)v_i(t)w_i(t)c_i(t)b_i(t)\right]^T \tag{14}$$

$$U(t) = \left[C_i(t)B_i(t)\right]^T \tag{15}$$

$$D(t) = \left[d_i(t)\beta_i(t)\right]^T \tag{16}$$

$$Y(t) = \left[\rho_i(t)v_i(t)\right]^T \tag{17}$$

where, $X(t)$ is the system state vector consists of the density, mean speed, queue length, metering rate and VSL rate of all the freeway sections. $U(t)$ is the input (control) signal consists of the first derivative of the metering rate $C_i(t)$ and VSL rate $B_i(t)$. The on-ramp demands and the off-ramp exiting rates are disturbances of the system that are defined as the vector $D(t)$. Finally, we assume density and mean speed as the system outputs denoted with $Y(t)$. The functions $f(X)$,

$g(X)$ and $p(X)$ are defined as below:

$$f(X) = \begin{bmatrix} \frac{1}{\Delta_i \lambda_i} [\rho_{i-1}(t)v_{i-1}(t)\lambda_{i-1} + c_i(t)\alpha^* - \rho_i(t)v_i(t)\lambda_i]_{(N \times 1)} \\ \frac{V(\rho_i(t)) - v_i(t)}{\tau} + \frac{v_i(t)}{\Delta_i}(v_{i-1}(t) - v_i(t)) - \frac{v}{\tau \Delta_i} \frac{\rho_{i+1}(t) - \rho_i(t)}{\rho_i(t) + k} - \frac{\delta}{\Delta_i \lambda_i} \frac{v_i(t)c_i(t)}{\rho_i(t) + k} \alpha^*_{(N \times 1)} \\ -c_i(t)\alpha^*_{(N \times 1)} \\ 0_{(2N \times 1)} \end{bmatrix} \qquad (18)$$

$$\alpha^* = \alpha_{i1} Q_0 + \alpha_{i2} Q_0 \frac{\rho_{\max} - \rho_i}{\rho_{\max} - \rho_{cr,i}} + \alpha_{i3} \rho_{w_i} v_{f,i} \qquad (19)$$

$$g(X) = \begin{bmatrix} 0_{(3N \times 2N)} \\ I_{(2N \times 2N)} \end{bmatrix} \qquad (20)$$

$$p(X) = \begin{bmatrix} 0_{(1 \times N-1)} & \alpha_{i3} \frac{c_i(t)}{\Delta_i \lambda_i} & 0_{(1 \times N-1)} & \frac{-1}{\Delta_i \lambda_i} \rho_{i-1}(t)v_{i-1}(t)^{\vee}_{i-1} & 0_{(1 \times N-i)} \\ 0_{(1 \times i-1)} & \alpha_{i3} \frac{\delta}{\Delta_i \lambda_i} \frac{v_i(t)c_i(t)}{\rho_i(t) + k} & 0_{(1 \times 2N-i)} \\ 0_{(1 \times i-1)} & 1 - \alpha_{i3}c_i(t) & 0_{(1 \times 2N-i)} \\ 0_{(2N \times 2N)} \end{bmatrix} \qquad (21)$$

For the sake of simplicity and in order to make it practical to work with the minimum function on (10), we represent (19), where $\alpha_{i1}$, $\alpha_{i2}$ and $\alpha_{i3}$ are constant parameters that in every $t$ only one of them is 1 and the others are 0.

## 2.2 Relative degrees

Isidori (1995) presents the following theorem to calculate relative degrees of the systems.

*Theorem 1*

The nonlinear system has vector input relative degree $IRD = \{ird_1, \ldots, ird_m\}$ at the equilibrium point if

1. $L_{g_j} L_f^k h_i(X) = 0$ for all $1 \le i, j \le m$, for all $k < ird_i - 1$, and for all $X$ in the neighborhood of equilibrium point.
2. The following decoupling matrix is nonsingular at the equilibrium point.

$$A(X) = \begin{bmatrix} (L_{g_1}L_f^{ird_1-1}h_1(X) & \dots & L_{g_m}L_f^{ird_1-1}h_1(X) \\ \vdots & \ddots & \vdots \\ L_{g_1}L_f^{ird_m-1}h_m(X) & \dots & L_{g_m}L_f^{ird_m-1}h_m(X) \end{bmatrix}_{(m \times m)}$$

(22)

*Theorem 2*

The system has disturbance relative degree $DRD = \{drd_1, \dots, drd_m\}$ at the equilibrium point if

1. $L_{p_j}L_f^k h_i(X) = 0$ for all $1 \le i, j \le m$, for all $k < drd_i - 1$, and for all $X$ in the neighborhood of equilibrium point.
2. For each $i$ there exists at least one $j$ where $L_{p_j}L_f^{drd_i-1}h_i(X_0)$ 0.

The functions involved in (11) are summarized as below to avoid cumbersome calculation in this problem.

$$f(X) = \begin{bmatrix} f_{1(N \times 1)} \\ f_{2(N \times 1)} \\ f_{3(N \times 1)} \\ 0_{(2N \times 1)} \end{bmatrix}, \quad g(X) = \begin{bmatrix} 0_{(3N \times 2N)} \\ I_{(2N \times 2N)} \end{bmatrix}, \quad p(X) = \begin{bmatrix} p_{1(N \times 2N)} \\ p_{2(N \times 2N)} \\ p_{3(N \times 2N)} \\ 0_{(2N \times 2N)} \end{bmatrix}$$

(23)

$$\frac{\partial h(X)}{\partial X} = \begin{bmatrix} I_{(2N \times 2N)} & 0_{(2N \times 3N)} \end{bmatrix}$$

(24)

$$k = 0 \rightarrow \begin{cases} L_g h(X) = \frac{\partial h(X)}{\partial X} g(X) = [0_{(2N \times 2N)}] \\ L_p h(X) = \frac{\partial h(X)}{\partial X} p(X) = \begin{bmatrix} p_{1(N \times 2N)} \\ p_{2(N \times 2N)} \end{bmatrix} \end{cases}$$

(25)

From the above calculation, when $k = 0$ there is at least one nonzero element in each row of $L_p L_f^k h(X)$, therefore the disturbance relative degree is $DRD = \{1\}_{(1 \times 2N)}$. To find out the input relative degree we need to take further steps.

$$k = 1 \rightarrow L_g L_f h(X) = \frac{\partial L_f h(X)}{\partial X} g(X) = \frac{\partial}{\partial X}(\frac{\partial h(X)}{\partial X} f(X)) g(X) = \frac{\partial}{\partial X}\begin{bmatrix} f_{1(N \times 1)} \\ f_{2(N1)} \end{bmatrix} g(X) = \begin{bmatrix} \frac{\partial f_1(X)}{\partial c_i}_{(N \times N)} & \frac{\partial f_1(X)}{\partial b_i}_{(N \times N)} \\ \frac{\partial f_2(X)}{\partial c_i}_{(N \times N)} & \frac{\partial f_2(X)}{\partial b_i}_{(N \times N)} \end{bmatrix}$$

(26)

In this step, $L_g L_f^k h(X)$ is a lower triangular matrix with nonzero diagonal elements. Therefore, this matrix is nonsingular and the input relative degree is $IRD = \{2\}_{1 \times 2N}$.

## 2.3 Internal dynamics and zero dynamics

As we mentioned before, if $\sum ird_i < n$ a part of the nonlinear system with $n - \sum ird_i$ dimension is unobservable from the external input-output relationship, which is called internal dynamics. Therefore, it is essential to investigate the stability of the internal dynamics. For this reason, there is a concept called zero dynamics that is akin to the concept of zeros for linear systems. Zero dynamics is defined as the internal dynamics when the system output is zero. Local Asymptotic stability of the zero dynamics is a necessary and sufficient condition for local asymptotic stability of the internal dynamics.

In this problem, the sum of the input relative degree elements is $4N$ that is smaller than the number of system states which is $5N$. Therefore, we are encountered an internal dynamics, which its stability should be investigated. We can transform our nonlinear system into a linear system with a normal form by defining $\xi_k^i = L_f^{k-1} h_i(x)$ for all $1 \leq k \leq ird_i$, $1 \leq i \leq 2N$. The following equations represent the normal form where $\xi$ denotes the states of linearized model and $\eta$ denotes the vector of unobservable states, called internal dynamics.

$$\dot{\xi}_1^i = \xi_2^i + c_i(\xi, \eta)D \tag{27}$$

$$\dot{\xi}_2^i = b_i(\xi, \eta) + a_i(\xi, \eta)U + s_i(\xi, \eta)D \tag{28}$$

$$\dot{\eta}_j = q_j(\xi, \eta) + t_j(\xi, \eta)D \tag{29}$$

$$y_i = \xi_1^i \tag{30}$$

Suppose that $g_i$ denotes the vectors of $g(X)$, if $G = \text{span}\{g_1, \ldots, g_m\}$ is involutive it is always possible to choose the states of the internal dynamics in such a way that $L_{g_i}\eta_j = 0$ for all

$1 \leq i \leq m, 1 \leq j \leq N$. From (20), $G$ is obviously involutive therefore

$$L_{g_i}\eta_j = \frac{\partial \eta_j}{\partial X}g_i = 0 \Rightarrow \left[\frac{\partial \eta_j}{\partial C_k}, \frac{\partial \eta_j}{\partial B_k}\right] = 0, \ 1 \leq k \leq N. \tag{31}$$

On the other hand, $\Phi(X) = col(\xi_1^1, , \xi_1^{2N}, \xi_2^1, , xi_2^{2N}, \eta_1, \eta_N)$has to be nonsingular at the equilibrium point. As a result, we determine the internal dynamics as $\eta = w$.

## 2.4 Disturbance decoupling problem

As we discussed before, in this problem the disturbance relative degree is less than the input relative degree. One can interpret that in the METANET model disturbances affect outputs more directly than input (control) signals. We can rewrite the equations (27-29) as below:

$$\dot{\xi}_1^i = \xi_2^i + \sum_{j=1}^{2N} L_{p_j}h_i(\xi)D_j, \tag{32}$$

$$\dot{\xi}_2^i = L_f^2 h_i(\xi) + \sum_{j=1}^{m} L_{g_j}L_f h_i(\xi)U_j + \sum_{j=1}^{2N} L_{p_j}L_f h_i(\xi)D_j, \tag{33}$$

$$\dot{\eta}_j = L_f \eta_j + \sum_{j=1}^{2N} L_{p_j}\eta_i D_j \tag{34}$$

where $D_j$ and $U_j$ are the $j^{\text{th}}$ column of $D$ and $U$, respectively. In case of measurable disturbance, Daoutidis and Kravaris (1989) proposed a solution for disturbance decoupling problem under the same circumstance explained above. As it is shown below, the proposed control law contains anticipatory factor of disturbances.

$$U = (L_g L_f h(x))^{-1}\left(V - \sum_{j=0}^{2} L_f^j h(X) - \sum_{l=0}^{1} \frac{d^l(L_p L_f^{1-l}h(x)D)}{dt^l}\right) \tag{35}$$

Applying the above equation results in a linear representation of the METANET model where $V$ is the control signal for the linear closed loop representation of the system. $V$ can be derived from any available control methods for linear system such as pole placement.

# 3 Conclusion and Future Work

So far, we investigated the characteristics of the METANET model as a nonlinear system. METANET belongs to the group of non-affine systems with respect to the input signals. A useful method to deal with this type of nonlinear systems is adding an integral block before the input signals, which results in an affine representation of the system. The studies of relative degrees reveal the existence of the internal dynamics in this problem. Also, these studies indicate that the disturbances affect the system outputs more directly than the input signals. Consequently, to solve the disturbance decoupling problem, we need to add an anticipatory component to the control low that involves factors of disturbance anticipation in terms of derivatives.

For the future work, we aim to design a pole placement to control the closed loop linear representation of the system. As we mentioned before, due to the existence of the internal dynamics, applying a proper controller is not sufficient to the stability issue. Since the states of internal dynamics are unobservable from the external input-output relationship, we have to investigate the stability of the internal dynamics separately. After completing these steps, our methodology would be ready to be tested with real world freeway traffic datasets.

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