Urban mode and subscription choice - An application of the three-dimensional MFD

Andrew Schreiber

Allister Loder

Kay W. Axhausen

Institute for Transport Planning and Systems

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Andrew Schreiber
Department of Agricultural and Applied Economics
University of Wisconsin-Madison
330 North Orchard Street
Madison WI 53715, USA
phone:

schreiber@aae.wisc.edu

Allister Loder
IVT
ETH Zürich

CH-8093 Zürich
phone: +41-44-633 62 58
fax: +41-44-633 10 57
allister.loder@ivt.baug.ethz.ch

Kay W. Axhausen
IVT
ETH Zürich

CH-8093 Zürich
phone: +41-44-633 39 43
fax: +41-44-633 10 57
axhausen@ivt.baug.ethz.ch

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Abstract

In urban environments, people have the choice between private and public modes for travel. These choices are characterized by fixed and variable monetary and time costs which affect decision making through impacts on utility levels. Travel times depend on the interaction of private and public vehicles and their congestion externalities. A common economic mechanism used to correct for such inefficiencies is pricing. While the impact of pricing a single mode is well understood, it is less for simultaneously pricing multiple modes on a large urban scale.

In contrast to traditional mode choice models with travel times taken as exogenous, we propose a model which endogenizes travel times based on the three-dimensional macroscopic fundamental diagram (MFD). The integrated model combines a consumer utility maximization framework with the MFD which describes regional effects of fixed and variable cost policy changes to the transportation system. The model is empirically tested for the city of Zurich, Switzerland.

Keywords

Mode choice, season-ticket subscription, MFD, general equilibrium, urban economics

Preferred citation style

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1 Introduction

In an urban environment, besides the choice of residential location travelers typically have the choice between private and public transport modes. The latter mentioned individual choice bundles together aspects of extensive and intensive margin decisions. From a purely monetary perspective, the extensive margin choice can be characterized by incurring the fixed costs for a car or a public transit season-ticket. The intensive margin is characterized by the variable costs for fuel or for single ride public transport tickets. However, the choice for transport does not solely depend on monetary costs. One must also consider the time costs associated with a given mode choice which ultimately depend on the interaction of private and public vehicles in the network. All choices create utility and disutility. Utility is created by the benefits the traveler receives at his destination and disutility is created by travel costs and travel time incurred (Lerman and Ben-Akiva, 1976; Golob and Beckmann, 1971).

Having the choice between a private and public mode results in competition for scarce urban road space constrained by the capacity of the city (Tsekeris and Geroliminis, 2013). In order to improve upon the efficiency of the transportation network, the social costs of each mode must be considered. The social cost implications of one additional passenger depends on the mode of transport. Whereas each additional car driver imposes negative external costs to all other drivers, one additional public mode passenger will not impose negative external costs to other travelers unless the system is used to capacity (Small and Verhoef, 2007). This trade-off between the private and public mode raises the problem for transport engineers and economists to improve efficiency and equity of transport policies by creating incentives for travelers across competing modes. For engineers, this is often related to network extension and improvement of the level of service. For economists, this often means pricing. However, exactly how to price individuals remains an open debate, especially when considering individuals with heterogeneity in preferences and valuation of time (Chakirov, 2016). From the social planner's perspective, part of the objective function would consider traffic flow (average vehicle speeds) and marginal external costs. If the city's transportation system is unable to provide sufficient levels of traffic flow and speed, the increasing travel costs may outweigh the benefits individuals receive (Graham, 2007; Venables, 2007). Though, this may not be the only consideration. For instance, equity concerns about distributional consequences of policy across households could matter as well as public revenue consequences.

To approach optimality and, thus, to realize beneficial outcomes with pricing policies, the quantification of the relationships between traffic flow and the interaction of private and public transport vehicles on an urban scale must be understood (Smeed, 1968; Florian, 1977). Recently, Geroliminis and Daganzo (2008) have shown empirically that on a large urban scale vehicle density is linked to vehicle flow by a unique and reproducible curve called the macroscopic fundamental diagram (MFD). The MFD exhibits for each city the vehicle density in cars per kilometer at which the maximum in traffic flows occurs. The MFD can be extended to a three-dimensional MFD for mixed bi-modal networks in which the interaction of private and public vehicles is captured (Geroliminis et al., 2014). The three-dimensional MFD allows us to identify the degree of suboptimal vehicle flow in the urban transportation system.

Geroliminis and Daganzo (2008) argue that policy makers can make use of traffic control and pricing strategies based on the MFD in order to improve overall network performance and social welfare. However, the primary research application of the MFD is in traffic control and only minuscule in economics, e.g. by Zheng et al. (2012) for cordon pricing for private vehicles and by Zheng et al. (2016) for multimodal pricing. A comprehensive approach of pricing the fixed and variable cost of the private and public mode is not found in the literature. While there exists large amounts of theoretical literature on the optimal pricing of externalities, there is much less on numerical investigations of pricing policies (c.f. Section 2.1). In particular, investigating instances such as season-tickets which provide a different cost structure relative to textbook optimal fares is of interest. We use the advances in MFD theory to address the question of how pricing for public transport affects urban transportation systems and to quantify the impact of pricing on urban speeds.

In this paper, we propose an integrated general equilibrium mode choice model that combines a consumer choice utility maximization framework with the three-dimensional MFD. The residential location of households is assumed to be fixed in this model. An equilibrium is reached once representative agents of different household income levels totally arbitrage over their range of choices. In this model, the effects of fixed cost subscriptions to a mode and the actual mode use on the transport network performance can be assessed without any traffic simulation. A brief overview of theory and background material is presented in Section 2. Section 3 develops the model and applies this in an empirical case study to the city of Zurich in Section 4. This paper closes with a discussion on policy relevance and limitations in Section 5.

2 Background

2.1 Pricing

The seminal work of Smeed (1961, 1968) provides the starting place for quantifying the external costs of cars and buses on travel speed. The trade-off between the faster car and the higher transport capacity of buses with lower speeds outlined the research avenue for finding optimality in the allocation of urban resources. The network optimal allocation is related to the realized travel times and how much time losses/congestion externalities one additional user imposes on other users (Small and Verhoef, 2007). Economic textbook treatment that address congestion externalities concerns road pricing schemes. In this context, road users could pay a variable road toll (first best) or fixed road toll (second best) of the amount that charges for the congestion externalities. As discussed, these transport policies are purposed to address congestion alone. The economic rationale of addressing congestion externalities does not mean that congestion is completely mitigated but that the benefits and costs are optimized. Pricing can also help internalize external effects such as environmental pollution, accidents, and others outside the scope of the current paper. Besides of pricing, optimality can be achieved by expanding the infrastructure up to the optimum or impose driving restrictions (Anas and Lindsey, 2011).

However, most applications consider the implications of pricing private road users alone. The rationale for public transport pricing is less ubiquitous. The textbook rationale of providing public transport with some form of subsidy (sometimes not requiring a public budget balance) is related to enticing individuals to choose their transportation mode such that the number of private and public vehicles in the network equals the optimal traffic flow and that the subsidy equals the sum of all avoided time losses of private transport users in the presence of public transport (Small and Verhoef, 2007). However, this relationship can also be thought of in the converse. In presence of car travel and public transportation, the first best pricing for private vehicles sets a toll equal to the marginal social costs, while the public mode is priced according to the sum of average agency costs and the adjustments for scale economies of user and agency costs as one additional passenger reduces average costs. The mentioned scale economies and under-priced car travel in peak hours is a widely used argument for subsidizing public transport (Small and Verhoef, 2007).

Finding the optimal traffic flow is not the key objective in itself. In economic terms, the costs for the transportation of goods and persons should be minimal (Krugman, 1993). With minimal transportation costs, individuals can maximize their social and business interactions. More interactions create more value and increase productivity (Bettencourt, 2013). These agglomeration benefits are manifest in higher wages of workers and greater tax income for the public budget (Venables, 2007).

In practice, first-best road pricing and public transport fares are not implemented. Second best road pricing schemes are implemented reducing congestion levels, e.g. in London (Prud'homme and Bocarejo, 2005), Stockholm (Eliasson et al., 2009) and Singapore (Goh, 2002). Public transport operators offer users typically distance- or time-depending fares, daily travel passes and the subscription to season-tickets (Carbajo, 1988). Sherman (1967) emphasizes that car driving usually charges the marginal price whereas public transport charges the average costs. In order to overcome this misallocation and to achieve an equal choice between car and public transport, he suggests the concept of a two part tariff within a passenger club[1] where a fix price is charged for the fix costs and a usage charge that corresponds to the variable prices.

For public transport operators, the subscription to a season-ticket brings in a large upfront cash transfer and for public transport users the benefit of having no marginal travel costs. Contrary, the downside of a subscription system is that it is in competition with the polluter pays principle and, therefore, occasionally misaligned with the first-best pricing principle. In the literature on subscription based pricing, the emphasis is on the fare design from the public transport operators perspective (e.g. Carbajo (1988); White (1981)). The effects of subscription based public transport pricing on the transportation system is only present in analyzing the patronage of the public transport systems (e.g. FitzRoy and Smith (1998, 1999); García-Ferrer et al. (2006)).

2.2 The three-dimensional MFD

From an aggregate perspective, the fundamental diagram of a single road section links the vehicle density to the total vehicle flow by a unique curve with maximum flow at a nonzero density. The average speed on that link equals the ratio of flow and density. From the perspective of an individual traveler the fundamental diagram links travel distance to travel time. Geroliminis and Daganzo (2008) have shown with data from loop detectors and floating mobile probes in Yokohama, Japan, that this curve also exist on a large urban scale and they named it the macroscopic fundamental diagram (MFD). Figure 1(a) shows a stylized version of a MFD. The maximum of flow occurs at the so called critical density. Note that average speed is monotonically decreasing in the vehicle density. Daganzo and Geroliminis (2008) give an analytical approximation of the MFD.

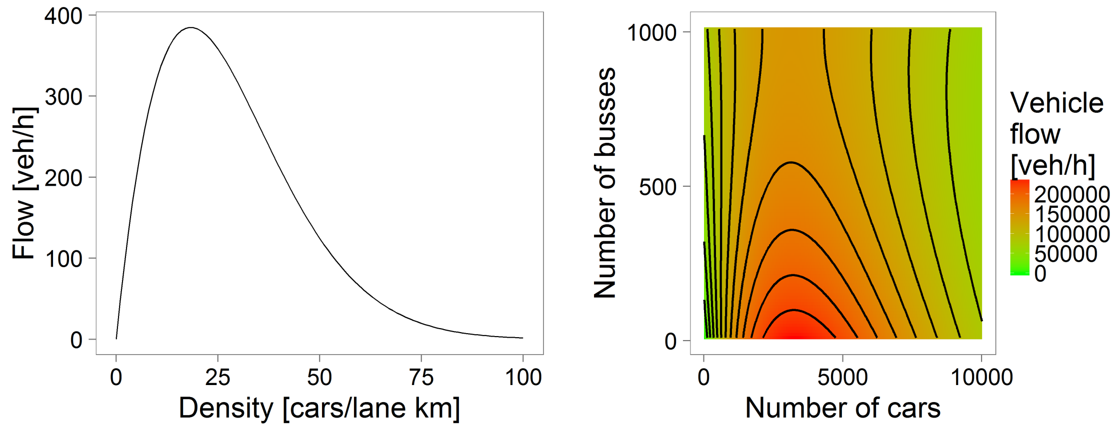
Geroliminis et al. (2014) propose that this relationship can be extended to the bi-modal case with the density of public transport vehicles in the third dimension. This three-dimensional MFD quantifies the interaction of private and public transport vehicles. Figure 1(b) shows a stylized version of the three-dimensional MFD. The maximum flow of vehicles occurs at zero public transport vehicles in the network. The more public transport vehicles in the network, the more the total flow decreases. If one considers the passenger flow instead of the vehicle flow, the maximum of flow occurs at non-zero public transport vehicle density. This draws the benefits of

[1]From the theory of pricing of club goods: "Whenever the utility derived by an individual from a specific good or service is depended on the size of the consumption group, then a club organization will supply the service efficiently while the market will not" (Buchanan, 1965). "If the size of the club is optimal, the subscription fee per capita equals the marginal social costs" (Berglas, 1976).

Figure 1: Illustration of the macroscopic fundamental diagram (MFD)

(a) Single mode fundamental diagram. The ratio of flow over density corresponds to the average travel speed

(b) Stylzed multi-modal MFD. The darker the colour the greater the total vehicle flow, taken from Geroliminis et al. (2014)



the operation of both private and public transport.

The MFD has been estimated for many other cities with a variety of methodologies. One approach considers collecting data from loop detectors and floating mobile probes in the network (e.g. Geroliminis and Daganzo (2008)). Other approaches include the use of three dimensional vehicle trajectories (Saberi et al., 2014), or estimate the MFD from traffic simulations (e.g. Ortigosa et al. (2015)). Findings suggest that the MFD is a property of the transport network itself and of the traffic control measures applied and ongoing research works on the link between the network and the MFD parameters (e.g. Leclercq and Geroliminis (2013)).

The basic assumption of the MFD is that the urban area is homogeneously congested (Geroliminis and Daganzo, 2008). Thus, one research avenue is to explore the impacts of inhomogeneities (e.g. Daganzo et al. (2011); Buisson and Ladier (2009)). In order to homogenize the traffic network, partitioning of the network is necessary (Ji and Geroliminis, 2012).

Currently, the primary application of the MFD in the literature is in traffic control as it identifies and quantifies the network state and the effects of signaling and other control measures (e.g. Keyvan-Ekbatani et al. (2015); Haddad (2015)). Economic applications of the MFD are minuscule in the literature. For example, the three-dimensional MFD contributes to the discussion of the allocation of urban road space (Geroliminis et al., 2014), and has been used for deriving optimal road tolls (Zheng et al., 2012, 2016).

2.3 Equilibrium modeling

Computable general equilibrium modeling has a long history in the economics literature. It was initially used to study trade policy (e.g. Harrison et al. (1997)) but has more recently been extended to cover a wide range of interdisciplinary topics. Specifically for our purposes, the use of spatial general equilibrium models have been popularized to study the economic effects of transportation policy. There are a variety of flavors of urban equilibrium models available. We can segment these model types as either more traditional urban spatial design equilibrium models and spatial general equilibrium models where agents sort according to fixed city limits. Within the context of pricing policies, both modeling types have been used. For instance, studies using equilibrium models akin to more traditional urban spatial design theory which allow for optimal city size depending on the bid-rent curve have looked at implications of second best pricing policies such as cordon tolls (e.g. Mun et al. (2005); Verhoef (2005)). More in line with our model are works such as Anas and Liu (2007), Rutherford (2008), and van Nieuwkoop (2014). Anas and Liu (2007) incorporates many more elements than of our immediate interests, but the model combines a general equilibrium model (RELU) with a transportation model (TRAN) for the Chicago area. The model is segmented into individual zones linked together through transport links. It allows for land use change, household sorting and transportation effects and solved using an iterative algorithm between the two sub-models. Rutherford (2008) and van Nieuwkoop (2014) use an integrated approach which allows for simultaneous solution of the economic and transportation models. Such studies use an Alonso-Muth-Mills (AMM) style sorting framework which allows households to fully arbitrate differences of different nodes in the exogenously defined urban network.

While our model is similar in respect to sorting conditions as proposed by Rutherford (2008), we differ in terms of scope. Both Rutherford (2008) and van Nieuwkoop (2014) model the economic effects of transportation policy by explicitly defining the transportation network specific to a given urban area (for instance, Zurich) and rely on the Wardopian network equilibrium framework for characterizing route choices. However, we rely as aforementioned on the city wide aggregated three-dimensional MFD. By using aggregated data for an entire city, we can study the overall outcomes of city-wide traffic policy with limited computational costs.

Our economic framework stems from Rutherford (2008) and Vandyck and Rutherford (2013). The basic construction of our economic decision model is rooted in utility maximization theory. A representative agent seeks to maximize total utility subject to a budget constraint. Leisure prices are allowed to fluctuate according to a market clearance condition which requires that supply be greater than or equal to demand. If supply is greater than demand, then the equilibrium price is zero. Leisure demand depends on achievable average network speeds as a result of

MFD relationships. We refrain from including endogenous production, but rather focus on demand side effects. This additional step will be the subject of future research[2]. Though we lack the circular flow economic structure, we define this model as general equilibrium because we model endogenous pricing for leisure as differing across time periods, income groups and mode sorting.

We formulate the model as a mixed complementarity problem (MCP) (Rutherford, 1995; Mathiesen, 1985). The concept of complementarity allows us to translate an optimization problem into a system of inequalities based on first order Karush-Kuhn-Tucker conditions. As a general example, the problem seeks to find $x \in \mathbb{R}^n$ such that for $F : \mathbb{R}^n \rightarrow \mathbb{R}^n$:

$$F_i(x) \geq 0, \quad x_i \geq 0, \quad x_i F_i(x) = 0$$

This can be concisely written using $\perp$ to denote the complementarity condition:

$$F_i(x) \geq 0 \quad \perp \quad x_i \geq 0$$

where $F_i(x)$ is zero, x_i equals zero, or both are zero. In the context of our economic model, this means that we explicitly treat the indirect utility functions of heterogeneous agents as opposed to specifying the utility maximization problem. The complementarity conditions represent both market clearing conditions and sorting conditions, i.e. comparable to a discrete choice modeling approach. Formulating the model as a complementarity problem provides advantages when integrating the economic decision framework with a transport model based on the MFD[3]. We are able to solve the entire model simultaneously rather than iterate between the economic and transport models to reach an optimal solution. We use the General Algebraic Modeling Software (GAMS) (GAMS Development Corporation, 2013) to solve the model.

3 Integrated model

Because we are interested in an aggregated economic/transportation model of an urban area, we refrain from specifying geography in the model. Rather, we are interested in distributional affects across different household types, h, modes, m, and subscription statuses, s. We allow for multiple household types, two modes (private vs. public) and two subscription statuses (season ticket holder vs. pay per use rider). We imagine the case where there are four possibilities that

[2] For instance, understanding to what extent public transportation pricing can influence regional wage and capital rates. However, because we focus on a single urban environment as opposed to a collection of urban areas linked through transportation networks, within urban area variation is assumed to be negligible.

[3] See Ferris et al. (1999) for another example of transportation modeling formulated as a mixed complementarity problem.

a given household can choose for their transportation needs. A household could choose the public mode with a subscription (season ticket holder), the public mode without a subscription (pay per use rider), the private mode without a subscription (e.g. car owners) or the private mode with a subscription to allow for the case that households choose to take both the public transportation and own a car. In order to differentiate between weekday commute and weekend travel we introduce the time of travel index, t, to indicate either the weekday or weekend. Our window of analysis is a complete week where agents are able to allocate time between leisure and travel during the week and weekends.

3.1 Economic model

The fundamental assumption in the model is that people will choose the mode, subscription status, and level of leisure which optimizes their level of utility. The demand for transportation is not explicitly represented in the utility function as a standard good. Rather, much more common is the demand for leisure, where the choice of transportation mode impacts the level of attainable leisure through time costs. The choice of transportation however, impacts the budget constraint of the household. Transportation demand is valued at the fixed and marginal monetary costs (that are exogenously specified) as well as by the leisure time implications which depend on the value of leisure for a given household. Thus, the two goods considered in the utility function are leisure demand and consumption of other goods.

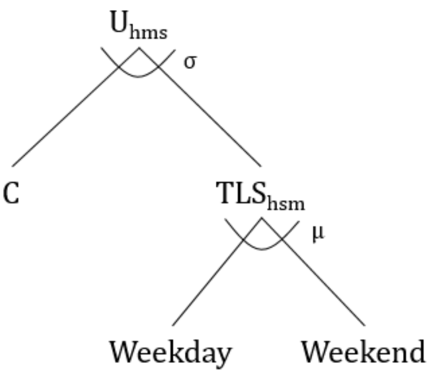
To capture the household trade-offs between consumption, c_{hsm}, and total leisure demand, TLS_{hsmt}, we use a calibrated share form nested constant elasticity of substitution utility function[4]. In this function, ψ is the elasticity parameter for time t and ρ is the elasticity parameter between consumption and leisure. We set the price of consumption to unity as the numeraire and thus do not treat consumption explicitly in the model equations[5]. Value shares for time dependent leisure consumption are denoted as γ_{hmt} for household h taking mode m in time t. θ_{hm} represents the value share of leisure relative to other consumption goods. In the following, variables and parameters with an overline denote benchmark values.

We assume the nested utility function will take the a form analogous to the tree diagram in Figure 2. Households gain utility from time dependent leisure demand and consumption. However, we exploit the nesting feature of CES functions by assuming the substitutability between leisure demand across time periods follows the elasticity of substitution μ where $\mu = 1 - 1/\psi$ and

[4]For more on the calibrated share form versions of standard CES functions, see Rutherford (2002).

[5]Because we formulate the model as a mixed complementarity problem, we explicitly define value functions which are in terms of *prices*. Therefore, because the other consumption good is the numeraire, its place is replaced by one.

Figure 2: Utility structure



substitutability between leisure and consumption follows the elasticity σ where $\sigma = 1 - 1/\rho$. For instance, it is quite likely that the ability to substitute leisure time on weekends for leisure time during the work day is small and different from say aggregate leisure to consumption.

Algebraically, households h with subscription s using mode m maximize the function:

$$U(c_{hsm}, TLS_{hsmt}) = \left(\theta_{hm} \left[\sum_t \gamma_{hmt} \left(\frac{TLS_{hsmt}}{\overline{tl}_{hmt}} \right)^{\psi} \right]^{\frac{\rho}{\psi}} + (1 - \theta_{hm}) \left(\frac{c_{hsm}}{\overline{c}_{hsm}} \right)^{\rho} \right)^{\frac{1}{\rho}}$$

where $\overline{tl}_{hmt}$ represents the benchmark value of leisure demand. Letting D_{hsmt} denote travel time and FC_{hsm} be the fixed cost of travel, agents are restricted via their budget constraint:

$$c_{hsm} + \sum_t \omega_{hsmt} TLS_{hsmt} = \overline{winc}_h + 2 \sum_t \omega_{hsmt} \left(\overline{l}_{hmt} - D_{hsmt} \right) - FC_{hsm} = I_{hsm}$$

It is assumed that agents have total time, $\overline{l}_{hmt}$ available each morning and evening to allocate to either leisure or travel and fixed exogenous weekly income levels $\overline{winc}_h$. ω_{hsmt} denote the equilibrium leisure prices for each sorting case for a given household. Value shares, θ_{hm}, are based on reference income levels $\overline{I}_{hm}$ and reference leisure prices $\overline{\omega}_{hmt}$[6]. Note that benchmark leisure prices are not simply benchmark wage rates, but rather depend on mode use, day of travel, etc. The value share of leisure relative to other consumption goods θ_{hm} is given by:

$$\theta_{hm} = \frac{2 \sum_t \overline{\omega}_{hmt} \overline{l}_{hmt}}{\overline{winc}_h + 2 \sum_t \overline{\omega}_{hmt} \overline{l}_{hmt}} = \frac{2 \sum_t \overline{\omega}_{hmt} \overline{l}_{hmt}}{\overline{I}_{hm}}$$

[6]See Appendix A.2 for our methods for discerning benchmark leisure prices.

The value share for each time period γ_{hmt} is given by:

$$\gamma_{hmt} = \frac{\overline{\omega}_{hmt}\bar{l}_{hmt}}{\sum_t \overline{\omega}_{hmt}\bar{l}_{hmt}}$$

The corresponding price of a unit of utility, with price of consumption set to unity is:

$$e_{hsm}(p) = \left(\theta_{hm} PLS_{hsm}^{1-\sigma} + (1-\theta_{hm})\right)^{\frac{1}{1-\sigma}}$$

where PLS_{hsm} is the composite price of leisure following the nesting structure in Figure 2. It is defined as:

$$PLS_{hsm} = \left[\sum_t \gamma_{hmt}\left(\frac{\omega_{hsmt}}{\overline{\omega_{hmt}}}\right)^{1-\mu}\right]^{\frac{1}{1-\mu}}$$

We write the level of indirect utility achieved as:

$$V_{hsm}(p,I) = \frac{I_{hsm}}{\overline{I_{hm}}e_{hsm}(p)}$$

The fundamental sorting condition of the model finds the maximum achievable equilibrium indirect utility level across all transportation options. Let U_h^* be the equilibrium level of indirect utility. We define the number of households h sorting into subscription s and mode m as N_{hsm} and $\overline{pop}_h$ as the benchmark number households from group h. The equilibrium conditions are

$$U_h^* \geq V_{hsm}(p,I) \quad \perp \quad N_{hsm} \geq 0 \quad \forall(h,s,m) \tag{1}$$

$$\sum_{sm} N_{hsm} \geq \overline{pop}_h \quad \perp \quad U_h^* \geq 0 \quad \forall h \tag{2}$$

Equation 1 ensures that the chosen subscription for household type h share the same level of indirect utility. If $U_h^* = V_{hsm}(p,I)$, then complementary slackness means that $N_{hsm} > 0$ which gives the number of households of type h choosing mode m with subscription s. If $V_{hsm}(p,I) < U_h^*$, then $N_{hsm} = 0$. Equation 2 requires that everyone of household type h must sort into some mode and subscription status when $U_h^* > 0$.

Rather than imposing strict caps on each time period on the number of people able to travel, we place limits on the amount of time a household can devote to leisure in a given period. The demand function for time dependent leisure follows from Roy's identity. Consider Appendix A.1 for the derivation. We allow for half of the leisure demand to apply in the mornings, and half in the evenings for a given time period t.

$$LS_{hsmt} = -\frac{\partial V_{hsm}(p,I)/\partial \omega_{hsmt}}{\partial V_{hsm}(p,I)/\partial I_{hsm}}/2 = \bar{l}_{hmt} V_{hsm}(p,I)\left(\frac{e_{hsm}(p)}{PLS_{hsm}(p)}\right)^{\sigma}\left(\frac{PLS_{hsm}(p)\overline{\omega}_{hmt}}{\omega_{hsmt}}\right)^{\mu} \quad (3)$$

The travel time costs D_{hsmt} follow from the three-dimensional MFD for given demand levels N_{hsm} as described in section 3.2:

$$D_{hsmt} = G\left[N_{hsm}, X_{hsmt}\right]$$

where G is a function describing the mapping between the number of people choosing to take a mode m with subscription s, the expected time costs, and X_{hsmt} which denotes a parameter set. The amount of time available then must be allocated to leisure and travel. Given this market clearing condition, we set ω_{hsmt} to be the shadow price of leisure:

$$\bar{l}_{hmt} = LS_{hsmt} + D_{hsmt} \quad \perp \quad \omega_{hsmt} \quad (4)$$

The equilibrium conditions of the demand side of the model are thus composed of equations (1), (2), (3) and (4).

3.2 Transport model

Our transportation sub-model uses an analytical representation of the three-dimensional MFD. Modal interactions drive how mode choice impacts aggregate network characteristics such as flow, density and average speeds achieved by private and public vehicle. In this equilibrium model, the three-dimensional MFD links the number of people choosing a mode N_{hsm}, to the resulting travel times D_{hsmt}.

In order to operationalize the three-dimensional MFD to characterize speeds, we use the analytical tools provided by Geroliminis et al. (2014). Specifically, we let the aggregate flow Q_t in the urban area be a function of the number of private and public vehicles. Let v^h_{mt} denote the density of vehicles of mode type m in time t, which is characterized as:

$$v^h_{mt} = \frac{\sum_{hs} \kappa_{mt} N_{hsm}}{\overline{ps}_m}/\overline{nw}_m \quad (5)$$

$\overline{ps}_m$ denotes the average number of passengers per vehicle for mode m, κ_{mt} gives the average occupancy rate for each mode and time period and $\overline{nw}_m$ gives the length of the aggregate network for a given mode. The shape of the three-dimensional MFD can be approximated by an exponential function. As in Geroliminis et al. (2014), we let flow be an exponential function of vehicle densities, allowing for interaction and nonlinear effects across vehicle types.

$$Q_t = a \left[\sum_m v^h_{mt} \right] e^{\left(\sum_m b_m (v^h_{mt})^2 + c \prod_m v^h_{mt} + \sum_m d_m v^h_{mt} \right)} \tag{6}$$

where (a, b_m, c, d_m) are the three-dimensional MFD parameters estimated with data via nonlinear least squares. Given this representation of flow, we can assume that the aggregate flow is characterized both by the flow of private and flow of public transport:

$$Q_t = \sum_m s_{mt} v^h_{mt}$$

where s_{mt} is the average speed in a given time period (weekday vs. weekends) for mode type m. In order to characterize mode specific speeds, an analytical relationship between speed levels across modes is necessary. To keep the model simple, we use a linear function as proposed by Geroliminis et al. (2014):

$$s_{pub,t} = s_{prv,t}\delta + \beta \tag{7}$$

where δ and β are parameters estimated outside of the model. Note, then that given these relationships, one can solve for mode specific speed explicitly. For private transport, this becomes:

$$s_{prv,t} = \frac{Q_t - \beta v^h_{pub,t}}{v^h_{prv,t} + \delta v^h_{pub,t}} \tag{8}$$

and for public transport:

$$s_{pub,t} = \frac{Q_t - \beta v^h_{pub,t}}{v^h_{prv,t} + \delta v^h_{pub,t}} \delta + \beta \tag{9}$$

We can therefore derive travel times from average network speeds and distances. We assume that households travel average distances $\overline{ds}_{ht}$. In order to allow for elastic demand for travel, we impose a calibrated demand function for distance which is dependent on leisure prices.

$$DS_{hsmt} = \overline{ds}_{ht} \left(1 - \epsilon_t \left(\frac{\omega_{hsmt}}{\overline{\omega}_{hmt}} - 1 \right) \right) \tag{10}$$

ϵ_t is the elasticity of distance with respect to leisure prices. We assume that the elasticity is zero for weekday travel (people will travel the same distance to work regardless of the mode or subscription) but allow for elastic demand on weekends[7]. The characterizations of speed from equations 8 and 9 allows us to determine travel times D_{hsmt} which factors into leisure demand.

[7] In subsequent versions of this paper, the value of this elasticity should be estimated from the data.

$$D_{hsmt} = DS_{hsmt}/s_{mt} + MC_{hsm}/\omega_{hsmt} \tag{11}$$

Marginal costs due to tickets, fuel costs etc., is included in equation 11 and is translated into time units using the price of leisure. The average speed in the network can be computed as just the weighted sum of vehicles traveling:

$$A_t^s = \frac{\sum_m s_{mt} v_{mt}^h}{\sum_m v_{mt}^h} \tag{12}$$

Equations (5), (6), (8), (9), (10) and (11) and (12) denote the transport side of the model equations.

3.3 Calibration procedure

There are two primary concerns when calibrating a model such as the one described above. First and foremost, we must specify a reference equilibrium which properly describes the reference sorting paradigm observable in the real world while allowing reference leisure prices to factor in travel times by a given mode m to satisfy the observable leisure demand conditions. Secondly, we must calibrate the three-dimensional MFD functional approximation to reference leisure and travel time trade offs.

In order to satisfy the first concern, we compute an idiosyncratic preference parameter which is household and mode specific and relates to unobservable differences in taste. This can be done in a variety of ways. For the sake of brevity, we describe an additive framework though having tested a multiplicative framework for robustness. Define a calibration parameter, ϕ_{hsm} such that:

$$V_{hsm}(p, I) = \frac{I_{hsm}}{\overline{I}_{hm} e_{hsm}(p)} + \phi_{hsm}$$

The idea behind this calibration routine allows ϕ_{hsm} to change in order to let the indirect utility level achieve a maximum which corresponds to benchmark sorting. Supposing we have a reference sorting equilibrium, $\overline{n}_{hsm}$, that gives the number of households sorting into mode m and subscription status s, ϕ_{hsm} is computed as:

$$N_{hsm} = \overline{n}_{hsm} \quad \perp \quad \phi_{hsm}$$

In words, when calibrating the model, we let ϕ_{hsm} fluctuate in order to satisfy reference data by serving as the complementary variable to an equation fixing sorting behavior. Moreover, the resulting price of leisure captures the reference travel times in the demand for leisure.

The second concern is considered via backward induction. We first estimate δ and β in our assumed modal speed relationship. Given these estimates, we can calculate the reference flow value which accords with our functional assumptions on the relationship of modal speeds. Finally, we use this reference value to estimate coefficients of our MFD specification by constraining possible solution values. We first line up the data by using a least squares balancing routine minimizing percent change in observable data to account for error in responder answers such that in the benchmark:

$$\min \quad \sum_{hmt}(D'_{hmt}/\overline{D}_{hmt} - 1)^2 + \sum_{hmt}(l'_{hmt}/\bar{l}_{hmt} - 1)^2 + \sum_{ht}(ds'_{ht}/\overline{ds}_{ht} - 1)^2 + \sum_{m}(s'_{mt}/\overline{s}_{mt} - 1)^2$$
$$\text{s.t.} \quad l'_{hmt} = D'_{hmt} + \overline{ls}_{ht} \quad \text{and} \quad s'_{mt}D'_{hmt} = ds'_{ht}$$

All notation with an apostrophe denotes variables in the optimization problem. This routine accounts for all inconsistencies in reported data. Given this relationship, δ and β could be estimated using constrained least squares where the estimated linear relationship must pass through benchmark speeds, s_{mt}. Given estimates for δ and β we then solve for observable flows given equations (9) and (10). Finally, equation (6) is estimated using constrained non-linear least squares to obtain parameter values which satisfy reference flow levels.

4 Case study

The model is applied to the city of Zurich based on empirical data. Zurich is the largest city in Switzerland with around 400,000 inhabitants in 2014 (Stadt Zürich, 2016c). The city has a road network length of approximately 740km and operates a public transport network with 14 tram lines and 31 bus lines on a total network length of around 280km. In 2014, the public transport operator counted around 305 million travelers entering its vehicles, which corresponds to approximately 800,000 persons a day (Stadt Zürich, 2016b,c).

One specialty of the Swiss public transport system is the option to subscribe to the nation-wide season-ticket *Generalabonnement* (GA) besides the regular option of subscription to local season-tickets. The GA provides for a fixed price unlimited access to the train, bus, ship and cable-car network with minor exceptions primarily for tourism. The subscription based access allows travelers with high travel demands access to public transport for a discounted price relative to the pay per use paradigm. In this case study, we pool all public transport subscriptions into the group called GA to have only one representative public transport subscription choice and analyze the effect of a varying GA subscription price on mode choice and average city speed.

4.1 Data

The data for the calibration of the household side of the proposed model stems from the 2010 Swiss mobility micro-census (Swiss Federal Statistical Office (BFS), 2012). Table 1 lists the variables extracted from this dataset and required reference prices in order to benchmark the model.

This data is complemented for the transportation side of the model with the three-dimensional MFD for the city of Zurich. The three-dimensional MFD stems upon data from the public transport operator, VBZ, and the traffic management authority of the city of Zurich, Dienstabteilung Verkehr (DAV) (Open Data Zürich, 2016a,b; Stadt Zürich, 2016a). In this paper, we estimate the three-dimensional MFD for the highlighted region in Figure 3. We explicitly note that the presented three-dimensional MFD is preliminary and that refinement of the model is in progress.

Demand side: Households

For this case study, we select households located in Zurich from the Swiss mobility micro-census. In the model, the notion of sorting and traveling at the household level is used. Some information in this dataset is only available for one person of the household, e.g. available leisure time or GA ownership. Therefore, to match the data to the model, we scale accordingly. Table 2 compares the subsample to the overall sample of the Swiss mobility micro-census.

Available leisure time is calculated from the trip diary of the micro-census. We subtract from 1440 minutes (one day) all travel times and all non-leisure activities, e.g. working, and assume 7 hours of sleeping. All home activities are defined as leisure. As aforementioned, all subscriptions to local season-tickets, GA and point-to-point season-tickets are merged into the variable GA ownership.

The reference weekly costs for mode subscription are computed by dividing the annual costs by the number of weeks per year. The annual car costs are computed by multiplying the number of cars and the annual mileage with the reference prices. The reference daily costs for GA

Table 1: Description of benchmark variables and parameters of the household side

Variable/Parameter	Description	Source
Household side		
Household income [CHF]	Monthly household gross income in CHF. Total income from working and other income sources.	Swiss Federal Statistical Office (BFS) (2012)
Number of cars and annual mileage	From the vehicle dataset. Required to calculate the annual car costs.	
Season-ticket ownership	Binary variable indicating the ownership of any kind of season-ticket	
Available leisure time [min]	From the trip diary, subtracting from 1440 min (one day) all travel times and non-leisure activties (working, shopping, business, delivery etc)	
Travel time private [min]	Extracted from the travel diary of the travel diary.	
Travel time public [min]	Extracted from the travel diary of the travel diary.	
Travel distance [km]	Extracted from the travel diary of the travel diary.	
Scale factors to whole population	Fraction of individuals in the dataset to the overall population in Switzerland to scale car densities to represent the whole population	Kanton Zürich (2016)
Benchmark prices		
Fix costs of car ownership	Accoring to the TCS,	Tanner and Bolduc (2014)
Variable car costs	Average fuel costs for the year 2010 are CHF 1.71	
Fix cost of season-ticket ownership	For each traveler the sum of all season-ticket expenses: GA 1st Class CHF 5970, GA 2nd Class CHF 3655, Zurich pass CHF 756, Fare reduction card CHF 185	SBB (2016); ZVV (2016a)
Variable public mode costs	Assuming each traveler traveling by public mode without season-ticket buys a daily pass for CHF 8.60, if he owns a fare reduction card CHF 6.00	ZVV (2016b)

Table 2: Comparing sample summary statistics of the full micro-census sample and the subsample for the city of Zurich

Variable	Unit	Micro-Census 2010	Zurich Sample
Households	-	59,971	2,420
Individuals	-	62,868	2,591
Households with cars	%	79.0	53.1
Travel indicators of interviewed household members			
Daily travel time	min	83.4	100.8
Daily travel distance	km	36.7	29.6
Daily car travel distance	km	24.3	12.2
Daily public transport travel distance	km	8.6	13.9

subscription are calculated as the weighted average subscription of all kind of season-tickets. The weights are the shares of the corresponding kind of season-ticket.

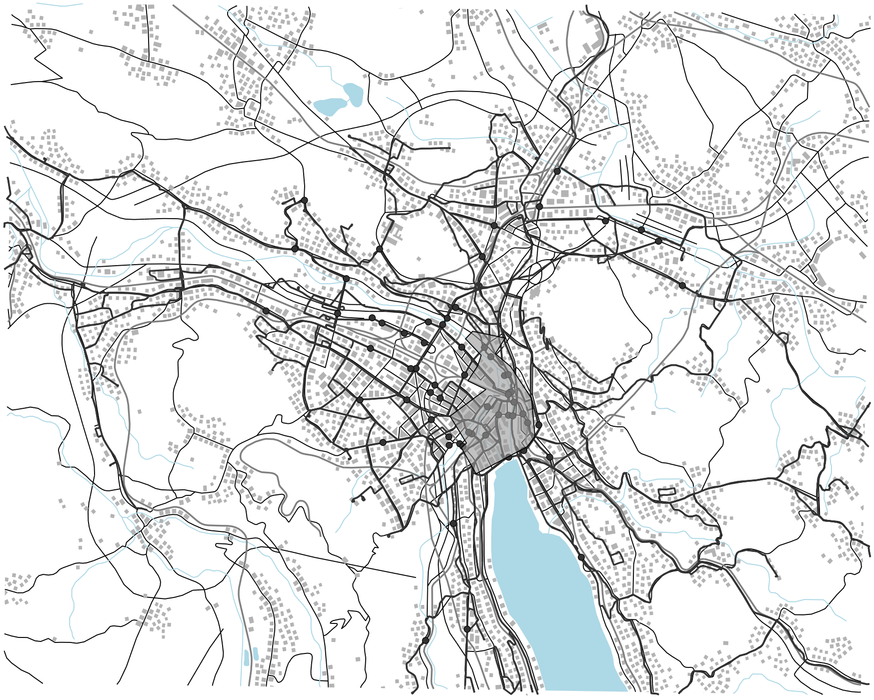
Transport side: Three-dimensional MFD

The data for private mode stems from loop detector data. The city of Zurich operates more 285 loop detectors at 192 locations. The loop detectors are usually located at intersections. The loop detector locations correspond to the black dots in Figure 3. The case study area covers 20 loop detector locations where the detector is located at least 20 m from the downstream traffic light. Each loop detector counts the passing vehicles and measures the time the vehicles spent on the detectors. The raw data gives the occupancy of the detector in percent. The vehicle density has to be inferred from the occupancy with the space-effective mean length of a car Geroliminis and Daganzo (2008). We use the value of 3 m to ensure a fit of the transportation and household side in the benchmark point.

The public transport operator VBZ operates around 470 vehicles and records the start and arrival time of each vehicle at each stop. From this data we are able to estimate the speeds and flows of circulating vehicles. The vehicle density is computed by dividing the number of circulating vehicles by the length of the public transport network (Schweizerisches Bundesamt für Landestopografie, 2011). In our case, the network length equals 32 km. The vehicle occupancy is calculated as the mean for weekdays and weekends (Open Data Zürich, 2016a)

The data for this analysis was recorded in the week from the 18th to the 24th of October, 2015.

Figure 3: Case study region. The gray shaded area corresponds to the urban center of the city of Zurich. The black dots are the intersections equipped with loop detector. The thick black lines correspond to the public transport network.



Source: Reproduced with the authorisation of swisstopo (JA100120) (Schweizerisches Bundesamt für Landestopografie, 2011). Loop detector locations from Stadt Zürich (2016a)

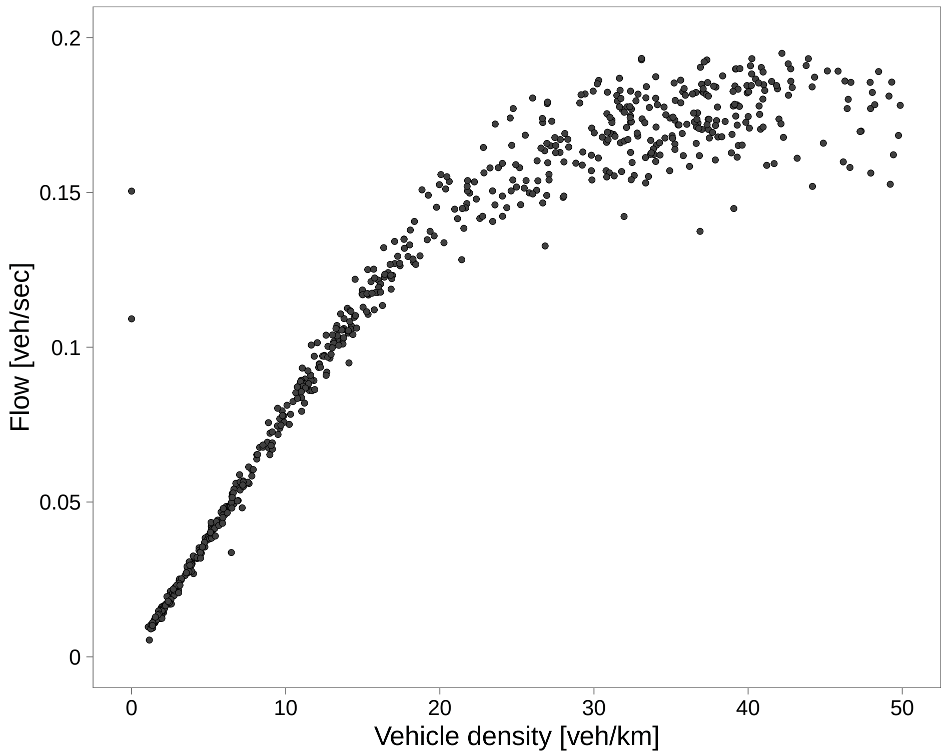
The aggregation interval has a length of fifteen minutes and is restricted by interval of the loop detectors. For the unimodal case, Figure 4(a) shows the MFD for the private transport and Figure 4(b) the time series of average public transport vehicle speed. At the moment, the loop detector data is unweighted by the link length which might explain the scatter in the MFD. The public transport data is weighted by the link length between two public transport stops. Our resulting three dimensional MFD is given in 3.

4.2 Benchmark model parameters

Consider Tables 4, 5 and 6 for the input data used for the model. Table 4 provides the basic household characteristics used to calibrate model equations. The table consists of weekly household income ($\overline{winc}_h$), estimated leisure prices (both $\overline{\omega}_{hmt}$ and $\overline{pl}_{hmt}$), observable morning or evening leisure availability ($\bar{l}_{hmt}$) and demand ($\overline{ls}_{ht}$), and value shares (θ_{hm} and γ_{hmt}) which

Figure 4: Scatter plots of the data used for the estimation of the three-dimensional MFD

(a) MFD from loop detector data for the private mode



(b) Average public transport vehicle speeds during the day

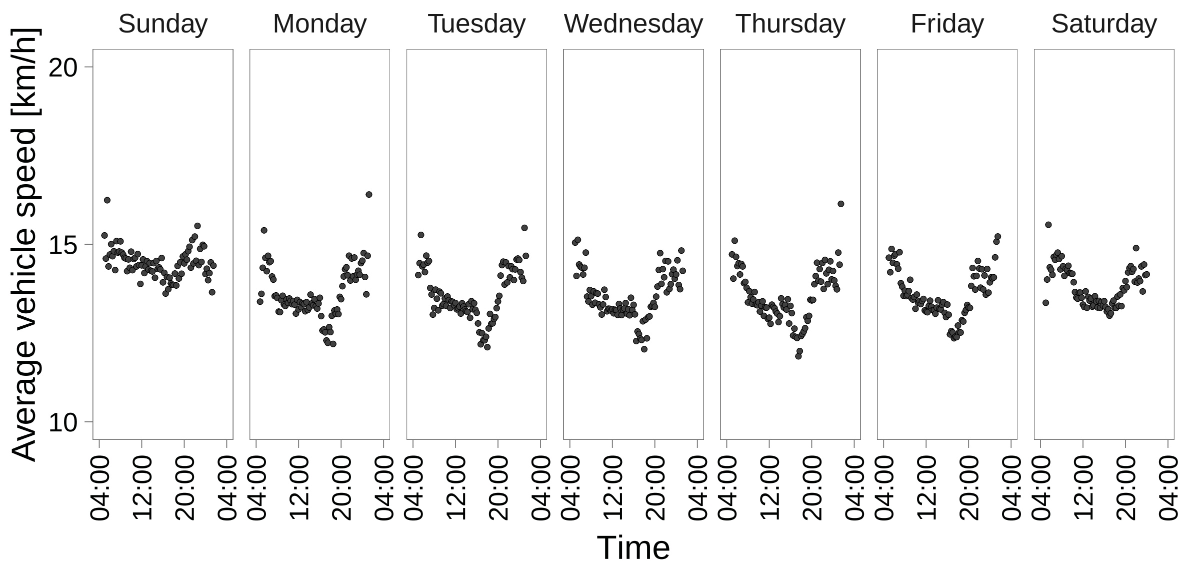
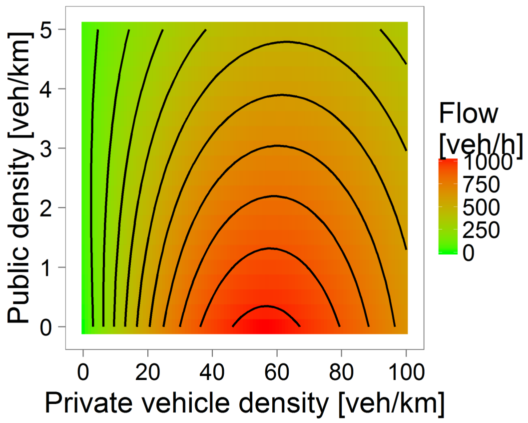


Table 3: Estimated three-dimensional MFD

(a) Parameter estimates

Parameter	Estimate
a	32.92
b	$-1.314 \cdot 10^{-4}$
c	-0.014
d	0.001
e	-0.003
f	-0.160

(b) Resulting three-dimensional MFD



Source: In the estimation of the MFD with non-linear least squares, extreme points corresponding to grid-lock situations are added to the dataset to improve the curve fitting.

are composed of the previously mentioned elements.

Estimates for benchmark leisure prices ($\overline{\omega}_{hmt}$) for household h taking mode m during time t comes from methods outlined in Axhausen et al. (2008). See Appendix A.2 for more detail on how the numbers were computed. Axhausen et al. (2008) make it clear that the value of travel time savings, or in our case, leisure depends on the type of mode, income level, and distance traveled. As is evident in Table 4, the level of benchmark prices for leisure generally increase with income, are larger for private versus public transport (with few exceptions which depend on distances traveled) and are larger during the week relative to the weekend. We assume that weekday travel is for commuting and weekend travel is for leisurely activities.

We assume in the initial calibration routine that $\overline{\omega}_{hmt}$ is the base benchmark leisure price, but it is then necessary to estimate altered leisure prices that are consistent with reference levels of leisure demand and travel times. As such, when estimating the idiosyncratic preference parameter, ϕ_{hsm}, we let the price of leisure fluctuate in order to satisfy the reference equilibrium. Levels of the resulting equilibrium leisure prices are $\overline{pl}_{hmt}$ which follow the same sorts of trends as $\overline{\omega}_{hmt}$ though with varying magnitudes. Reference levels of leisure availability $\overline{l}_{hmt}$ are calculated from the micro-census data as total time in the day, less time for sleep, work and less observable travel times τ_{hmt} (in Table 5). Notably, these levels denote morning or evening availability and are much higher for lower income classes with a potential reason being that lower income groups don't work as long of hours as the higher income groups (per the micro-census data). The final data elements are value shares, and computed as outlined above. As is evident, the value share for leisure is much higher for the lower income groups. Moreover, given the relative valuations

Table 4: Benchmark household data

	Unit	**Sets**		**Household Types**								
		Mode	*Time period*	1	2	3	4	5	6	7	8	9
$\overline{winc}_h$	[CHF/week]			250	750	1250	1750	2250	2750	3250	3750	5000
$\overline{\omega}_{hmt}$	[CHF/h]	*Public*	*Weekday*	25.45	28.05	37.55	38.82	42.42	43.14	43.36	44.12	51.43
			Weekend	12.92	12.84	15.58	14.75	14.92	17.64	19.13	16.56	17.92
		Private	*Weekday*	30.11	32.33	45.95	47.16	52.24	52.86	52.77	53.59	64.40
			Weekend	20.12	20.04	22.99	22.11	22.29	25.10	26.58	24.00	25.38
$\overline{pl}_{hmt}$	[CHF/h]	*Public*	*Weekday*	30.75	32.65	51.62	53.34	63.34	62.06	61.95	63.62	80.35
			Weekend	12.28	12.42	14.80	13.76	13.05	19.32	23.16	17.11	18.91
		Private	*Weekday*	36.54	37.76	63.50	65.22	78.69	76.32	75.55	77.65	101.23
			Weekend	20.03	20.18	23.48	22.17	21.13	29.82	34.63	26.96	29.29
$\bar{l}_{hmt}$	[h]	*Public*	*Weekday*	36.03	34.78	29.94	27.83	25.63	25.98	24.42	23.93	26.48
			Weekend	11.77	13.47	11.30	9.98	9.79	9.05	8.78	8.76	8.82
		Private	*Weekday*	36.09	34.83	30.01	27.90	25.71	26.06	24.49	24.01	26.57
			Weekend	11.91	13.60	11.55	10.19	10.00	9.42	9.26	9.06	9.21
$\overline{ls}_{ht}$	[h]		*Weekday*	33.09	32.49	25.87	24.03	21.30	21.86	20.56	20.15	21.41
			Weekend	11.36	13.06	10.55	9.36	9.14	7.93	7.34	7.85	7.65
θ_{hm}	[share]	*Public*		0.90	0.75	0.68	0.58	0.52	0.48	0.43	0.39	0.38
		Private		0.91	0.79	0.72	0.64	0.58	0.54	0.49	0.45	0.44
γ_{hmt}	[share]	*Public*	*Weekday*	0.86	0.85	0.86	0.88	0.88	0.88	0.86	0.88	0.90
			Weekend	0.14	0.15	0.14	0.12	0.12	0.12	0.14	0.12	0.10
		Private	*Weekday*	0.82	0.81	0.84	0.85	0.86	0.85	0.84	0.86	0.88
			Weekend	0.18	0.19	0.16	0.15	0.14	0.15	0.16	0.14	0.12

Source: Swiss Micro-census. Data calculations available upon request. h denotes hours and share denotes a value ranging between zero and one.

Table 5: Benchmark travel data

	Unit	Sets		Household types								
		Mode	*s/t*	1	2	3	4	5	6	7	8	9
$\overline{n}_{hsm}$	[1000s of hh]	*Public*	*GA*	2.31	15.84	19.05	10.83	5.41	3.01	1.80	1.00	1.60
			noGA	2.11	7.92	6.42	4.61	2.81	1.00	0.90	0.50	0.50
		Private	*GA*	0.40	3.41	8.52	7.32	5.71	5.61	3.01	1.80	5.41
			noGA	1.00	6.52	14.74	11.13	7.22	6.22	4.11	1.80	5.21
ϕ_{hsm}		*Private*	*GA*	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01
			noGA	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01
τ_{hmt}	[h]	*Public*	*Weekday*	2.94	2.29	4.06	3.80	4.32	4.12	3.86	3.79	5.07
			Weekend	0.41	0.40	0.75	0.63	0.65	1.11	1.44	0.91	1.17
		Private	*Weekday*	3.00	2.34	4.14	3.87	4.40	4.20	3.93	3.86	5.17
			Weekend	0.54	0.53	0.99	0.83	0.86	1.48	1.92	1.21	1.56
$\overline{ds}_{ht}$	[km]		*Weekday*	45.76	35.67	63.24	59.13	67.27	64.10	60.07	58.94	78.88
			Weekend	11.14	10.92	20.36	17.08	17.71	30.35	39.35	24.76	31.90

Source: Swiss Micro-census. Data calculations available upon request. hh denotes households, h denotes hours, km represents kilometers and share denotes a value ranging between zero and one.

of leisure time, the value share for weekdays is larger than that of weekends. Our assumed elasticity values for the substitutability of leisure across time periods and between leisure and consumption of other goods are 0.2 and 0.5 respectively.

Table 5 provides reference travel data. $\overline{n}_{hsm}$ denotes the reference sorting levels in thousands of households into the four categories of subscription status and mode choice (no GA public option, GA public option, no GA private option and GA private option). Due to the population distribution for Zurich, most of the households sorting into the GA come from the middle income groups. Moreover, it is unsurprising that many of the poorest group sort into the no GA public option while many of the richest sort into either the GA private option or no GA private option. ϕ_{hsm} denotes the calculated idiosyncratic preference parameter where we fix its level to zero for the Public GA option to tie down our reference sorting patterns[8]. τ_{hmt} represents reference travel times either in the morning or evening for a given household traveling using mode m in time t. This reference travel time fluctuates depending on distances DS_{hsmt} which follows a calibrated elastic demand relationship. Table 6 represents the estimated parameters for the transport model outlined above. The elasticity of demand for distance with respect to the value of leisure is given by ϵ_t. Reference speeds are given by $\overline{s}_{mt}$ which depend on mode and time of travel. Given the benchmark sorting paradigm, reference speeds for public transport are

[8]We fix ϕ_{hsm} for a certain sorting group to zero for the additive case and one for the multiplicative calibration case.

Table 6: Transport model data

	Unit	Mode	Time period	
			Weekday	*Weekend*
κ_{mt}	[share]	*Public*	0.09	0.06
		Private	0.11	0.06
$\overline{s}_{mt}$	[km/h]	*Public*	15.57	27.25
		Private	15.27	20.49
$\overline{v}^{h}_{mt}$	[veh/km]	*Public*	2.75	1.75
		Private	15	8
$\overline{Q}_t$	[veh/h]		271.88	211.57
ϵ_t			0	1
δ			2.24	2.24
β			-18.65	-18.65

Source: Swiss Micro-census and vehicle counting data. Data calculations available upon request. h denotes hours, km represents kilometers, veh denotes vehicles and share denotes a value ranging between zero and one.

actually slightly higher than for private. κ_{mt} provides the share of households that travel at a given moment and calculated based on an assumed benchmark density $\overline{v}^{h}_{mt}$ which is taken from the vehicle count data. $\overline{Q}_t$ denotes reference flows computed through our calibration routine, as are δ and β. Flows are higher during the week relative to the weekend.

4.3 Policy scenario

The main policy we consider refers to sorting and speed responses to the level of the subscription fee imposed for a GA. Specifically, we hold fixed the average levels of expenditures for ticket costs and car related costs, and allow the fixed fee for a GA subscription to vary. We can therefore study the characteristics of how the resulting equilibrium responds to changes in the fixed cost of GA subscription.

Consider Table 7 for an overview of the average weekly costs due to travel for each sorting category and household[9]. All data is in CHF. We let x denote the imposed level of the subscription fee for a GA in the simulation. Note that x factors into the GA public option as well as the GA private option where the household may find it advantageous to have both a GA and private

[9] We let average costs change with households to reflect the idea that richer households will buy different types of vehicles than poorer households. Note that the poorest income group spends quite a bit on private travel. This is largely due to a large portion of this income group being composed of retirees and students.

Table 7: Reference costs in Swiss Franks

	Households	GA		noGA	
		Public	*Private*	*Public*	*Private*
	1	x	251.42+x	1.49	244.88
	2	x	193.00+x	2.21	197.24
	3	x	214.9.0+x	4.65	197.79
$\overline{fc}_{hsm}$	4	x	213.18+x	6.96	219.97
	5	x	234.14+x	7.06	238.44
	6	x	243.00+x	4.60	241.63
	7	x	232.37+x	15.33	249.41
	8	x	223.82+x	3.16	229.71
	9	x	255.21+x	10.91	269.31
	1			14.02	11.25
	2	0.34		13.93	6.91
	3		0.64	20.21	7.56
$\overline{mc}_{hsm}$	4			15.57	8.54
	5			16.11	6.16
	6			24.03	9.65
	7			26.24	8.13
	8			22.49	8.61
	9			31.79	10.73

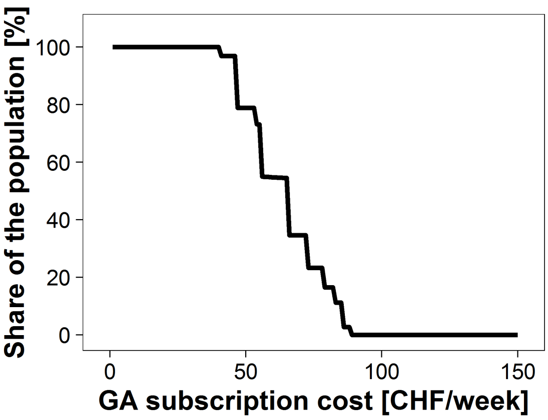
Source: Based on the data given in Table 1. Values in CHF/week. $\overline{fc}_{hsm}$ represents benchmark fixed costs and $\overline{mc}_{hsm}$ denotes benchmark marginal costs. Data calculations available upon request.

vehicle. The level of the private expenditure is different on average for those sorting into the GA private option relative to the no GA private option in the benchmark. We let $x \in [0, 150]$ in CHF increase monotonically in increments of CHF 1. Note that very small marginal costs exist in the data for subscription to public transportation as a few individuals have to buy extra tickets where the GA does not apply.

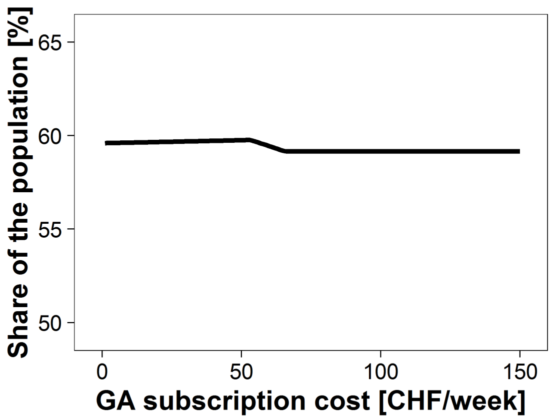
Households must choose one of the following categories: no GA public (pay per use of public transportation), GA public (one time fixed fee for public transportation), no GA private (drive a person vehicle and forgo public transportation use) and a GA private option (use both private and public transportation). The benefits of subscribing to a GA means that marginal costs of transportation are essentially zero. However, one is faced with a high fixed cost. The decision therefore, will depend on a given household income level, the level of the fixed cost and the value of leisure in a given time period.

Figure 5: Simulation results of the model by varying the GA subscription price

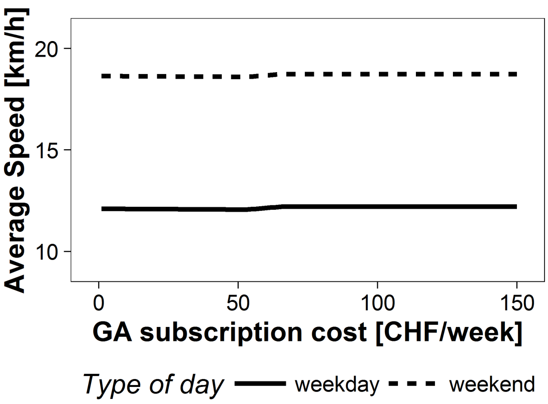
(a) Share of GA subscription



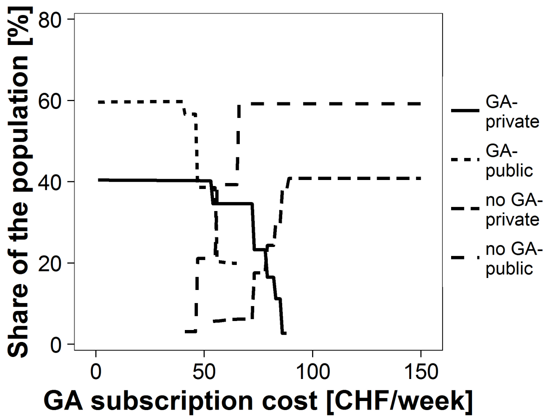
(b) Share of public transport use



(c) Average speed in the city



(d) Categorical choices of households for sorting into subscription and mode



4.4 Results

Figure 5(a) shows the change in the share of households choosing a GA subscription and Figure 5(b) the change in the the public mode choice with respect to changes in the fixed cost of subscription status for a GA. The share of households choosing to opt into the GA subscription remains at 100% until 40 CHF (weekly subscription cost). Thereafter, the share of households choosing a GA declines until roughly 90 CHF per week where no one finds it advantageous to sort into a membership. The right side of the figure notes that the share of households choosing public transportation increases by a quarter of a percent until 53 CHF, falling 1% thereafter once the share of GA subscribers falls to zero. The shift in public transport demand is only slight because the model indicates that individuals that would have chosen the GA for public use only transition into the no GA public option (ticketing).

Figure 6: Average distance traveled on weekends

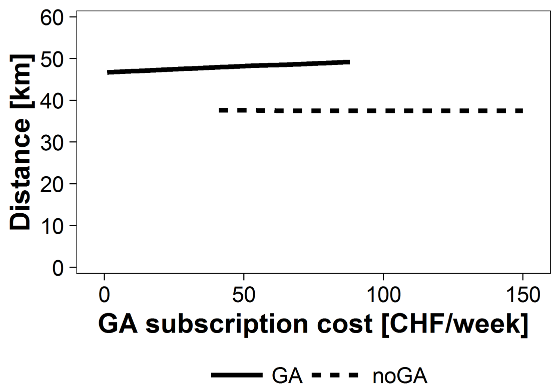


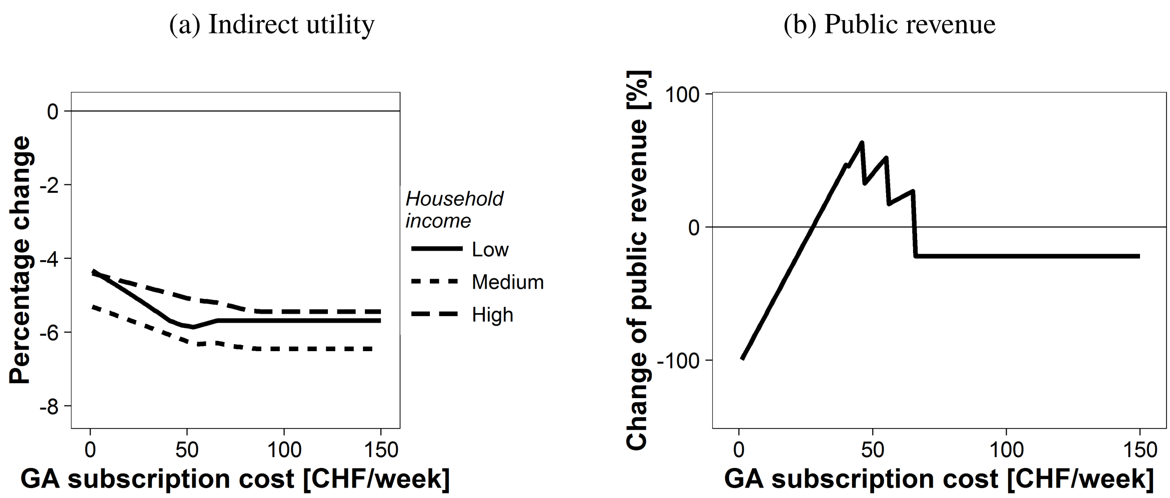
Figure 5(c) represents the average speed achieved in the network for both during the work week and weekends where individuals travel for leisure. The computed levels of speed largely stay unchanged from approximately 19 km/hr during the weekends and 12 km/hr during the week. The reason can be seen in Figure 5(d), which gives the share of each household choosing each subscription and mode option. Initially, everyone either chooses the GA public or private option. Once the level of the subscription costs to the GA becomes too expensive, households find it advantageous to switch to the non-GA public or private option. However, as is evident, the proportion of people choosing the public or private mode options stays largely unchanged.

We allow for elastic demand for travel on the weekends. As a result, Figure 6 shows the average distance traveled by households between a GA versus no subscription on the weekends. Distance is fixed to the benchmark for weekday travel by assuming an elasticity of demand of zero. When the GA is cheap, everyone is sorting into this option, hence no distance traveled for the no GA case. As the fixed cost of subscription increases, the value of leisure decreases, leading to increases in the distance traveled and thus travel times. Once households begin to make the transition to their non-subscription choices, the distance traveled decreases due lesser increases in the value of leisure for the pay-per-use choices. Once sorting behavior ends for high fixed costs of subscription, the distance traveled remains constant.

While understanding the effects of pricing on individual choice of membership status and mode choice is an interesting interaction, one needs to be concerned with the burden such choices have on individuals and the public authority whom is responsible for providing the funds for public infrastructure. Consider Figure 7 for the effects of varying levels of fixed cost can have on household indirect utility levels (Figure 7(a)) and public revenue (Figure 7(b)).

As a measure of distributional consequences, we plot the percentage change in indirect utility

Figure 7: Effects of GA subscription pricing on households utility and public budget



levels for aggregate household types in Figure 7(a). Low income is classified as the first three poorest income groups, middle income the next three groups and upper income as the three richest representative households. The change in utility will always be negative in this case because the model is calibrated to zero fixed costs of transportation (treated as a sunk cost in calibrating to household's observable choices). Therefore, any positive fixed cost will negatively impact households from the benchmark in forcing them to choose between mode and membership categories. As to be expected, richer households face less of a burden than poorer households because the level of the fixed cost is much smaller relative to total income. However, middle income households are burdened relative to benchmark indirect utility levels, the most.

The right side of Figure 7 gives the percentage change in public revenue received through public transportation from the benchmark (Figure 7(b)). Initially, if the fixed cost of GA membership is zero, everyone sorts into this membership status. Therefore, no public revenue is raised. The percentage change in revenue rises steadily with increases in the fixed cost of public transportation as the level of households sorting into a GA remains constant. The change in total public revenue attributed to transportation reaches a maximum at 46 CHF/week and falls thereafter as households begin to no longer opt for membership status. The percentage change in public revenue remains negative once no household choose to opt into a GA membership at approximately -25% of benchmark revenue. Note that in such case, public revenues are still accrued by the no GA public case, in which individuals choose to pay per use rather than through subscribing.

5 Discussion

Based on recently developed theory in the transportation literature on the MFD in urban networks, we link a calibrated general equilibrium model with an estimated three-dimensional MFD to study the implications of pricing public transport season-ticket subscription on speeds, household mode and subscription choice and public revenue. This research adds to the literature on the external costs due interactions between private and public transport vehicles and the benefits of the provision of public transport (e.g. Smeed (1961); Adler and van Ommeren (2016)). We find that speeds in the urban network can be improved given certain levels of subscription costs for public transportation. However, higher speeds come at a higher cost to poorer relative to richer households. Moreover, higher subscription prices generally result in more public revenue until the subscription price becomes too high to warrant participation in membership.

Policy implications

This model allows us to derive implications for public transport pricing in the context of average urban speed and the interaction of private and public transport vehicles. A number of things can be learned given the simulation outcomes. We have detailed on how the equilibrium response changes given different subscription pricing for public transportation. The social planner's problem of determining the optimal level of the subscription price is another matter entirely. Determining the optimal level of public transport pricing will be specific to the urban area and depend on achievable speeds, utility costs, and public revenue. In precisely what fashion these considerations factors into the social planner's problem is a matter of debate. For instance, the highest average speed is reached at roughly 70 CHF/week for a GA subscription. However, indirect utility changes show that flat public transportation pricing is regressive, in that it disproportionately burdens poorer households. Indirect utility levels for the poorest household reaches a minimum at roughly 50 CHF/week, with other households reaching minimums at roughly 80 CHF/week. Additionally, public revenue reaches a maximum at roughly 40 CHF/week. While this is just public revenue associated with transportation, one must also consider if this additional revenue can be used to improve transportation infrastructure or elsewhere in the economy to lessen burdensome taxes leading for a potential double dividend. The optimal public transport pricing strategy thus depends on the weighting of all these interactions and lies in the duty of the social planner. Implementing these interactions in a general equilibrium model to find the optimal pricing strategy is subject to further research.

Notably, we considered only one city (Zurich) in our model but pricing private and public transport is a nation-wide affair. The car and the GA can be used everywhere and individuals commute and drive into cities and between cities. Pricing within Zurich will not only depend on the implications in Zurich but in other Swiss cities as well. For instance, if it becomes more expensive to commute to Zurich from nearby smaller cities, whom have sizable portions of the local population that commute to Zurich for work, the price of public transportation will affect job and residence locational choices. Moreover, there could be agglomeration effects. Further research should explore the implications of including many urban areas with separate three-dimensional MFDs to characterize the optimal pricing of the subscription based public transportation.

The proposed model has limitations worth mentioning. First, the model has limited temporal resolution, except for weekend and weekday travel, that does not allow travelers to reschedule or reroute their trips but rather adjust distance traveled on the weekend. The model may be improved by introducing multiple time slots for departing and arriving. Second, the current formulation does not explicitly treat car ownership, which is also important for policy makers (e.g. tax income and pollution). An extension of the model could also consider the concept of car-sharing to avoid large sunk costs. Third, the model assumes all vehicle crowding, public transport timetables, etc. are embedded in the estimated MFD curve. More attention to this may improve the model fit. Forth, the model does not distinguish between the nation-wide season-ticket GA and a local season-ticket. As both give different utilities for travelers and have different prices, considering both will improve the model. Important to mention, some companies subsidize the commute of employees, which has not been considered yet and will be more of interest if production is introduced to the model. Finally, the framework can be extended by solving for speed maximizing prices and taxes directly.

Our modeling efforts also differ in scope from past studies. We use a macroscopic transportation model, from a city level perspective, linked with an equilibrium choice model to understand the implications of public transportation pricing on speeds, public revenue and utility levels of city constituents. The model is solved as a mixed complementarity problem for simultaneous solution of both the economic model and transportation model. As a result, our model is solved with very little computational costs due to our macroscopic focus. Moreover, we avoid heuristic algorithms for solving the model which limits the possibility for mistakes.

Data limitations

A few data limitations bear mentioning. At the moment, the three-dimensional MFD has at least three limitations: Too few loop detectors with a resolution of 15 min intervals, the single measurements are unweighted, and occupancy to density scale factor is not calibrated for the city of Zurich. Further research must overcome this to provide a three-dimensional MFD from which quantified policy recommendations can be drawn. This requires an increase in the coverage of measurement points and in the measurement interval, e.g. around five minutes length (Ortigosa et al., 2015; Geroliminis and Daganzo, 2008).

In addition, this paper could be improved by estimating MFDs for homogeneously congested neighborhoods within a city. One of the basic assumptions of the MFD is that it emerges in such environments. With this extension of multiple MFDs for a city, the macroscopic model will result in more robust results.

On the household side of the model, the leisure demand can improved by using synthetic populations generated from census data. This approach would overcome the limitation that we assume an average number of individuals per household and scale accordingly. The elasticities used for this model should be estimated from Swiss data in order to capture the trade-offs in the decisions of households.

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A Appendix

A.1 Deriving Demand Functions

In order to derive the demand functions in the model (specifically, equation 4), we employ Roy's identity. This identity relies on the duality between utility maximization and expenditure minimization and provides the Marshallian demand functions by dividing the derivatives of the indirect utility function with respect to prices and income respectively. Therefore, the demand for leisure is written:

$$l = -\frac{\partial V(p,I)/\partial w}{\partial V(p,I)/\partial I}$$

In our case, the indirect utility takes the form:

$$V_{hsm}(p,I) = \frac{I_{hsm}}{\overline{I}_{hm} e_{hsm}(p)} = \frac{I_{hsm}}{\overline{I_{hm}}} e_{hsm}(p)^{-1}$$

where the price of a unit of utility is:

$$e_{hsm}(p) = \left(\theta_{hm} PLS_{hsm}^{1-\sigma} + (1-\theta_{hm})\right)^{\frac{1}{1-\sigma}}$$

and the composite price of leisure is written as:

$$PLS_{hsm} = \left[\sum_t \gamma_{hmt} \left(\frac{\omega_{hsmt}}{\overline{\omega}_{hmt}}\right)^{1-\mu}\right]^{\frac{1}{1-\mu}}$$

Computing each derivative (and canceling out terms), we get:

$$\frac{\partial V_{hsm}(p,I)}{\partial \omega_{hsmt}} = \frac{I_{hsm}}{\overline{I_{hm}}} \left(\frac{1}{\sigma-1}\right) e_{hsm}(p)^{\sigma-2} \theta_{hm} \left(\frac{1-\sigma}{1-\mu}\right) PLS_{hsm}^{\mu-\sigma} (1-\mu) \gamma_{hmt} \left(\frac{\omega_{hsmt}}{\overline{\omega}_{hmt}}\right)^{-\mu} \overline{\omega}_{hmt}^{-1}$$

$$= -I_{hsm} \overline{I}_{hm}^{-1} e_{hsm}(p)^{\sigma-2} \theta_{hm} PLS_{hsm}^{\mu-\sigma} \gamma_{hmt} \left(\frac{\omega_{hsmt}}{\overline{\omega}_{hmt}}\right)^{-\mu} \overline{\omega}_{hmt}^{-1}$$

$$\frac{\partial V_{hsm}(p,I)}{\partial I_{hsm}} = \overline{I}_{hm}^{-1} e_{hsm}(p)^{-1}$$

Remembering the minus sign and dividing the first by the second, total leisure demand in time period t is denoted as:

$$TLS_{hsmt} = \frac{\theta_{hm}\gamma_{hmt}}{\overline{\omega}_{hmt}} \frac{I_{hsm}}{e_{hsm}(p)} \left(\frac{e_{hsm}(p)}{PLS_{hsm}}\right)^{\sigma} \left(\frac{PLS_{hsm}\overline{\omega}_{hmt}}{\omega_{hsmt}}\right)^{\mu}$$

where TLS_{hsmt} denotes the total amount of leisure demanded in time period t. Remembering that:

$$\theta_{hm} = \frac{2\sum_t \overline{\omega}_{hmt}\bar{l}_{hmt}}{\overline{winc}_h + 2\sum_t \overline{\omega}_{hmt}\bar{l}_{hmt}} = \frac{2\sum_t \overline{\omega}_{hmt}\bar{l}_{hmt}}{\bar{I}_{hm}} \quad \text{and} \quad \gamma_{hmt} = \frac{\overline{\omega}_{hmt}\bar{l}_{hmt}}{\sum_t \overline{\omega}_{hmt}\bar{l}_{hmt}}$$

we cancel out terms to get:

$$TLS_{hsmt} = 2\bar{l}_{hmt}V_{hsm}(p, I)\left(\frac{e_{hsm}(p)}{PLS_{hsm}}\right)^{\sigma} \left(\frac{PLS_{hsm}\overline{\omega}_{hmt}}{\omega_{hsmt}}\right)^{\mu}$$

Note, then that:

$$TLS_{hsmt}/2 = LS_{hsmt} = \bar{l}_{hmt}V_{hsm}(p, I)\left(\frac{e_{hsm}(p)}{PLS_{hsm}}\right)^{\sigma} \left(\frac{PLS_{hsm}\overline{\omega}_{hmt}}{\omega_{hsmt}}\right)^{\mu}$$

where LS_{hsmt} is the leisure demanded either in the morning or evening of a given time period t.

A.2 Leisure Prices

Axhausen et al. (2008) estimate the value of travel time savings econometrically for Switzerland. They use an elasticity based formulation in order to calculate how the value of travel time savings is affected by varying income levels and distances traveled. From their paper, we use the following relationships for benchmark leisure prices. Notably, the paper shows that the value of travel time savings depends on the purpose of a trip. Therefore, we take estimates for the weekday to be commuting trips and estimates for the weekend to be leisure trips. Thus, for the private mode (pr) on a weekday (wd):

$$\overline{\omega}_{h,pr,wd} = 30.64\left(\frac{\overline{yinc}_h}{84656}\right)^{0.1697} \left(\frac{\overline{ds}_{h,wd}}{23.22}\right)^{0.5949} \left(\frac{\overline{ds}_{h,wd}}{23.22}\right)^{-0.1321}$$

where $\overline{yinc}_h$ denotes annual income. For the private mode on a weekend (we), there is no longer a statistically significant income effect according to the paper:

$$\overline{\omega}_{h,pr,we} = 29.2\left(\frac{\overline{ds}_{h,we}}{60.27}\right)^{0.5949}\left(\frac{\overline{ds}_{h,we}}{60.27}\right)^{-0.3744}$$

For the public (pu) transport mode on the weekday:

$$\overline{\omega}_{h,pu,wd} = 27.81\left(\frac{\overline{yinc}_h}{84656}\right)^{0.1697}\left(\frac{\overline{ds}_{h,wd}}{23.22}\right)^{0.5949}\left(\frac{\overline{ds}_{h,wd}}{23.22}\right)^{-0.2368}$$

And for the public mode on a weekend (no statistically significant income effect as in the private case):

$$\overline{\omega}_{h,pu,wd} = 21.84\left(\frac{\overline{ds}_{h,we}}{60.27}\right)^{0.5949}\left(\frac{\overline{ds}_{h,we}}{60.27}\right)^{-0.2837}$$