Estimation of macroscopic traffic variables in cities with big but sparse multi-sensor data

Raphaël Lamotte

Anastasios Kouvelas

Nikolas Geroliminis

STRC

LUTS, EPFL

Estimation of macroscopic traffic variables in cities with big but sparse multi-sensor data

Raphaël Lamotte, Anastasios Kouvelas, Nikolas Geroliminis
Urban Transport Systems Laboratory
École Polytechnique Fédérale de Lausanne
GC C2 389, Station 18, 1015 Lausanne, Switzerland
phone: +41-21-693 24 81
fax: +41-21-693 53 97
{raphael.lamotte,tasos.kouvelas,nikolas.geroliminis}@epfl.ch

May 2016

Abstract

As urban centers become persistently denser and wider, research on realistic macroscopic models has steadily gained momentum over the last decades. Many efforts have been carried out on the modeling of traffic in urban environments, but also on the suitability of such models for real time congestion management and their potential policy implications. Surprisingly however, there are very few works in the literature that aim at estimating the macroscopic variables that are used as inputs for all other applications (e.g. real-time control, dynamic routing).

Keyvan-Ekbatani *et al.* (2013), Ortigosa *et al.* (2013) and Leclercq *et al.* (2014) have studied how the number of detectors in a city or their location within the links can influence the quality of estimates for different applications. While this is certainly useful in the case where new detectors can be added in a network, in some other cases cities already have their roads instrumented with detectors and might not be willing to invest in a significant expansion of their network coverage. Fortunately, the emergence of smartphones and embedded systems nowadays provide rich amounts of traffic data that can be used as an additional source of measurements.

On one hand, probe vehicles (also called Lagrangian sensors) travel over the entire network but have a highly time-dependent penetration rate; this creates a significant uncertainty for this type of measurements. On the other hand, inductive loop detectors (Eulerian sensors) are in fixed locations and therefore only provide a partial image of the network. Thus, the challenge of the current work is to combine both sources of data in order to provide low-uncertainty and unbiased estimates of macroscopic traffic variables (i.e. vehicle accumulation and flow) in real-time. There are a few works that deal with traffic estimation in highways (Patire *et al.*, 2015), or with travel time estimation in urban networks (Hofleitner *et al.*, 2012) but to the best of our knowledge, the combination of Eulerian and Lagrangian sensors to estimate accumulation and flow in urban networks remains unstudied. The methodology presented here applies a classical Bayesian framework for real-time data fusion of urban multi-sensor data. Finally, the estimation scheme is tested in a micro-simulation environment for which the ground truth is known.

Keywords

Data fusion, Arterial networks, Eulerian sensors, Lagrangian sensors, Production, Density

1 Introduction

The last decade there has been an increasing attraction of interest in topics related to the Macroscopic Fundamental Diagram (MFD) in the research community. Network-wide or region-wide MFDs provide a functional relationship between macroscopic traffic variables that has been used both for modelling the dynamics of traffic in cities and also designing control methodologies for efficient traffic management. It postulates a significantly useful tool for zone-based traffic control (Keyvan-Ekbatani *et al., 2012,* Geroliminis *et al., 2013,* Aboudolas and Geroliminis, 2013) and network-wide performance measures (see e.g. Zheng and Geroliminis (2013), Tsekeris and Geroliminis (2013)). On the other hand, the measurement or observation of the required variables in real test cases has been often overlooked in the literature. Most of the existing works related to MFD consider mostly fixed sensors (Eulerian) and the main question that they address is to determine the required network coverage. However, in the real world, detectors are usually already installed in the studied networks and the main questions that arise when collecting data from a network are the following: a) if there is a possibility to add new loop detectors in a city, what would be the optimal locations for them? b) is it cost effective to invest in installing more loop detectors or the city should prioritize investments in probe data (Lagrangian sensors)? and c) how someone can evaluate the level of information that the installed sensors provide and is this sufficient for network-wide real-time traffic management?[1]

As a matter of fact, there exists a vast literature on the topic of multi-sensor data fusion for traffic state estimation (El Faouzi *et al., 2011*). Common applications in the field of intelligent transportation systems include traveller information systems, automatic incident detection, driver assistance, motorway or intersection control, road safety, traffic demand estimation, traffic forecasting and monitoring and position estimation. Nevertheless, there is no work addressing the three aforementioned questions. Leclercq *et al.* (2014) compares different kinds of data sources for estimating the MFD parameters. Also, there are a couple of works that explore the data fusion of Lagrangian and Eulerian sensors data to estimate travel times for arterial networks (Hofleitner *et al., 2012,* Hunter *et al., 2013*) and motorways (Patire *et al., 2015*). In (Wu *et al., 2015*) a convex optimization formulation is derived to estimate route flows on an arterial network by utilizing Lagrangian traffic sensors. Note that Lagrangian sensors are not necessarily representative of the entire network flow (e.g. expensive cellphone contracts are more likely to appear in some neighbourhoods than others, some services are more likely to be used by young people rather than by the elderly, etc.). As a consequence, comparison of loop detector measurements and probe measurements might help to determine whether such a bias exists. In Yuan *et al.* (2014) the authors combine Lagrangian and Eulerian sensors to estimate

---

[1]Essentially, this depends on the data requirements of the applied control strategy, but currently there is no automated way to assess this problem and in reality application-specific engineering judgment is used.

densities and speeds on a motorway stretch which also includes ramps.

Another important issue for the research community and practitioners is the selection of locations for installing new detectors. Currently, cities use human judgement and engineering experience in order to decide how to instrument their transportation infrastructure. However, this could be modelled as an optimization problem and try to maximize the estimation "power" provided by a given infrastructure. There are some studies related to this topic (Ortigosa *et al.,* 2013) but there is no solid methodology that can answer this question. If for example we have a city that does not have any sensors and we have a budget to instrument some percentage of the roads, how can someone define the optimal locations given an objective function? This is a crucial issue and the estimation method that is developed in this work can provide some useful insights in this direction. In Keyvan-Ekbatani *et al.* (2013) an MFD-based gating controller is tested under different detector coverage in a city and it is shown that if this selection is done in a smart way, the controller can be efficient even with a very small number of detectors. However, in this case the selection of the measurements locations is made by people that have a very good knowledge of the network and not by an automated procedure. Nevertheless, in a large-scale network with multiple congestion pockets and various daily traffic patterns this procedure cannot be done manually. A notable work in this direction is Tsekeris and Stathopoulos (2006), where a principle component analysis methodology has been applied to identify the critical measurement points in a network. However, this approach assumes that the data from different points is already available and the selection is based on the measurements themselves and not some attributes of the network.

In the current work we focus on the estimation of variables that are needed in order to apply MFD-based control in real networks. Given the data availability in realistic field implementations (e.g. based on our experience with a study in the city of Geneva) we study the problem of estimating production (total travelled distance in veh·km)[2] and accumulation at the network level (the same method can be applied at the zone level). Control methodologies that have been proposed recently (Ramezani *et al.,* 2015*,* Kouvelas *et al.,* 2015) assume that these measurements are available in real-time, but this is not the case in most of the big cities around the world; as a consequence, an estimation layer is necessary towards the successful implementation of these strategies in real life. Usually, traffic operators decide to install detectors in the most congested links of the networks, and as a result a simple aggregation of the loop detectors data might not be sufficient for the state of the network. An average of the flows of the most congested links is not representative of the network flow and the same stands for occupancy measurements; there is a bias in the estimation of the state of the system that depends on the location of the detectors. The use of probe data (which are assumed to be homogeneously distributed in the network) can

---

[2]Note to the reviewer: the current version of the document only includes the estimation of accumulation. Estimation of production will be added in a later version.

enhance the estimation and help correct this bias.

Another objective of our work is to demonstrate the estimation efficiency for different penetration rates of probe data and loop detector coverage and analyse the trade-off between investing in the one sensor or the other. This can provide a support system to traffic operators and authorities and guide their future decision making about network instrumentation. Intuitively, a way to achieve that is to try to quantify the standard deviation of our predictions and investigate the impact of different parameters (e.g. probes penetration rate, number of detectors) on this measure of effectiveness. Locally, we can do this numerically (i.e. try different penetration rates and coverage percentages and provide a lookup table); however, an analytical approach also seems possible. In the current paper we present the estimation methodology and some preliminary results. Synthetic data for probe vehicles and loop detectors from a microscopic simulation have been used, where the ground truth (i.e. total network accumulation and production) is known and used for validation purposes, but similar analysis can be conducted with real data.

2 Methods and models

2.1 Methodology

In contrast to the vast literature on data fusion, the methodology followed in this work can be seen as a rather straightforward Empirical Bayes approach. The quantity to be estimated (i.e. accumulation or production) is not directly measured but it is stochastically related to various other quantities that can be easily measured (e.g. occupancy, counts, or speeds of probe vehicles). Hence, it is considered here as a latent variable.

The three main steps of the proposed methodology consist of (i) formulating the observation equations, i.e. the functional forms for the stochastic relations between these quantities, (ii) specifying the prior distributions, and (iii) estimating the maximum likelihood estimates (MLE) for the parameters values of observation equations, given the observed data. Let x and y denote respectively the vector of variables that are observed and that need to be estimated (i.e. the latent variable). The first step consists in finding a function p_{obs} such that $p_{obs}(x|\beta, y)$ is a good approximation of the probability of observing x given the state of the system y and the parameters β. The second step consists in specifying a priori the probability density functions $p(y|\beta)$ (note that all probability density functions are denoted by p for simplicity, except the

observation equation). Then, the MLE estimates of the parameters β are given by:

$$\beta_{\text{MLE}} = \underset{\hat{\beta}}{\arg\max} \prod_{i \in H} p(x_i \,|\, \hat{\beta}) = \underset{\hat{\beta}}{\arg\max} \prod_{i \in H} \int p_{obs}(x_i \,|\, \hat{\beta}, y_i) p(y_i \,|\, \hat{\beta}) dy_i, \tag{1}$$

where H is the set of times for which historical data is available. Although this is an approximation, this assumes that all realizations of the experiment are independent.

Once these two steps are completed, the observation equations associated with the MLE estimates are assumed to be the true model[3]. Hence, for any time sample at which the observations are available, the quantity to be estimated should be distributed according to

$$p(y \,|\, \beta_{MLE}, x) = p_{obs}(x \,|\, \beta_{MLE}, y) \frac{p(y \,|\, \beta_{MLE})}{p(x \,|\, \beta_{MLE})}, \tag{2}$$

where $p(x \,|\, \beta_{MLE})$ is simply a normalizing constant.

2.2 Utilized synthetic data

The data used in this work is synthetic and was obtained using the micro-simulation environment AIMSUN and a pre-calibrated replication of a central area of the city of Barcelona, Spain. Virtual loop detectors are emulated on all links, at mid-distance between each extremity and produce for each measurement interval of 90 s noise-free measurements of the occupancy rate, the total flow and the flow of probe vehicles recorded within this time period. Every time a vehicle is generated, it has some constant probability of being a probe vehicle (Bernoulli trial). To obtain various penetration rates (i.e. various percentages of probe vehicles) without running the simulation again, different types of probe vehicles were generated. In addition to the flows of probe vehicles recorded by the detectors, probe vehicles provide two other measurements with the same sampling period: the instantaneous number of probe vehicles and the total distance traveled by each vehicle since it entered the zone. Finally, probe vehicles also provide the total distance they travelled inside the zone of interest when they exit. The total distance travelled by all types of probe vehicles for each measurement interval can then be derived from these different measurements of distance traveled. Hence, all the measurements used are for the same periods of 90 s, which represent the realisations of the stochastic process studied. Although this is not strictly true, consecutive experiments are considered independent to avoid modeling the dynamics of the phenomena involved.

In addition to these different measurements, the true accumulation and production of a large sam-

---

[3]A possible extension could be to consider a fully Bayesian framework in which the parameters of the models would only be known probabilistically.

ple of vehicles (10%) is known precisely and serves as ground truth for validation purposes.

2.3 Observation equations for the estimation of accumulation

Four of the previous measurements are useful in order to estimate the accumulation $n(t)$ of vehicles within the zone of interest at all times t: the average occupancy rate from all detectors in the period preceding t: $O(t)$, the number of probe vehicles in the zone at time t: $n_p(t)$, the total flow measured in the preceding period: $f(t)$, and the flow of probe vehicles $f_p(t)$, also measured in the preceding period by the loop detectors. For brevity, the variable t will simply be omitted hereafter when there is no ambiguity.

The purpose of this section is to specify a functional form for the probability density function $p_{\text{obs}}(n_p, O, f, f_p \,|\, n, \beta)$. To simplify the derivations, some of these variables can be assumed to be independent *given the accumulation n*. More specifically,

$$p_{\text{obs}}(n_p, O, f, f_p \,|\, n, \beta) = p(f_p \,|\, n, \beta, n_p, O, f)p(n_p \,|\, n, \beta, O, f)p(O \,|\, n, \beta, n_p, f)p(f \,|\, n, \beta) \text{ (Bayes' rule)}$$
$$\approx p(f_p \,|\, n, \beta, n_p, f)p(n_p \,|\, n, \beta)p(O \,|\, n, \beta, n_p).$$

The most arguable simplifications are certainly $p(O \,|\, n, \beta, n_p, f) \approx p(O \,|\, n, \beta, n_p)$ and $p(f \,|\, n, \beta) \approx 1$. In fact, given n and β, the flow and the occupancy are most likely positively correlated and given the accumulation, the flow should only be known via a stochastic MFD. These relationships however are more related to the spatial heterogeneity of congestion, hence to its second-order moment, than to its mean. Hereafter, only the three remaining probability density functions are studied.

2.3.1 Loop detector occupancy

If all sections of the network were as likely to be monitored by loop detectors and if the stretch of road monitored by a detector can be occupied by only one vehicle, the probability that k loops are occupied at a given time can be modeled by the binomial distribution of n independent Bernoulli trials with probability of success P_c, where P_c is the proportion of the network that is monitored by loop detectors. Let l denote the effective length (in meters) of a single loop detector and v denote the average vehicle speed (m/s) in the network. During the measurement interval, the vehicles travel in average over $\frac{90v}{l}$ intervals of length l. Hence, taking the average occupancy over 90 s essentially amounts to doing $\frac{90v}{l}n$ independent Bernoulli trials, still with the same probability of success P_c. Then, for such a big number of trials, the binomial distribution $\mathcal{B}(n, p)$ is very well approximated by a Gaussian with mean np and variance $np(1 - p)$. The

occupancy is simply given by the number of successes, divided by the product of the number of detectors (N_d) times the number of time intervals, $\frac{90*v}{l}$. Hence, the occupancy should follow a Gaussian distribution with mean $\frac{P_\mathrm{c}}{N_\mathrm{d}}n$ and variance $\frac{(P_\mathrm{c})(1-P_\mathrm{c})}{N_d^2}\frac{l}{90v}n$. Finally, in order to dissociate the estimation of the production and the accumulation of the accumulation, we simply replace here the average vehicle speed v by an affine function of the accumulation ($v = v_f(1 - \frac{n}{n_{jam}})$), known as the linear MFD of speed.

There are 4297 one-lane links in the network (one link with three lanes is counted three times), which measure in average 80m. The effective length of a loop detector is about 8m. Hence, the considerations above lead to the conclusion that the slope of the relation between accumulation and occupancy should be about $\frac{8}{80*4297} = 2.3 \times 10^{-5}$. Based on simulation results however, the true slope is about 1.4×10^{-5}. This discrepancy is most likely due to the fact that in the simulation, loop-detectors are not randomly distributed but always located at mid-distance between the two extremities of each link. Since vehicles spend significantly more time near the stop-lines than anywhere else, the slope is over estimated when queues rarely reach the middle of the blocks. Ideally, in order to account for this particularity, a S-shaped curve would be necessary, such that occupancy increases slowly with accumulation as long as the queues do not reach the middle of the links, then increases more rapidly, and less rapidly again when the accumulation is so big that queues never completely empty. To avoid introducing too many parameters however, we chose in the present work to model only the range of accumulation close to the critical one (i.e. the one that leads to the maximum flow in the MFD), such that an affine relationship is enough. Similarly, the variance in practice was often found to be larger than expected. Hence, the following functional form was chosen:

$$p(O\,|\,n, a, \mu, \sigma, b) \sim \mathcal{N}\left(a + \mu n, \frac{\sigma^2 n}{1 - bn}\right), \tag{3}$$

where a, μ, σ and b are parameters to be estimated.

2.3.2 Instantaneous penetration rate

In the simulation, the instantaneous accumulation of probe vehicles n_p at any time follows a true Binomial distribution with n as the total number of trials and r the probability of success, which is a setting of the scenario considered. The only approximation in this case stems from the assumption that consecutive measurements are independent. Again, assuming that n is sufficiently big, this binomial distribution can be approximated by a Gaussian:

$$p(n_p\,|\,n, r) \sim \mathcal{N}\left(nr, nr(1 - r)\right). \tag{4}$$

2.3.3 Measured penetration rate

At every time period, the penetration rate can be estimated by dividing the flow of probe vehicles f_p over the loop detectors by the total flow f. If we consider the total flow as a function of the accumulation, the flow of probe vehicles can be modeled by a binomial distribution with probability of success the instantaneous penetration rate $\frac{n_p}{n}$ and with total number of trials f. By applying the Gaussian approximation again:

$$p(f_p \,|\, n_p, n, f) \sim \mathcal{N}\left(f\frac{n_p}{n}, f\frac{n_p}{n}\left(1 - \frac{n_p}{n}\right)\right). \tag{5}$$

Empirically however, the variance was often found to be slightly larger than predicted by equation (5), probably due to local heterogeneities in the density of detectors. Hence, an additional parameter α was introduced, such that

$$p(f_p \,|\, n_p, n, f, \alpha) \sim \mathcal{N}\left(f\frac{n_p}{n}, \alpha f\frac{n_p}{n}\left(1 - \frac{n_p}{n}\right)\right).$$

2.4 Prior probability density functions

As explained in the methodology section (2.1), a prior joint probability density functions should be specified for all realizations of the latent variable, given the parameters. As we assumed independence of the latent variables for different times, the prior that should be defined is a function $p(y \,|\, \beta)$, or with the variables used in this particular example: $p(n \,|\, a, \mu, \sigma, b, r, \alpha)$.

In the spirit of Empirical Bayes methods, we wish to rely as much as possible on the data and avoid introducing any bias via the prior distribution. In such cases, the prior distributions are often simply ignored. In the case at hand however, the use of a latent variable forces us to postulate some non-trivial distribution for the different parameters of the observation equations. Indeed, assuming some distribution for n given the parameters (a, μ, σ, b) implies that given the observation of occupancy O, $p(O \,|\, a, \mu, \sigma, b) = \int p(O \,|\, n, a, \mu, \sigma, b) p(n \,|\, a, \mu, \sigma, b) dn$ is also defined and varies with the set of parameters (a, μ, σ, b). Hence, even though theoretically the sole observation of the occupancy cannot provide any knowledge of the accumulation, the prior assumption on n would make some values of parameters more likely than others. To avoid this bias, it was decided here to define the prior distribution $p(n \,|\, a, \mu, \sigma, b)$ *after* collecting the observations. More precisely, it was set such that if $\hat{n}$ is within some positive interval $[0, n_max]$,

then

$$p(\hat{n} \mid a, \mu, \sigma, b) \propto \frac{1}{\displaystyle\int p(O \mid n, a, \mu, \sigma, b)\,dn}, \tag{6}$$

and $p(\hat{n}) = 0$ otherwise. In other words, given only the observations of occupancies, all values associated to the parameters of the loop detector occupancy are as likely.

3 Results

3.1 Optimization set-up

The optimization problem (Eq. (1)) to be solved to obtain the *MLE* estimates of the parameters is highly non-convex. Indeed, the objective function involves the product of as many integrals as experiments carried out, and each integral involves all of the decision variables in a non-trivial way. The decision space being continuous, this problem can only be solved approximately, using heuristics for global optimization. Since this problem can be solved off-line, computational efficiency is not critical and the built-in MATLAB function "particleswarm" was used. After some trials and errors, we decided to run the optimization algorithm with a swarm size of 200 particles and replicated the operation five times, to ensure that the results do not vary too much from one iteration to the other.

3.2 Validation of the methodology

Using the MLE estimates of the different parameters, the accumulation at all times can then be estimated probabilistically by applying Eq. (2). In practice, these estimates would be difficult to validate. In a micro-simulation environment however, the ground truth can be measured exactly and diverse statistics can be calculated. In the present work, we use the relative root-mean-squre error (RMSE) given by:

$$\text{relative RMSE} = \sqrt{\sum_{i \in H} \left(\frac{n_i - \hat{n}_i}{n_i} \right)^2}, \tag{7}$$

where $\hat{n}_i$ is the expected value of the accumulation estimated with Eq. (2) for time i and n_i is the true accumulation. The values of relative RMSE obtained for different proportions of links monitored by loop detectors and for different fleet penetration rates are listed in tables 1 and 2.

		Fleet penetration rate				
		0%	0.5%	1%	2%	5%
links monitored	0%	N.A.	11.69%	9.18%	6.53%	4.16%
	5%	12.05%	6.57%	5.93%	5.02%	3.58%
	10%	6.61%	4.59%	4.63%	4.12%	3.10%
	20%	3.11%	3.16%	3.64%	3.19%	2.50%

Table 1: Relative RMSE in case only the links with the highest average occupancy are monitored for different fleet penetration rates and different proportion of links that are monitored by loop detectors.

		Fleet penetration rate				
		0%	0.5%	1%	2%	5%
links monitored	0%	N.A.	11.69%	9.18%	6.53%	4.16%
	5%	7.12%	4.99%	4.94%	4.30%	3.33%
	10%	4.89%	3.85%	4.06%	3.67%	2.98%
	20%	4.22%	2.94%	3.32%	2.91%	2.44%

Table 2: Relative RMSE in case the monitored links are randomly chosen, for different fleet penetration rates and different proportion of links that are monitored by loop detectors.

The values corresponding to the cases with 0% penetration rate or 0% of links monitored are only meant to serve as references. Indeed, with only one of these sensors, it is a priori impossible to estimate the accumulation, either because the scale factor is unknown (if only probe data is available) or because both the bias and the scale factor are unknown (is only loop detectors are available). Here, the average accumulation over all experiments was assumed to be known and a multiplying coefficient was applied, such that the average predicted accumulation is correct.

To produce these statistics, the MLE parameters chosen were those that maximized the objective function (1) over all replications of the optimization process. Note that since the objective function and the validation criterion are different, some other values of these parameters might provide smaller errors. However, it is noteworthy that by combining the two types of sensors, large gains in accuracy can be obtained, especially for small penetration rates and network coverage. The only exceptions to this observation are for proportion of links monitored of 10% and 20% and fleet penetration rates of 0.5% and 1%. In these particular cases, increasing the penetration rate reduced the accuracy of the estimated values. This surprising observation might be explained by the complexity of the optimization problem, which might have been solved only locally but not globally for fleet penetration rates of 1%.

4 Conclusions and future work

In conclusion, it should be emphasised that the degree of accuracy observed in this study is specific to the network used. More specifically, the accuracy obtained for a given penetration rate is expected to improve with the average accumulation in the network studied.

It should also be highlighted that this is a report written at a very early stage of the project and that consequently, many questions remain to be answered and many extensions need to be developed. Examples of extensions include a full Bayesian treatment (instead of simply relying on the MAP estimates), the inclusion of the dynamics of congestion into the model and the possibility of time or space-dependent fleet penetration rates.

5 References

Aboudolas, K. and N. Geroliminis (2013) Perimeter and boundary flow control in multi-reservoir heterogeneous networks, *Transportation Research Part B*, **55**, 265–281.

El Faouzi, N.-E., H. Leung and A. Kurian (2011) Data fusion in intelligent transportation systems: Progress and challenges – A survey, *Information Fusion*, **12** (1) 4–10.

Geroliminis, N., J. Haddad and M. Ramezani (2013) Optimal perimeter control for two urban regions with macroscopic fundamental diagrams: A model predictive approach, *IEEE Transactions on Intelligent Transportation Systems*, **14** (1) 348–359.

Hofleitner, A., R. Herring, P. Abbeel and A. Bayen (2012) Learning the dynamics of arterial traffic from probe data using a dynamic bayesian network, *IEEE Transactions on Intelligent Transportation Systems*, **13** (4) 1679–1693.

Hunter, T., A. Hofleitner, J. Reilly, W. Krichene, J. Thai, A. Kouvelas, P. Abbeel and A. Bayen (2013) Arriving on time: estimating travel time distributions on large-scale road networks, *arXiv preprint arXiv:1302.6617*.

Keyvan-Ekbatani, M., A. Kouvelas, I. Papamichail and M. Papageorgiou (2012) Exploiting the fundamental diagram of urban networks for feedback-based gating, *Transportation Research Part B*, **46** (10) 1393–1403.

Keyvan-Ekbatani, M., M. Papageorgiou and I. Papamichail (2013) Urban congestion gating control based on reduced operational network fundamental diagrams, *Transportation Research Part C: Emerging Technologies*, **33**, 74–87.

Kouvelas, A., M. Saeedmanesh and N. Geroliminis (2015) Feedback perimeter control for heterogeneous urban networks using adaptive optimization, paper presented at the *18th IEEE International Conference on Intelligent Transportation Systems*, 882–887.

Leclercq, L., N. Chiabaut and B. Trinquier (2014) Macroscopic fundamental diagrams: A cross-comparison of estimation methods, *Transportation Research Part B: Methodological*, **62**, 1–12.

Ortigosa, J., M. Menendez and H. Tapia (2013) Study on the number and location of measurement points for an MFD perimeter control scheme: a case study of Zurich, *EURO Journal on Transportation and Logistics*, **3** (3) 245–266.

Patire, A. D., M. Wright, B. Prodhomme and A. M. Bayen (2015) How much GPS data do we need?, *Transportation Research Part C: Emerging Technologies*, **58**, 325–342.

Ramezani, M., J. Haddad and N. Geroliminis (2015) Dynamics of heterogeneity in urban networks: aggregated traffic modeling and hierarchical control, *Transportation Research Part B*, **74**, 1–19.

Tsekeris, T. and N. Geroliminis (2013) City size, network structure and traffic congestion, *Journal of Urban Economics*, **76**, 1–14.

Tsekeris, T. and A. Stathopoulos (2006) Measuring variability in urban traffic flow by use of principal component analysis, *Journal of Transportation and Statistics*, **9** (1) 49–62.

Wu, C., J. Thai, S. Yadlowsky, A. Pozdnoukhov and A. Bayen (2015) Cellpath: Fusion of cellular and traffic sensor data for route flow estimation via convex optimization, *Transportation Research Part C: Emerging Technologies*, **59**, 111–128.

Yuan, Y., H. Van Lint, F. Van Wageningen-Kessels and S. Hoogendoorn (2014) Network-wide traffic state estimation using loop detector and floating car data, *Journal of Intelligent Transportation Systems*, **18** (1) 41–50.

Zheng, N. and N. Geroliminis (2013) On the distribution of urban road space for multimodal congested networks, *Transportation Research Part B: Methodological*, **57**, 326–341.