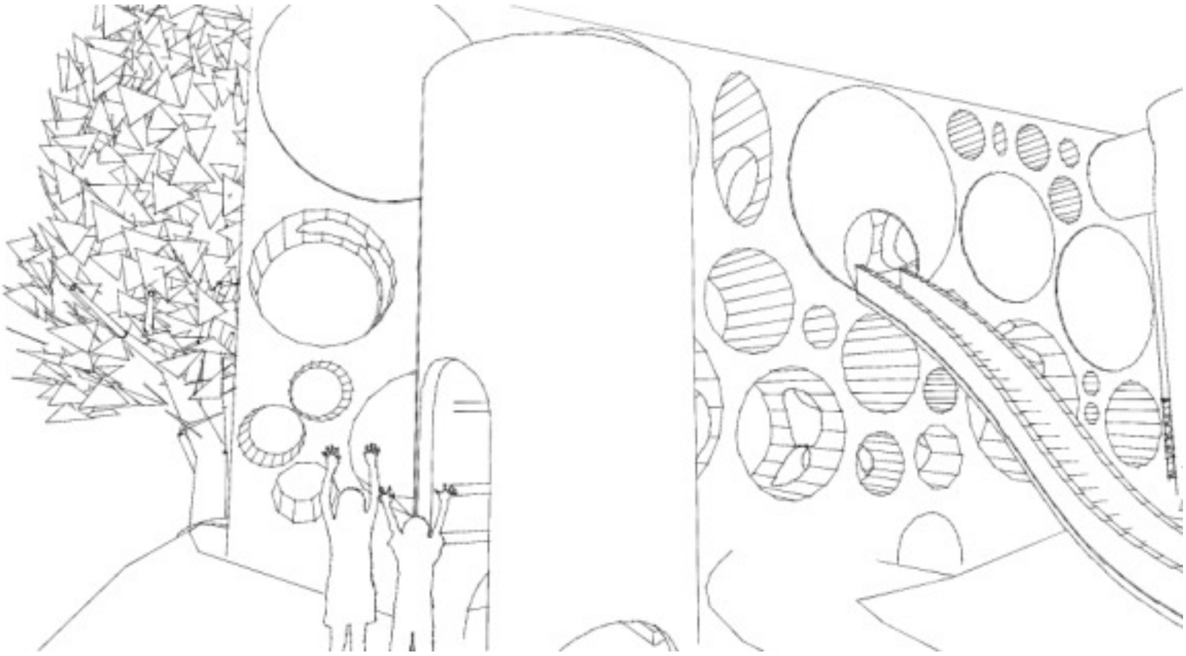


Assessing complex route choice models using mental representations

Evanthia Kazagli

Michel Bierlaire

TRANSP-OR, EPFL

May 2016

STRC | 16th Swiss Transport Research Conference

Monte Verità / Ascona, May 18 – 20, 2016

TRANSP-OR, EPFL

Assessing complex route choice models using mental representations

Evanthia Kazagli
Transport and Mobility Laboratory,
School of Architecture, Civil &
Environmental Engineering
École Polytechnique Fédérale de Lausanne
CH-1015 Lausanne
phone: +41 21 693 24 29
fax: +41 21 693 80 60
evanthia.kazagli@epfl.ch

Michel Bierlaire
Transport and Mobility Laboratory,
School of Architecture, Civil &
Environmental Engineering
École Polytechnique Fédérale de Lausanne
CH-1015 Lausanne
phone: +41 21 693 25 37
fax: +41 21 693 80 60
michel.bierlaire@epfl.ch

May 2016

Abstract

In Kazagli *et al.* (2015), we introduced the concept of the Mental Representation Item (MRI) as a modeling element and we presented the methodology for the derivation of operational random utility models for route choice analysis based on MRIs. The use of the MRI as a modeling element enabled us to obviate the need for choice set generation and sampling of choice sets.

In this work, we extend the MRI approach in order to apply it to more complex models such as the cross nested (CLN) and the recursive (RL) logit. The CNL and RL models are of interest as they tackle the two main challenges related to route choice modeling; namely the correlation of the alternatives and the choice set generation. Their estimation for large networks is cumbersome. We are interested in (i) investigating the potential of the MRI approach to break down the combinatorial complexity of these models and (ii) comparing their performance under the two representational approaches (MRI and path), using real case studies.

Keywords

Route choice, Random utility models, Mental representation item (MRI), Cross nested logit, Recursive logit

1 Introduction

Route choice is one of the key travel demand problems. Given an origin (o), a destination (d) and a transportation mode, we are interested in understanding and predicting how a traveler selects the route that brings her from o to d. Route choice models provide a useful tool to describe the distribution of travelers on the network. This distribution allows to identify congestion and enables for transportation planning as well as real-time operations.

The need to go beyond the shortest and fastest path models (reference) has triggered a great deal of research. Route choice analysis is commonly performed within the discrete choice modeling framework. Discrete choice models (DCMs) allow for a great deal of explanatory variables to be considered and for the behavioral heterogeneity across the population to be explicitly captured. This transition from efficiency to behavioral realism entails additional model complexity though. Classical route choice models assume a network representation $G = (A, V)$, defined in terms of nodes (vertices) V and links (oriented arcs) A, and a number of attributes associated with each link. Following this network representation, a route choice is defined as a sequence of links connecting the o and the d of the trip. This is denoted as a *path* alternative.

Letting data issues aside, the two main challenges concerning the path representation are (i) the enumeration of paths between each o and d and (ii) the structural correlation among the path alternatives that overlap. Different approaches that deal with the choice set generation (reference) and tackle the correlation of alternatives (reference) have given rise to various discrete choice models. Most of these models are path based.

Recently, Fosgerau *et al.* (2013) presented the recursive logit model (RL), where the path choice problem is formulated as a sequential link choice problem in a dynamic framework. The proposed technique avoids the full enumeration of paths and does not require sampling. The first work to use a Multivariate Extreme Value (MEV) model with sampling of alternatives and addressing the correlation in the stochastic part of the utility function, is the one by Lai and Bierlaire (2015). The authors specify a cross nested logit (CNL) model and adopt the Metropolis-Hastings algorithm proposed by Flötteröd and Bierlaire (2013) with a new expansion factor inspired by Guevara and Ben-Akiva (2013) in order to avoid the enumeration of paths.

The general trend in the literature in order to deal with the challenges pertaining to the path representation is to propose more and more complex models (reference). In Kazagli *et al.* (2015), we aimed at simplifying the route choice problem by modeling the strategic decisions of people, represented by the *mental representations* of their itineraries, instead of the operational ones, represented by *paths*. We introduced the concept of *Mental Representation Item* (MRI) as a modeling element and we presented the methodology for the definition of route choice models based on MRIs.

In this work, we extend the MRI model and we adapt the approach for the specification of CNL and RL models. Our objective is to evaluate the potential of the MRI approach in simplifying the estimation of these models and rendering their application, to big real networks, feasible. In this context, we evaluate the performance of the models under the two route representational approaches, i.e. the MRI and the path.

The paper is organized as follows. In Section 2 we review the MRI modeling framework. Section 3 reviews the CNL and RL models and their adaptation to the MRI framework. In Section 4 we present the available case studies. The last section summarizes the goals of the present study.

2 Route choice with MRIs

In this section, we briefly recapitulate the MRI approach and we present a way to generalize the model for application to CNL and RL specifications. For more details regarding the definition of the MRI elements and the specification of MRI-based models, we refer the reader to Kazagli *et al.* (2015).

2.1 The MRI approach

According to the MRI formulation, a route is defined as an origin, a sequence of MRIs in order and a destination. A MRI is a modeling element that captures any perceived items influencing a route choice, such as a central part of a city, a bridge or a highway. Each MRI is characterized by a name, a description, a geographical span and a list of representative points (Fig. 1). The first two components capture the conceptual aspects, and the last two are needed for the model to be operational, by enabling the association of the MRIs with more objective representations, such as a map or a network (Kazagli *et al.* (2015)).

2.2 The MRI network

In Kazagli *et al.* (2015), we presented the simplest possible case of a MRI model where each alternative involved exactly one MRI. The choice set consisted of four labeled alternatives,

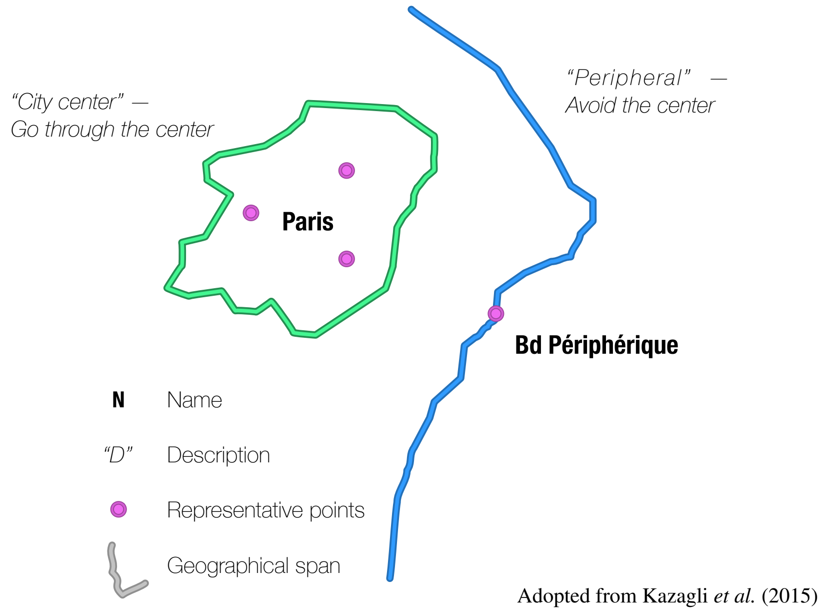


Figure 1: Examples of two MRIs and their components.

common for all travelers. As an extension of this methodology, we propose the definition of the MRI network according to the following steps:

1. Define the MRIs and the origin o and destination d zones for the case study of interest.
2. For each MRI r creat a node.
3. For each o and d zone determine the centroid s of the zone and create a node corresponding to it.

   The number of vertices of the MRI network equals the summation of the number of MRIs $\mathcal{R}$ and zone centroids $\mathcal{S}$.
4. For each pair of nodes in the MRI network create a link (edge) ℓ if the transition from one node to another is allowed.

An example of a MRI network with 6 MRI nodes and 6 *od* zones is illustrated in Fig. 2. It is inspired by the case studies that we are utilizing, where the presence of a river splitting the cities in two sides entails a bridge choice (MRI 5 and 6 in the blue polygon correspond to bridges) for some *od* pairs.

Ideally, the MRI approach should allow for enumeration of all the MRI sequences between each *od* pair. In the opposite case, the classical choice set generation procedures can be used.

The MRI network serves as the basis for the specification of the CNL and RL models presented in the following section.

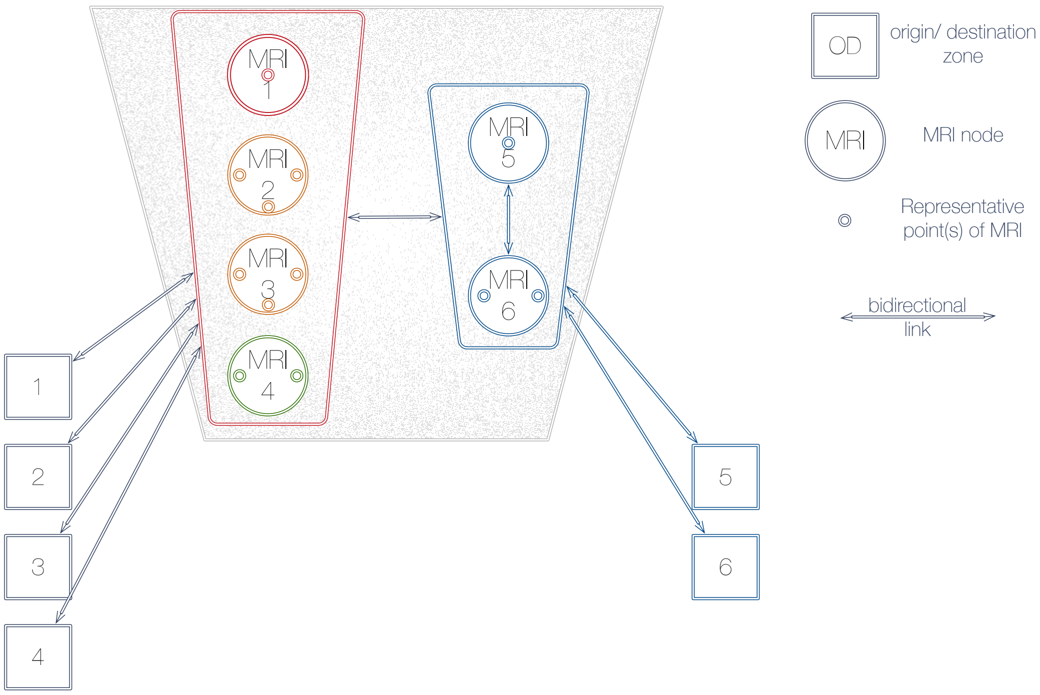


Figure 2: The MRI graph.

3 Cross Nested and Recursive Logit with MRIs

3.1 The cross nested logit

The CNL model is a generalization of the nested logit model with a more flexible correlation structure. It allows each alternative to belong to more than one nest. The CNL belongs to the MEV family that was first proposed by McFadden (1978). Let $i = 1, 2, ..., J$ be an alternative in the choice set and $m = 1, 2, ..., M$ be a nest, the choice probability generating function (CPGF) of the CNL is then given by:

$$G(y) = \sum_{m=1}^{M} \left(\sum_{j=1}^{J} \alpha_{jm}^{\frac{\mu_m}{\mu}} y_j^{\mu_m} \right)^{\frac{\mu}{\mu_m}}, \tag{1}$$

where α_{jm} are parameters indicating the degree of membership of an alternative j to a nest m (also called inclusion coefficients), μ is the scale for the model, μ_m is the scale parameter associated with nest m, and y_j is equal to 1 if alternative i is chosen, and 0 otherwise. The following conditions need to be satisfied (Bierlaire (2006)): (i) $\alpha_{im} \geq 0, \forall i, m$, (ii) $\sum_m \alpha_{im} > 0$ $\forall i$, and (iii) $0 < \mu < \mu_m$, $\forall m$. Finally, for the estimation of the inclusion coefficients the normalization $\sum_{m=1}^{M} \alpha_{im} = 1, \forall i = 1, ..., J$ is applied.

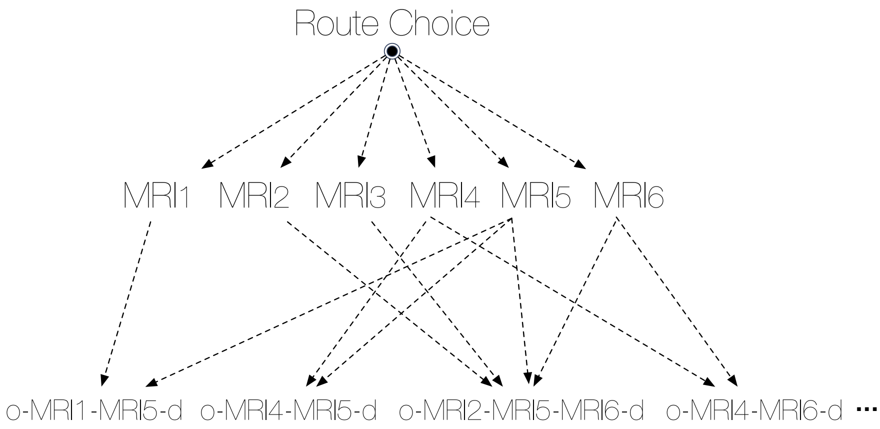


Figure 3: The underlying MRI nesting structure.

The cross nested logit probability is given by:

$$P_n(i) = \sum_{m=1}^{M} \frac{(\sum_{j\in C_n} \alpha_{jm}^{\frac{\mu_m}{\mu}} e^{\mu_m V_{jn}})^{\frac{\mu}{\mu_m}}}{\sum_{p=1}^{M}(\sum_{j\in C_n} \alpha_{jm}^{\frac{\mu_p}{\mu}} e^{\mu_p V_{jn}})^{\frac{\mu}{\mu_p}}} \frac{\alpha_{im}^{\frac{\mu_m}{\mu}} e^{\mu_m V_{in}}}{\sum_{j\in C_n} \alpha_{jm}^{\frac{\mu_m}{\mu}} e^{\mu_m V_{jn}}}, \tag{2}$$

that corresponds to:

$$P_n(i) = \sum_{m=1}^{M} P_n(m|C_n)P_n(i|m), \tag{3}$$

where $P_n(i|m)$ is to be interpreted as the probability of an alternative i conditional to nest m and $P_n(m|C_n)$ as the probability of nest m given the choice set C_n.

3.1.1 CNL with MRIs

The choice set of the MRI model consists in MRI sequences. As the alternatives are sequences of MRIs, the correlation structure can be captured by a CNL model, where each MRI is a nest. An alternative i belongs to a nest m if MRI r appears in the sequence i (Fig. 3). This model specification is similar to the link nested model proposed by Vovsha and Bekhor (1998). The difference is that the nests correspond to MRIs instead of links, reducing the number of nests and allowing for a level of complexity which is much lower, and fully under the control of the modeler. The estimation of nest specific scales is possible[1].

[1] Note that Ramming (2002) estimated this model for a network of 34,000 links, where the estimation of nest-specific coefficients was impossible.

3.2 The recursive logit

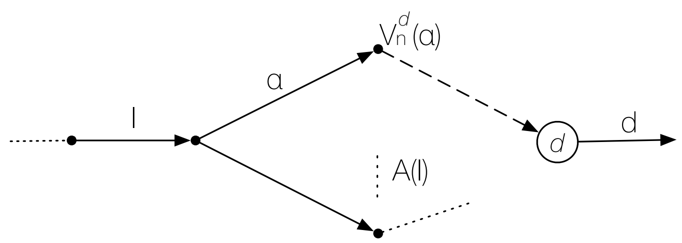
Unlike most of current state-of-the-art models the RL model (Fosgerau *et al.* (2013)) is not path based. Inspired by previous works developed in the context of traffic assignment (e.g. Akamatsu (1996); Baillon and Cominetti (2008)), the RL model decomposes the route choice problem into a sequential link choice problem that is equivalent to a finite multinomial logit model. The problem is then solved in a dynamic discrete choice framework, where at each node the individual chooses the outgoing link that maximizes her utility. The link utilities consist of an instantaneous cost, the expected maximum utility to the destination and independently and identically distributed extreme value type I error terms. The next link choice probabilities are given by a multinomial logit model.

Following the formulation proposed by Fosgerau *et al.* (2013), given a directed connected network $G = (A, V)$, where A is the set of links and V is the set of nodes, we denote links $\ell, \alpha \in A$ and $A(\ell)$ the set of outgoing links from the sink node of link ℓ. Each link pair (ℓ, α) is characterized by a deterministic utility component $v_n(\alpha|\ell)$, which is a function of the observed attributes $x_{n,\alpha|\ell}$ of the link pair and that may also include the characteristics of the individual z_n.

In the context of dynamic programming, ℓ is a state and α is a feasible action given state ℓ. In the formulation proposed by Fosgerau *et al.* (2013), a stochastic process with the Markov property is used to identify the next chosen link given the current state. It is assumed that, at each current state ℓ the individual observes random utility terms $\varepsilon_n(\alpha)$ and chooses the α that maximizes the sum of instantaneous utility $u_n(\alpha|\ell)$ and expected downstream utility. The latter is given by the value function $V^d(\alpha)$, and is computed by means of the Bellman equation (Bellman (1957)) as:

$$V^d(\ell) = E[max_{\alpha \in A(\ell)}(v_n(\alpha|\ell) + V_n^d(\alpha) + \mu\varepsilon_n(\alpha))] \forall \ell \in A. \tag{4}$$

An absorbing state needs to be defined for each destination link. This is done by adding to the destination node a dummy link d without outgoing links. The set of links is then defined as



Adopted from Fosgerau *et al.* (2013).

Figure 4: Illustration of the RL model notation.

$\tilde{A}^{\mathrm{d}} = A \bigcup \mathrm{d}$. $v_n(\mathrm{d}|\ell) = 0$ for all ℓ having the destination node as sink node. Figure 4 is adopted from Fosgerau *et al.* (2013) to illustrate the notation.

The probability of choosing link α conditional on stake ℓ is given by the multinomial logit model

$$P_n^d(\alpha|\ell) = \frac{e^{\frac{1}{\mu}(v_n(\alpha|\ell)+V_n^d(\alpha))}}{\sum_{\alpha'\in A(\ell)} e^{\frac{1}{\mu}(v_n(\alpha'|\ell)+V_n^d(\alpha'))}}. \tag{5}$$

The value function is the logsum

$$V_n^d(\ell) = \begin{cases} \mu ln \sum_{\alpha\in A} \delta(\alpha|\ell) e^{\frac{1}{\mu}(v_n(\alpha|\ell)+V_n^d(\alpha))} & \forall \ell \in A \\ 0 & \ell = \mathrm{d}, \end{cases} \tag{6}$$

where $\delta(\alpha|\ell) = 1$ if $\alpha \in A(\ell)$ and zero otherwise. The maximum of an empty set is zero, hence $V(d) = 0$. A path σ is consequently realized as a sequence of link choices, with probability

$$P(\sigma) = \prod_{i=0}^{I-1} e^{\frac{1}{\mu}(v(\ell_{i+1}|\ell_i)+V(\ell_{i+1})-V(\ell_i))} = e^{-\frac{1}{\mu}V(\ell_0)} \prod_{i=0}^{I-1} e^{\frac{1}{\mu}v(\ell_{i+1}|\ell_i)}. \tag{7}$$

Letting $v(\sigma) = \sum_{i=0}^{I-1} v(\ell_{i+1}|\ell_i)$

$$P(\sigma) = \frac{e^{\frac{1}{\mu}v(\sigma)}}{e^{\frac{1}{\mu}V(\ell_0)}} = \frac{e^{\frac{1}{\mu}v(\sigma)}}{\sum_{\sigma'\in\Omega} e^{\frac{1}{\mu}v(\sigma')}}, \tag{8}$$

where Ω is the set of all paths, which is infinite.

3.2.1 RL with MRIs

As soon as the MRI network is defined, it is straightforward to apply the formulation proposed by Fosgerau *et al.* (2013) to it. The only difference lies in the underlying network that provides the input for the RL model. Figure 5 provides one example of the application of the RL model to a MRI network.

The MRI approach reduces the state space of the RL model with the potential to improve its efficiency in cases of large networks.

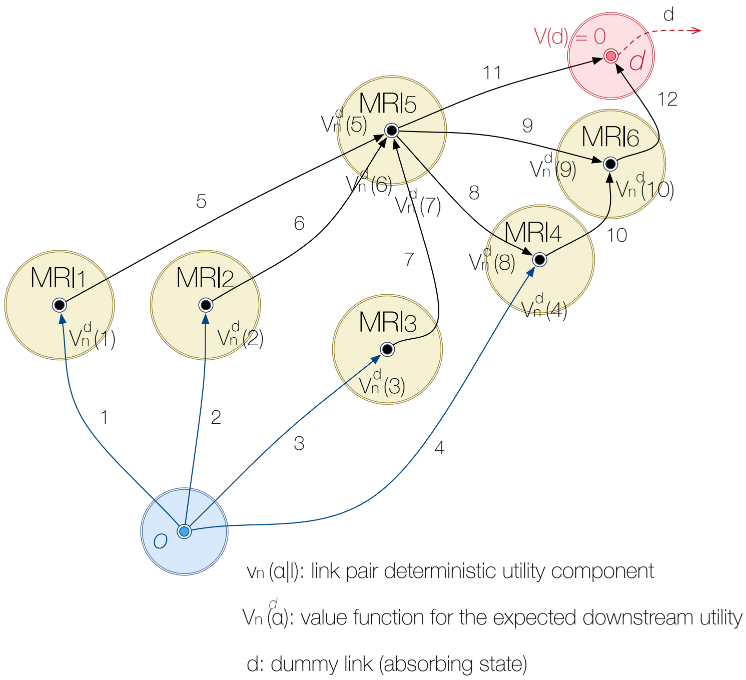


Figure 5: Illustration of the RL model on the MRI network.

4 Playground

The objective of this work is to investigate the potential of the `MRI` approach in facilitating the application of the CNL and RL models to large networks. The goal is to specify and compare these models using real data (Table 1). The logit model is used as a benchmark. By the end of the day, we need to show that the logit and the RL model are equivalent (blue color coding ⊕ in Table 1). The estimation of the logit and CNL models with paths is omitted due to the complexity that it entails, which is in contrast with the current objective. Yet this estimation is feasible with the RL model (blue color coding ⊕ in Table 1).

Table 1: Scope of analysis for the comparison of the models

model type	MRI	path
logit	⊕	–
CNL	⊕	–
RL	⊕	⊕

The analysis aims at identifying the trade-offs among model fit, complexity and computational burden. With respect to the model fit, the comparison is conducted directly, by comparing the resulting path probabilities, and indirectly, by comparing the resulting link flows. In Kazagli *et al.* (2015), we present a simple methodology for the application of the MRI model to traffic assignment. It can be used for the prediction of link flows given the CNL model specification. Fosgerau *et al.* (2013) also provide a way of predicting link flows as a result of the application of the RL model to traffic assignment.

It is also interesting to compare the resulting probabilities, as well as the computational savings, given the RL model and the two representational approaches, i.e. the MRI and the path. For this comparison, it is necessary to render the disaggregate alternatives of the specification on the general network[2] and the aggregate ones on the MRI network, compatible (Fig. 6). As several paths may correspond to the same MRI sequence, the task consists in defining a measurement model that maps each path to one MRI sequence[3]. Then, by aggregating the probabilities of paths associated with an MRI alternative we are able to compare the estimation results

For the purpose of the analysis outlined above, as well as for the comparison of the computational times for the CNL and RL models under the MRI approach, we use two case studies for which GPS data is available. The first one concerns a relatively small city in Sweden, while the second one concerns the city of Quebec, in Canada. We briefly present the two case studies in the following paragraphs.

[2] With the term general network we refer to the road network, represented by a directed connected graph $G = (A, V)$, as defined above.

[3] We refer the reader to Kazagli *et al.* (2015) for examples of how to use the geographical span of the MRIs to build the measurement model.

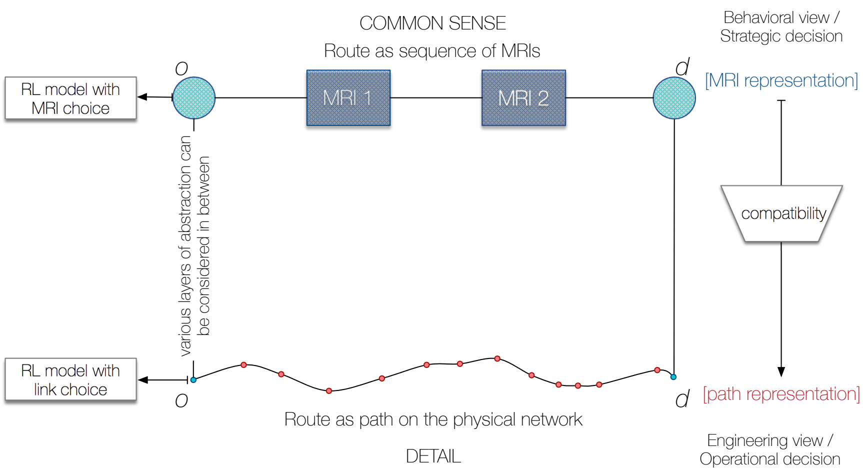


Figure 6: From MRI to path.

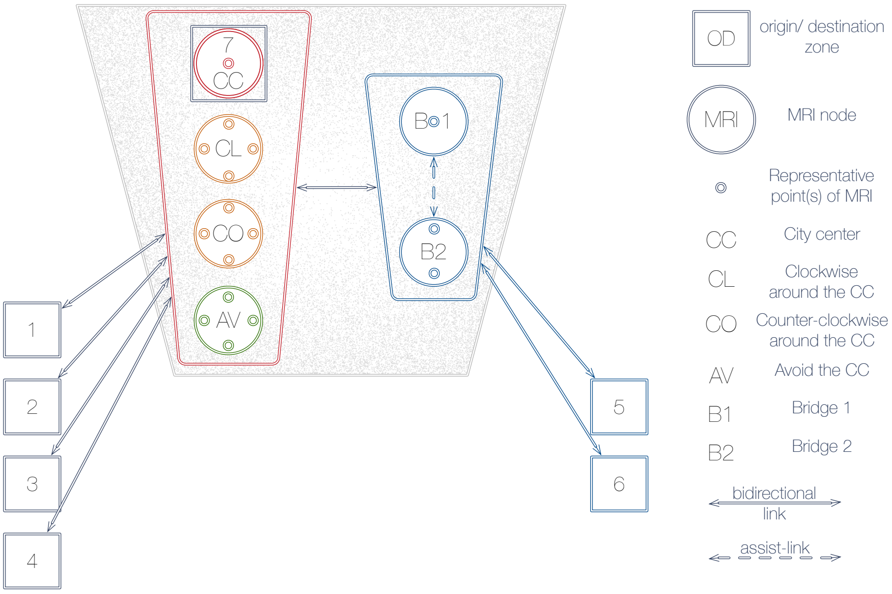


Figure 7: The MRI graph.

4.1 Borlänge case study

This case study exploits a GPS dataset from the city of Borlänge in Sweden. It was used as a proof of concept in Kazagli *et al.* (2015). It is extended here by including two more MRIs in the choice context and by defining the MRI network according to Section 2.2. The general network consists of 3077 nodes and 7459 unidirectional links.

The city is divided in seven zones, including the city center that serves both as an *od* zone and a MRI. There are six MRIs as depicted in Fig. 7. The MRI network consists in 12 nodes and 58 links, allowing for the enumeration of all the routes on the graph for each *od* pair. The maximum number of alternatives for an *od* pair in this network is 36.

4.2 Quebec case study

We are currently preparing the case study for route choice analysis in the city of Quebec in Canada. A GPS dataset is available which has been previously processed to obtain map-matched trajectories. Additional information regarding the users and the trip purposes is available[4]. The general network consists of $\sim$ 20000 nodes and $\sim$ 40000 unidirectional links. The dataset contains more than 20000 trips.

The city of Quebec is favorable for investigating several route choice contexts under the MRI

[4]We refer to Miranda-Moreno *et al.* (2015) for more details regarding the dataset and data collection process.

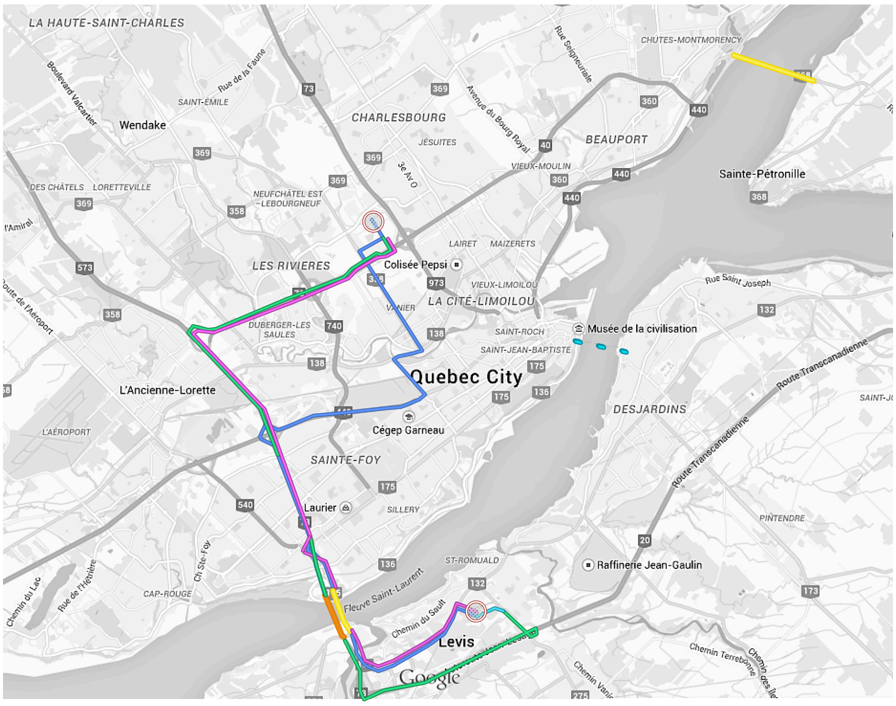


Figure 8: Example of MRIs in Quebec.

assumption; such as the bridge or ferry choice, the choice of autoroute, as well as combinations of them. Figure 8 shows some indicative examples. The level of complexity depends on the application. As the objective of the present work is to investigate the potential of the MRI-based models, we increase the level of complexity by introducing higher number of MRIs in the model. This allows us to identify the gains from incorporating the MRI representation in the CNL and RL formulations, as opposed to the path representation.

5 Conclusion

In this paper, we investigate the potential of the previously proposed MRI approach to simplify complex models, such as the CNL and RL. Instead of an algorithmic approach, the MRI model exploits the behavioral rationale in order to facilitate the application of these models to large networks and real case studies. This is accomplished by reducing the number of nests for the CNL model and the state space for the RL model. Eventually, we are interested in comparing the performance of the two models under the MRI approach. As the analysis is ongoing we do not provide any results.

6 References

Akamatsu, T. (1996) Cyclic flows, markov process and stochastic traffic assignment, *Transportation Research Part B: Methodological*, **30** (5) 369–386.

Baillon, J.-B. and R. Cominetti (2008) Markovian traffic equilibrium, *Mathematical Programming*, **111** (1–2) 33–56.

Bellman, R. (1957) *Dynamic Programming*, 1 edn., Princeton University Press, Princeton, NJ, USA.

Bierlaire, M. (2006) A theoretical analysis of the cross-nested logit model, *Annals of Operations Research*, **144** (1) 287–300.

Flötteröd, G. and M. Bierlaire (2013) Metropolis-Hastings sampling of paths, *Transportation Research Part B: Methodological*, **48**, 53–66, February 2013, ISSN 01912615.

Fosgerau, M., E. Frejinger and A. Karlstrom (2013) A link based network route choice model with unrestricted choice set, *Transportation Research Part B: Methodological*, **56** (0) 70 – 80, ISSN 0191-2615.

Guevara, C. A. and M. E. Ben-Akiva (2013) Sampling of alternatives in multivariate extreme value (mev) models, *Transportation Research Part B: Methodological*, **48** (0) 31 – 52, ISSN 0191-2615.

Kazagli, E., M. Bierlaire and G. Flötteröd (2015) Revisiting the route choice problem: A modeling framework based on mental representations, *Technical Report*, **TRANSP-OR 150824**, Transport and Mobility Laboratory, Ecole Polytechnique Fédérale de Lausanne.

Lai, X. and M. Bierlaire (2015) Specification of the cross-nested logit model with sampling of alternatives for route choice models, *Transportation Research Part B: Methodological*, **80**, 220 – 234, ISSN 0191-2615.

McFadden, D. L. (1978) Modelling the choice of residential location, in A. K. et al. (ed.) *Spatial Interaction Theory and Residential Location*, 75–96, North Holland, Amsterdam, The Netherlands.

Miranda-Moreno, L. F., C. Chung, D. Amyot and H. Chapon (2015) A System for Collecting and Mapping Traffic Congestion in a Network Using GPS Smartphones from Regular Drivers, paper presented at the *Proceedings of the 94th Annual Meeting of the Transportation Research Board*.

Ramming, M. S. (2002) Network knowledge and route choice, Ph.D. Thesis, Massachusetts Institute of Technology (MIT).

Vovsha, P. and S. Bekhor (1998) Link-Nested Logit Model of Route Choice: Overcoming Route Overlapping Problem, *Transportation Research Record: Journal of the Transportation Research Board*, **1645**, 133–142.