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Abstract

Unpredictability of travel behaviors and high complexity of accurate physical modeling have challenged researches to discover implicit patterns of congestion propagation and distribution in large urban networks. However, real traffic data in urban cities reveals the spatial correlation of congestion in adjacent roads and its spatiotemporal finite propagation speed. Spatial data mining and clustering in particular can help us discover spatial patterns that may exist implicitly in distribution of congestion. In fact, clustering allows us to partition heterogeneous networks into homogeneous regions and identify boundaries. In addition, we are capable of chasing spatiotemporal growth of congestion which is crucial for real-time traffic control schemes specifically hierarchical perimeter control approaches. In this paper, we develop and solve a binary quadratic optimization model for partitioning heterogeneous networks taking into account contiguity and size constraints for clusters. The proposed approach utilizes set of distinct and robust homogeneous components in the network called *'snake'*. '*Snake*' is a sequence of links created by adding new adjacent links iteratively based on their similarity to join previously added links. Hence, firstly, snakes corresponding to all different initial points grow in a way that they have the highest possible homogeneity. Based on robust behaviour observed in sub-regions with different level of congestion, we come up with the idea to select a sub-set of distinct and non-similar snakes which cover most parts of the network and reduce our search space. These robust components show potential connected clusters with similar traffic conditions in the network. Secondly, an optimization framework is designed to select sectors from different snakes and assign them as different clusters by minimizing heterogeneity index. This leads to quadratic binary optimization with linear constraints which can be solved with existing optimization solvers (e.g. GUROBI, CPLEX). Finally, a fine-tuning step is used to assign links that are not associated with any clusters to proper clusters. The proposed clustering framework is applied in heterogeneous real network and the promising obtained results reveal the ability of finding directional congestion within a cluster, robustness with respect to parameters' calibration and its good performance for networks with low connectivity and missing data.

Keywords

Graph partitioning – Conegstion propagation – Binary Quadratic Programming - MFD

1. Introduction

Large-scale urban traffic modeling aims to simplify micro-simulations of link-level traffic dynamics to have a better understanding of aggregated-level traffic behavior. This modeling could be utilized in development of efficient real-time traffic management and control schemes. In the link level, fundamental diagrams (FDs) express the relation between flow and density using free-flow and congested states for low and high values of density respectively (1). Early study toward extension of FD concept to network level with an optimum accumulation belongs to Godfrey (2) and later similar approaches were re-introduced by Herman and Prigogine (3). The first empirical verification about the existence of relation between aggregated traffic indicators was presented in (4). This research showed that by spatially aggregating the highly scattered plots of flow vs. density from individual detectors (e.g. 1 min data), the scatter almost disappeared and a well-defined MFD exists between space-mean flow and density. Existence of MFD was also reported using simulation data in (7)-(9) but definitely, real-life data can shed more light in this direction than computer simulations that try to approximate with a large number of parameters, individuals' complex behaviors.

In previous work (5), homogeneous network structure was assumed as a preliminary condition to have well-defined MFDs. It was demonstrated using empirical data of Yokohama that homogeneity is not necessary condition and spatial distribution is a key point (10). Recent studies have also investigated the conditions under which low scattered diagrams exist. They show that networks with an uneven and inconsistent distribution of congestion may exhibit traffic states that are much too scattered to line along an MFD (6)-(11). In fact, they claim that outflow of the network is a function of both average and variance of link densities.

In case of heterogeneously loaded cities with multiple centers of congestion, a possible way to take advantage of well-defined MFD is to partition network into a number of homogeneous smaller regions (12). The objectives of partitioning are to obtain (i) small variance of link densities within a cluster, which increases the network flow for the same average density and (ii) spatial compactness of each cluster which makes feasible the application of perimeter control strategies (13). The work in (13) has laid a solid foundation for static partitioning in grid-type networks with similar level of congestion on both traffic directions of the same road. It also requires a connected graph of the network with data and missing values or malfunctioning detectors might create difficulties in application of the method. Nevertheless, many congested urban networks in central business districts (CBD) of cities might experience strong directional flows during different times of the day, e.g. high demand for directions towards the CBD in the morning peak and the opposite during the evening. The network topology might not be symmetric (grid-type) but some areas might experience higher connectivity than others. The current paper tries to overcome the aforementioned difficulties and develop a clustering methodology that (i) is able to find directional congestion within a cluster, (ii) is robust with respect to parameters' calibration and (iii) has good performance for networks with low connectivity and missing data.

Recent findings from the concept of MFD were important for modeling purposes, as details in individual links are not needed to describe the congestion level of cities. It can also be utilized to introduce simple control strategies to improve mobility in homogeneous city centers building on the concept of an MFD, like in (14), (15). The main logic of the strategies is that they try to decrease the inflow in regions with points in the decreased part of an MFD.

The remainder of this paper is organized as follows: The clustering approach consisting of three consecutive steps of distinct snake space recognition, snake-segmentation and fine-tuning is described in details in section 2. Section 3 analyses the quality of partitioning results in a real network and comparing results for different number of clusters. This paper concludes with a discussion about the results and ideas for further research.

2. Methodological Framework

Let us assume an urban network with uneven and inconsistent distribution of congestion which happens frequently over a day in networks with high demand. Our main objective is to partition a network with a large set of heterogeneous spatial objects into a number of spatially connected sub-regions while optimizing a homogeneity index of the derived regions. An ideal sub-region is a relatively homogeneous area unit defined by set of following criteria: (1) spatial connectivity which facilitates effective traffic management strategies; (2) it has minimum size that should not contain less than a certain number of links. In fact, homogeneity and contiguity constraint are two conflicting objectives that need to be taken into account at the same time.

Based on aforementioned goals, our approach to deal with multi-objective problem is to consider heterogeneity as a main objective function and explicitly impose spatial connectivity as a constraint. To guarantee connectivity in the clusters, we limit our optimization search space to a subset of local homogeneous connected components. Hence, we first find best connected local components for each individual link in the network. Then, to limit the search space, a distinct set of components is specified, with respect to values of similarity, which can cover different parts of the network. Secondly, a binary quadratic optimization is solved in order to assign each link to one of the components in a way that a heterogeneity index is optimized. Finally an optimization based fine tuning step is applied to assign remaining links to proper clusters. Different steps of the proposed partitioning mechanism are described in details in the following sections.

2.1 Distinct snake-space

The aim of this step is to find a subset of distinct local homogeneous regions in the network that cover main parts of the network. Output of this step will be utilized later as a search space in the optimization framework. To achieve this, first we look for all homogeneous local components around different roads in the network and based on a similarity measure, a distinct subset of them are chosen in a way that they cover different parts of the network. Basically, in the first step, a sequence of links is built iteratively for each individual link in the network with an objective to minimize the variance of all the chosen links density (or speed). It can be considered as a "snake" that starts from a link and grows by attracting the most similar adjacent links in each step. In the other words, a link with the closest value to the average of the values of the values in the snake is added iteratively. Obviously, this step needs graph information about connectivity as only adjacent links are added at each step. Evolution of the variance curves of these sequences with the size of the links reveals interesting non-linear patterns. The fact that some parts of the graph are more similar than others can very well explain this observation.

Based on the obtained sequences starting from all different links in the network, a subset of distinct local components are identified that potentially represents homogeneous sub-regions in different parts of the network. The main reasons of finding a distinct subset and reducing the search space are: (1) to avoid having repeated sequences that belong to the same local component; (2) to guarantee that selected local components are able to cover different parts of the network. To attain this, a measure of similarity is defined to determine and select distinct snakes using a similarity measure. As it was mentioned, the sequence of the numbers in the arrays not only represents the links with the close values, but also has some information about the spatial connections of the links. Based on these properties of the snakes, we propose a method that identifies snakes corresponding to the links that belong to different local homogeneous components. This has been done by putting more weight on the snakes that have more common links in the first steps and converge to each other. The similarity measure is defined as follows:

$$w(i,j) = \sum_{k=1}^{N} intersect\ (S_{ik}, S_{jk}) \tag{1}$$

where S_{ik}, S_{jk} are the subsets containing k first elements in the arrays of the snakes starting from links i and j respectively and 'N'denotes the total number of links in the network. In equation (1), function 'intersect' calculates the number of common elements between two subsets with equal size. To calculate the similarity matrix $\mathrm{w(i,j)}$ a snake runs for each link of the network with a size equal to the whole network. Then for each pair of links, the number of common links is estimated for all possible snake sizes and a cumulative metric based on equation (1) is developed. Based on the similarity matrix, we find a subset of distinct snakes with an iterative approach as follow. Firstly, we find the pair which has the lowest similarity and then iteratively add one snake that has the lowest summation of similarities to all previous selected snakes. In this way, we guarantee that the network is well-covered by these snakes.

2.2 Snake segmentation

By the first step, a subset of snakes representing distinct local connected components has been obtained, in which contiguity constraints will be satisfied if optimal solution is chosen from segment of snakes. Hence, the problem is formulated in a way to minimize heterogeneity index by segmenting snakes. In this way, cluster connectivity is guaranteed and different constraint (minimum, maximum) on size of clusters could be defined. The problem is formulated as a quadratic binary optimization with linear inequality constraints. Different types of constraints are specified to impose connectivity, minimum size of clusters, and minimum number of links that should be covered by the clusters. Since we want to have at least a certain number of links to be included in clusters, it might happen that some links are assigned to more than one cluster. To avoid having many links belong to multiple clusters, a set of constraints is specified that illustrate the maximum allowable number of such links. In this part, we present the mathematical formulation of the proposed optimization framework for both predefined and unknown number of clusters. We first define the sets and indices used to describe the model as well as the variables and parameters (see table 1). Detailed mathematical optimization model is presented and its objective function and constraints are described afterwards.

Table 1: Definition of parameters and sets

N	Number of links in the network
N_s	Number of distinct snakes in the subset
l	Set of all links in the network
S	Set of selected distinct snakes
$N_{min}(i)$	Minimum size of cluster i
$N_{max}(i)$	Maximum size of cluster i
A_i	Cluster i
x_{ij}	Binary variable indicating if link j belongs to cluster i or not
$R_i(j)$	Location of link j in snake i
x''_j	Binary variable indicating if link j is assigned to multiple clusters
x'_j	Binary variable indicating if link j is assigned to at least one cluster
a'	Minimum percentage of links that should be associated to clusters
a''	Maximum percentage links that could be associated to multiple clusters
$d(l_i, l_j)$	Normalized distance (dissimilarity) between values of links i , j
ε	A number between 0 and 1

2.2.1 Unknown number of clusters

$$\min \sum_{i=1}^{N_s} \text{Dissim.}(A_i, A_i) + \sum_{i=1}^{N_s} \sum_{\substack{j=1\\ i\neq j}}^{N_s} \text{Sim.}(A_i, A_j) = \sum_{i=1}^{N_s} \sum_{j=1}^{N_{A_i}} \sum_{k=1}^{N_{A_i}} d(l_i, l_j) + \sum_{i=1}^{N_s} \sum_{\substack{j=1\\ i\neq j}}^{N_s} \sum_{m=1}^{N_{A_i}} \sum_{n=1}^{N_{A_j}} (1 - d(l_m, l_n))$$

$$= \sum_{i=1}^{N_s} \sum_{j=1}^{N} \sum_{k=1}^{N} x_{ij} d(l_i, l_j) x_{ik} + \sum_{i=1}^{N_s} \sum_{\substack{j=1\\ i\neq j}}^{N_s} \sum_{m=1}^{N} \sum_{n=1}^{N} x_{im} \big(1 - d(l_m, l_n)\big) x_{jn} \quad (2)$$

$$(a)\ \left(\left(\sum_j x_{ij}\right) - R_i(j) + \varepsilon\right) - N x_{ij} < 0 \quad (b)\ \left(\left(\sum_j -x_{ij}\right) + R_i(j) - \varepsilon\right) + N x_{ij} - N < 0 \quad \forall i \epsilon S, j \epsilon l \quad (3)$$

$$(a)\ \left(\left(\sum_i x_{ij}\right) - 1 + \varepsilon\right) - N_s\, x'_j < 0 \qquad (b)\ \left(\left(\sum_i -x_{ij}\right) + 1\right) + N_s\, x'_j - N_s < 0 \qquad \forall j \epsilon l \qquad (4)$$

$$\sum_j x'_j \geq a' \times N \qquad (5)$$

$$(a)\ \left(\left(\sum_i x_{ij}\right) - 1 - \varepsilon\right) - N_s\, x"_j < 0 \qquad (b)\ \left(\left(\sum_i -x_{ij}\right) + 1\right) + N_s\, x"_j - N_s < 0 \qquad \forall j \epsilon l \qquad (6)$$

$$\sum_j x"_j \leq a" \times N \qquad (7)$$

$$x_{ij} - x_{ij_i^*} - \varepsilon \ < 0\ \{\forall i, j | R_i(j) \leq N_{min}(i)\}, \qquad j_i^* = \{j | R_i(j) = 1\} \qquad (8)$$

$$\sum_j x_{ij} \leq N_{max}(i) \qquad \forall i \in S \qquad (9)$$

The problem formulation is described in Eq. (2)-(9). The objective function (Eq. (2)) consists of the summation of intra-dissimilarity and inter-similarity of clusters that need to be minimized. Note that number of terms in the objective function should not be changed by the size and number of clusters in order to have precise and fair comparison. Hence, both intra-dissimilarity and inter-similarity functions should be included in the objective function with the same order of magnitudes. This is the reason why normalized similarity and dissimilarity functions (between 0 and 1) are used in the objective function.

Constraints (3a) and (3b) ensure that connected segments of different snakes are selected as initial clusters. Since, snakes grow by adding adjacent links iteratively, connectivity is guaranteed if n first consecutive cells are selected from a candidate snake. Constraint (5) ensures that a certain percentage of the links in the network should be at least assigned to one cluster. To achieve this, a binary auxiliary variable (x'_j) is defined in constraints (4a) and (4b) for each individual link to specify if it is selected at least in one of the snakes or not. To have a minimum percentage of links associated to clusters, it is likely that some links are repeated in multiple clusters. Binary auxiliary variable (x''_j) in constraints (6a) and (6b) indicates whether a link is assigned to multiple clusters or not. Constraint (7) restricts the number of links that associated with more than one cluster. Constraints (8)-(9) ensure that clusters have a size between minimum and maximum values that are denoted by N_{min} and N_{max} respectively. N_{min} is normally defined by the user while N_{max} can be determined based on the variance of obtained snakes. For instance, we can consider sharp increasing jumps in the evolution of the variance as a truncation point for that snake. Nevertheless, in this work, we assume a fixed number as a maximum allowable size for all the snakes.

2.2.2 Fixed number of clusters

In this approach, number of desired partitions in the network is assumed to be known as a pre-defined value. There are three main reasons that we are interested in partitioning network into certain number of clusters. Firstly, it allows us to compare our results with other clustering approaches for different number of clusters. Secondly, it allows us to keep number of clusters constant over a period of time where clustering framework is applied dynamically. Finally, since this framework has a fine-tuning step to assign remaining links, it is possible that optimal solution of the current step does not lead to a best solution after final step. Hence, we keep track of the best results corresponding to different number of clusters and minimum coverage rate (a') in the current step. Basically, for all different combinations of arrays in the distinct sub-space, an optimization problem is solved. In each optimization, the first links in the arrays are automatically assigned to the clusters to satisfy minimum size of clusters and decision variables are limited to the rest of the links in the arrays. The problem formulation is described in Eq. (10)-(15):

$$\min \sum_{i=1}^{N_S}\left(\sum_{\substack{\{j|R_i(j)>N_{min}(i)\}\\ \{k|R_i(k)>N_{min}(i)\}}}\left(x_{ij}d(l_i,l_j)x_{ik}\right)\right)+\sum_{i=1}^{N_S}\sum_{\substack{j=1\\ i\neq j}}^{N_S}\left(\sum_{\substack{\{j|R_i(j)>N_{min}(i)\}\\ \{k|R_i(k)>N_{min}(i)\}}} x_{im}\left(1-d(l_m,l_n)\right)x_{jn}\right)+$$
$$\sum_{i=1}^{N_S} 2\times\left(\sum_{\{j|R_i(j)>N_{min}(i)\}} x_{ij}\times\left(\sum_{\{k|R_i(k)\leq N_{min}(i)\}} d(l_j,l_k)+\sum_{\substack{m=1\\ m\neq i}}^{N_S}\left(\sum_{\{n|R_m(n)>N_{min}(m)\}}\left(1-d(l_j,l_n)\right)\right)\right)\right) \qquad (10)$$

$$\begin{aligned}
&(a)\left(\left(\sum_{\{j|R_i(j)>N_{min}(i)\}} x_{ij}\right)+N_{min}(i)-R_i(j)+\varepsilon\right)-Nx_{ij}<0 \\
&(b)\left(\left(\sum_{\{j|R_i(j)>N_{min}(i)\}} -x_{ij}\right)-N_{min}(i)+R_i(j)-\varepsilon\right)+Nx_{ij}-N<0
\end{aligned} \qquad \forall i,j|R_i(j)>N_{min}(i) \qquad (11)$$

$$(a)\ \left(\left(\sum_{\{i|R_i(j)>N_{min}(i)\}} x_{ij}\right)+\left(\sum_{\{i|R_i(j)\le N_{min}(i)\}} 1\right)+\varepsilon\right)-N_s\, x'_j<0$$
$$(b)\ \left(-\left(\sum_{\{i|R_i(j)>N_{min}(i)\}} x_{ij}\right)-\left(\sum_{\{i|R_i(j)\le N_{min}(i)\}} 1\right)+\varepsilon\right)+N_s\, x'_j-N_s<0 \qquad \forall j\in l \qquad (12)$$

$$\sum_j x'_j \ge a'\times N \qquad (13)$$

$$(a)\ \left(\left(\sum_{\{i|R_i(j)>N_{min}(i)\}} x_{ij}\right)+\left(\sum_{\{i|R_i(j)\le N_{min}(i)\}} 1\right)-1-\varepsilon\right)-N_s\, x"_j<0$$
$$(b)\ \left(-\left(\sum_{\{i|R_i(j)>N_{min}(i)\}} x_{ij}\right)-\left(\sum_{\{i|R_i(j)\le N_{min}(i)\}} 1\right)+1+\varepsilon\right)+N_s\, x"_j-N_s<0 \qquad \forall j\in l \qquad (14)$$

$$\sum_j x"_j \le a"\times N \qquad (15)$$

The objective function (Eq. (10)) has both quadratic and linear terms. The quadratic term refers to the similarity and dissimilarity of the unassigned links (decision links) to each other while the linear term considers the similarity and dissimilarity of unassigned links to the pre-assigned links. Constraints in Eq. (11) - (15) are equivalent to the ones presented in Eq. (3) - (7).

2.3 Fine-tuning

After completing the first two steps, a fine tuning approach is applied to all feasible optimal solutions of previous step to assign the remaining links to proper clusters. As there are some links that are repeated in more than one cluster, we should reassign them only to one cluster in this step. At the same time, since we want to keep connectivity inside clusters, we define the biggest connected component, consisting of the links that only assigned to one cluster, as a core of that cluster. Hence, links that either belong to none of the clusters or to multiple clusters are considered as decision variables in the fine-tuning step. Decision variables and parameters used in the current optimization step are defined in table 2 as follows:

Table 2: Definition of parameters

N	Number of disassociated links (remaining links)
N_s	Number of clusters
x_{ij}	Binary variable showing that j connects to cluster i or not
N_i	Number of links associated to cluster i at the beginning
$r_{i,j}$	Distance of disassociated link j to cluster i
c_{ij}	Binary parameter indicating if links i and j are adjacent meaning that they are connected to the same intersection.

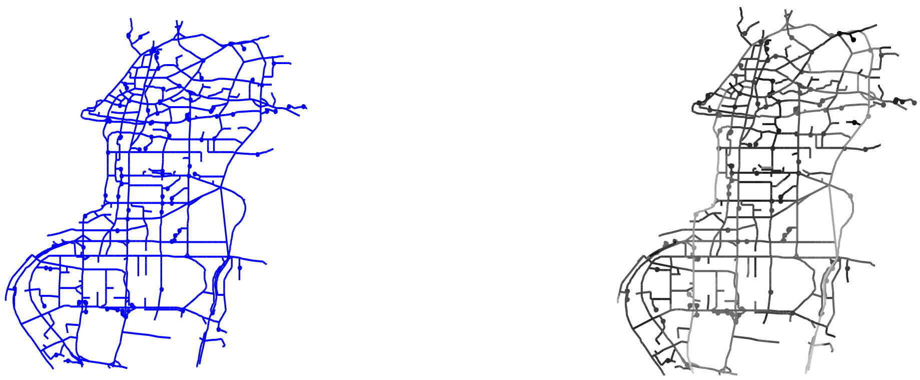
This approach utilizes the same objective function as presented in Eq. (10). To ensure connectivity, for each disassociated link in order to be connected to a cluster, it has to have at least one adjacent link closer to that cluster that has been assigned to it. This has been achieved using constraints written in Eq. (16) for each individual disassociated links.

$$x_{ij} \leq \sum_{\begin{Bmatrix} c_{jm}=1 \\ r_{i,m}<r_{i,j} \end{Bmatrix}} x_{im} \tag{16}$$

3. Case study and results

It would be challenging to investigate the proposed method in a large network with hierarchical structure and complex connectivity since the way traffic evolves and congestion propagates is much different from the simulated network. The case study is the megacity of Shenzhen which is a major city in the south of Southern China's Guangdong Province, situated immediately north of Hong Kong. The dataset contains network structure and daily GPS data of taxis for one month. Average speed of each links is used as a representative values which computed using map-matching algorithm. Upper part of Shenzhen, which has about 2000 links, is analyzed in this study, since more data is available for that part and the speed estimation is more reliable. Fig. 1a shows the upper part of the network and estimated value of speed for different links are depicted as a gray scale image in Fig. 1b. Note that this network has no grid structure with many hierarchical components and different spatial connectivity, which makes the application of the methodology challenging.

Figure 1: (a) Shenzhen network, (b) Grey scale speed profile of different links



(a) (b)

We apply proposed clustering algorithm in Shenzhen network during the peak period of congestion and examine the performance with different number of clusters. A distinct sub-space consisting 8 distinct snakes are obtained in the first step and an optimization is solved for each combination of 2 to 4 snakes in the second step (fixed number of clusters). The values for the parameters applied in the model are presented in Table 3.

Table 3

N_{min}	200
N_{max}	2000
a'	0.8 - 0.7 - 0.6
a''	0.05

The results of the clustering method with histogram of speeds for different clusters are depicted in Fig. 2. By comparing average values of speed in different clusters presented in Table 4, we could easily see that our method has the ability to differentiate between clusters with different speed values. The partitioning process produces a series of different partitioning with different number of clusters. Hence a metric is used to evaluate different partitioning and estimate the optimal number of clusters (Eq. (17)).

$$TV = \sum_{i=1}^{\#Number\ of\ clusters} N_i \times var(C_i) \tag{17}$$

where N_i is the size of cluster C_i.

Figure 2: (a) Shenzhen network, (b) Grey scale speed profile of different links

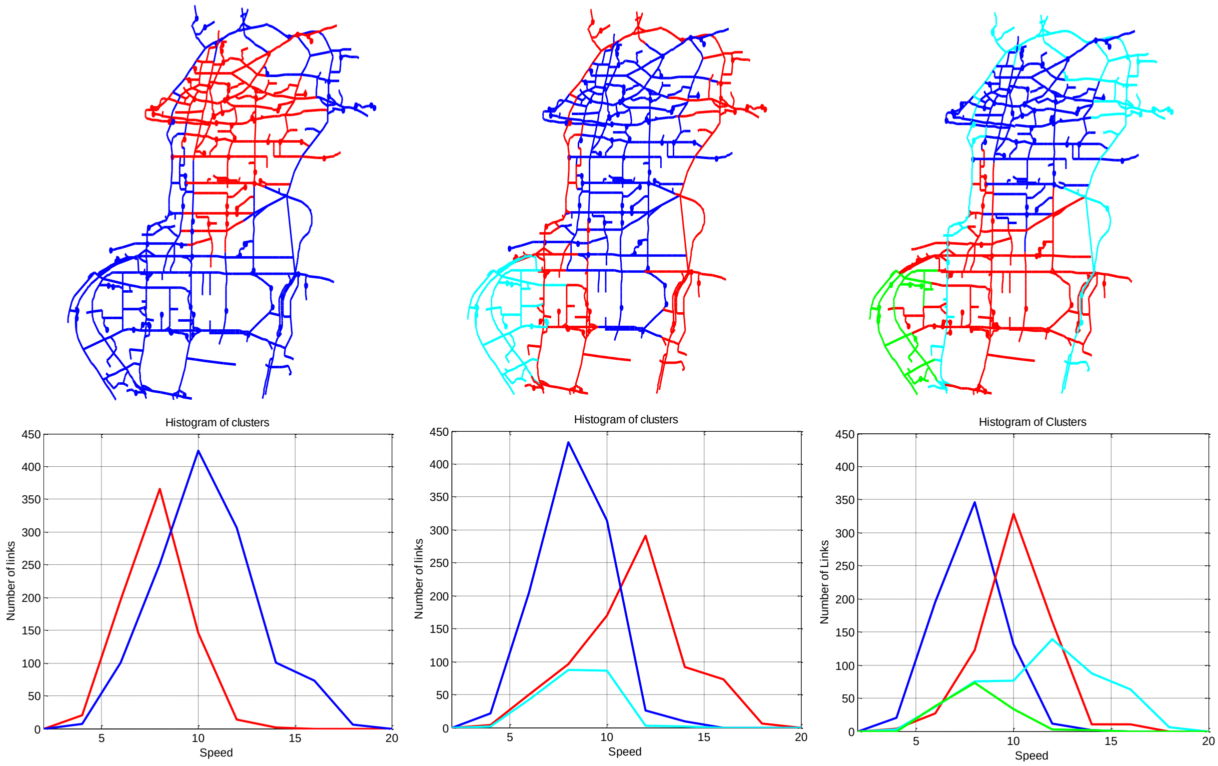


Table 3: Average values and standard deviations of link speeds for different clustering results [m/s]

Mean/Standard deviation	Blue	Red	Cyan	Green	TV($\times 10^4$)
2	10.42/2.51	7.86/1.57	-	-	0.98
3	8.03/1.69	11.29/2.60	8.44/1.64	-	0.87
4	7.79/1.56	10.10/1.68	11.48/3.00	8.10/1.67	0.84

4. Conclusion and future work

A three step optimization framework model for partitioning a heterogeneous network into connected homogeneous sub-regions was developed and tested in a large-scale real-world network. The proposed model takes into account the dependencies between adjacent links, value of the links and size of clusters in three steps. The main advantage of the proposed method is that it could deal with networks with different structures changing from perfect grid to the networks with low connectivity. Moreover, it could capture very well directional congestion which happens in many in different direction in morning and evening peak hours. The estimated method can be utilized in perimeter control which works based on the concept of MFDs to improve network performance since homogeneous clusters have low scatter MFDs.

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