Approximating Equilibrium Conditions with Macroscopic Fundamental Diagrams

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Abstract

Real-time coordinated traffic management strategies that benefit from parsimonious models with aggregated network dynamics, provide a new generation of smart hierarchical strategies to improve network capacity and performance. However, this raises the question of route choice behavior in case of heterogeneous urban networks, where different parts of the city are subject to different types of control. Traffic equilibrium phenomena have not been thoroughly investigated in these models. Approximate traffic equilibrium conditions can be integrated within the parsimonious traffic models to develop regional routing strategies, while detailed route choice strategies can be incorporated at a later stage in a hierarchical framework. In this study, we develop an aggregated and approximate DTA procedure to be incorporated in the macroscopic fundamental diagram (MFD) dynamics, and establish dynamic user equilibrium (DUE) conditions. The methodology consists of two main components; stochastic network loading and a fixed-point solution method. Loading procedure is designed to handle stochastic components in the model such as trip length uncertainty, variation of speeds across the links, perception error of travelers. The results taken from this procedure are averaged through the well-known method of successive averages (MSA) to reach fixed-point solution for the system. Real-time route guidance strategies can be revisited towards a "system of systems" approach.

Keywords

DTA, MFD, user equilibrium, MSA

1 Introduction

Real-time coordinated traffic management strategies (e.g. perimeter control, gating) that benefit from parsimonious models with aggregated network dynamics, provide promising results towards a new generation of smart hierarchical strategies to improve network capacity and performance, see for example (Keyvan-Ekbatani *et al.,* 2012, Geroliminis *et al.,* 2013, Aboudolas and Geroliminis, 2013). However, this raises the question of route choice behavior in case of heterogeneous urban networks, where different parts of the city are subject to different types of control. The developing optimization framework will not be realistic if it does not consider that people might adapt their route choices as a function of control. For example people might choose longer but more reliable routes when heavy congestion occurs. Additionally, traffic management that aims to establish equilibrium conditions by routing vehicles to the minimal travel time paths can significantly improve the performance. Equilibrium conditions are critical in this framework to produce a fair and efficient system, where users experience comparable travel times. Otherwise, the system can easily be disregarded by the users because of the unfair travel times suggested to them.

Dynamic traffic assignment (DTA) has been extensively studied in this sense to satisfy dynamic equilibrium conditions. Various traffic performance models spanning from link travel time functions to detailed microscopic simulation models have been incorporated in DTA. In this study, we incorporate a DTA model into the macroscopic fundamental diagram (MFD) framework, and establish dynamic equilibrium conditions. Given the heterogeneous distribution of congestion in a city and the multiple MFD regions, a region-based route choice model is developed, which can be exploited to test the real performance of dynamic traffic management systems or to develop a regional routing strategy.

A homogeneous urban region (with small spatial link density heterogeneity) can be modeled with MFD, which provides a unimodal, low-scatter, and demand-insensitive relationship between network vehicle density and space-mean flow (Geroliminis and Daganzo, 2008). However, urban transportation networks exhibit uneven distribution of congestion which leads to a scattered flow-density relationship. Heterogeneity in congestion distribution can affect the shape/scatter or even the existence of MFD (Buisson and Ladier, 2009, Geroliminis and Sun, 2011). By using a grid network and considering variance of link density as independent variable, Mazloumian *et al.* (2010) shows that MFD remains well-defined in sub-regions of the urban network. These results are very critical, because MFD concept can be useful for heterogeneously loaded cities, if the network can be partitioned into a small number of homogenous regions. The effect of heterogeneity has been recently studied by many researchers with similar conclusions with empirical data and simulation, see for example (Knoop *et al.,* 2013, Mahmassani *et al.,* 2013, Geroliminis and Sun, 2011). In addition, Gayah and Daganzo (2011), Mahmassani *et al.* (2013) and Leclercq and Geroliminis (2013) thoroughly investigate the effect of driver adaptation and route choice on the shape of MFD. To deal with heterogeneous urban networks, Ji and Geroliminis (2012) develop a partitioning mechanism to minimize the variance of link densities while maintaining a spatially compact shape. Resulting sub-regions can be used to develop macroscopic traffic control strategies; e.g. perimeter control. Haddad *et al.* (2013) develop cooperative control strategies for a large scale mixed transportation network that consists of one freeway and two homogeneous urban regions. They implement model predictive control schemes to determine the optimal inter-transfer flows at the boundary of two regions and the metering rate at the freeway entrance. In order to account for the change in route choice decision in response to the new control policy, they develop a dynamic simple route choice model, in which travelers choose the route with minimum instantaneous travel time. However, this model does not satisfy the equilibrium conditions. The exemplary multi-region urban network presented in Figure 1 shows a case where there are multiple paths connecting the same OD regions. In this framework, demand assigned to these alternative paths must be known in advance to predict traffic conditions in the network and to take better control decisions accordingly. A traffic assignment procedure that establishes equilibrium conditions is the only way to obtain the demand level on the alternative paths. Note that previous empirical studies observed time-invariant trip lengths for homogeneously congested networks (Geroliminis and Daganzo, 2008). In this work, we show that in case of heterogeneous networks with multiple MFD regions, regional trip lengths can vary over time because drivers have the ability to choose a different sequence of regions to decrease their travel times.

In this paper, we incorporate a DTA model into the MFD, and establish DUE conditions regarding experienced travel costs. The approach presented in this study can be deployed as a next step to evaluate the real effect of dynamic traffic management strategies using MFD dynamics or to develop regional routing strategies aiming for equilibrium conditions. The outline of the paper is as follows. The next section introduces the methodology that establishes DUE conditions in a multi-region MFD framework. The following section presents the results of a case study, and the last section gives the discussion and the conclusion of the study.

2 Methodological Framework

Differently than the existing DTA studies, we try to determine the sequence of regions between different origin-destination pairs instead of the exact sequence of links. While this approach might sound computationally lighter, it creates additional challenges in the modeling part (from an assignment point of view), as for example that trips which start later in a region can finish

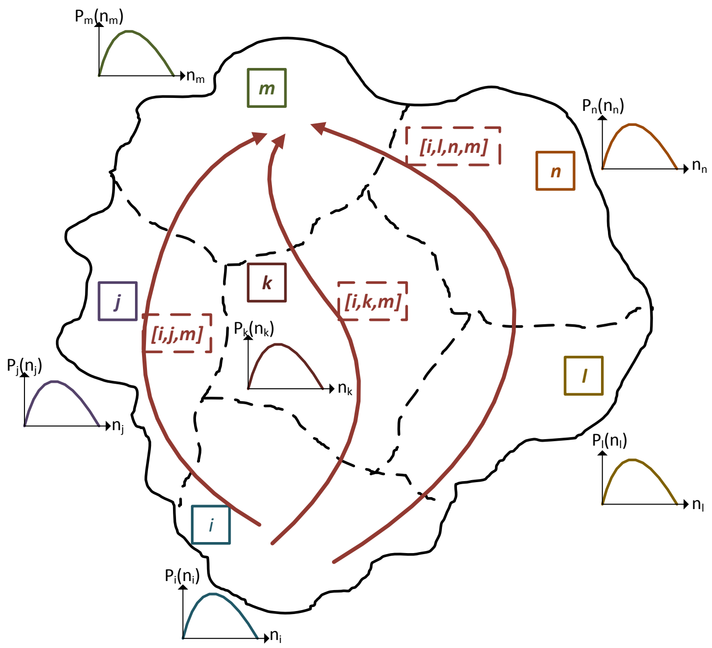

Figure 1: A multi-region urban network

earlier, because of variant trip lengths. Additional challenges (from an MFD point of view) is that this approach requires aggregate modeling of traffic flow within urban regions, and it describes traffic flow propagation within or between the regions using average trip lengths. However, average trip length is not informative enough to build a route choice framework; instead, trip length distributions within each region and for each regional OD (i.e. origin and destination regions) must be considered to ensure DUE conditions in the system. This study does not explicitly determine trip length distributions, but it deals with them in an iterative way within stochastic network loading (SNL) procedure. The results taken from SNL are processed through the well-known fixed-point solution algorithm called method of successive averages (MSA). MSA is an effective solution heuristic which is highly implemented in simulation-based DTA (Peeta and Mahmassani, 1995). Given that it does not require derivative information for the flow-cost mapping function, MSA can be easily adapted to simulation studies. MSA uses the traffic model (i.e. MFD dynamics in this case) in each iteration, to project future traffic information as part of the direction finding mechanism in searching for a solution. Figure 2 depicts a flowchart of the methodological framework, whose details will be presented in the text.

Figure 3 presents two representations of the same network that we investigate in this paper. Link-level representation is valid for most of the conventional traffic performance models (e.g. outflow models, mesoscopic models, etc.). However, detailed information (e.g. signal settings, lane configuration, etc.) is needed to accurately model link and node dynamics. On the other

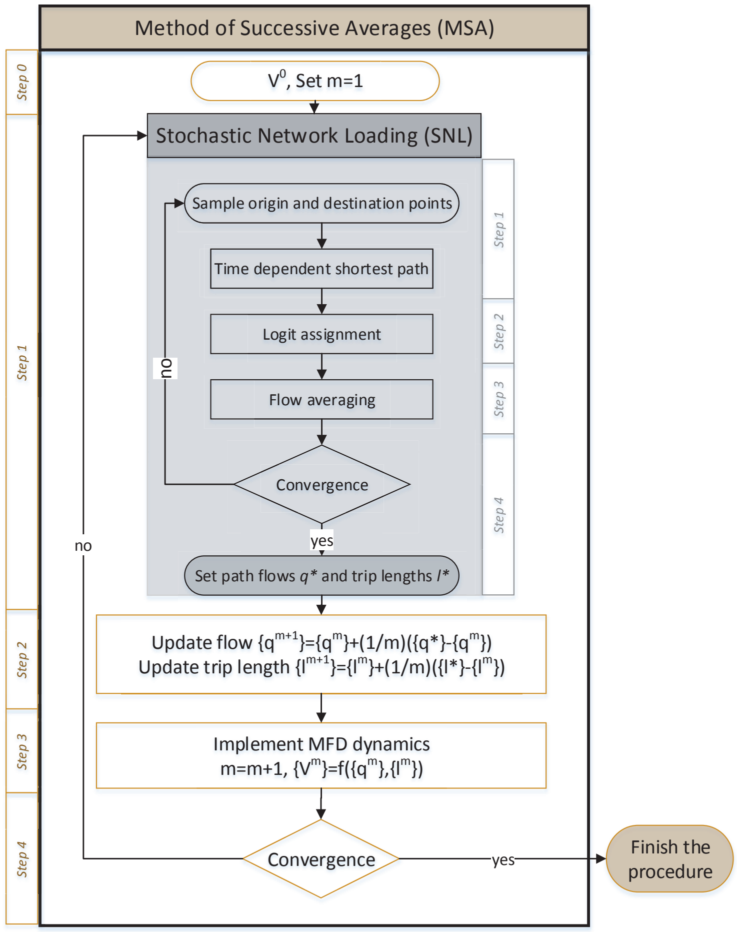

Figure 2: Methodological Framework

hand, regional representation, which can be employed in MFD dynamics, consists of partitions with separate MFD functions and boundaries connecting them. The assignment model developed in this paper makes use of both representations. The graph structure that builds the link-level representation (without any link-level traffic dynamics) is employed in SNL to produce aggregate traffic parameters, while regional representation is needed to run MFD dynamics.

SNL does not assign any explicit travel time distribution on the paths. Instead, by assuming origins and destinations to be randomly distributed in the corresponding regions, the model generates start and end points for trips within the same region or across regions. The procedure, then, deploys the time dependent shortest path algorithm developed by Chabini (1998) on each alternative regional path for the generated start and end points. This algorithm provides the optimal run solution for all-to-one dynamic shortest path problems. Note that regional path defines the sequence of regions to be followed to reach the destination point, not the detailed sequence of links (see Figure 4a). However, shortest path calculation is done on the graph which represents the connectivity between all the links (see Figure 4b). Shortest path taken from

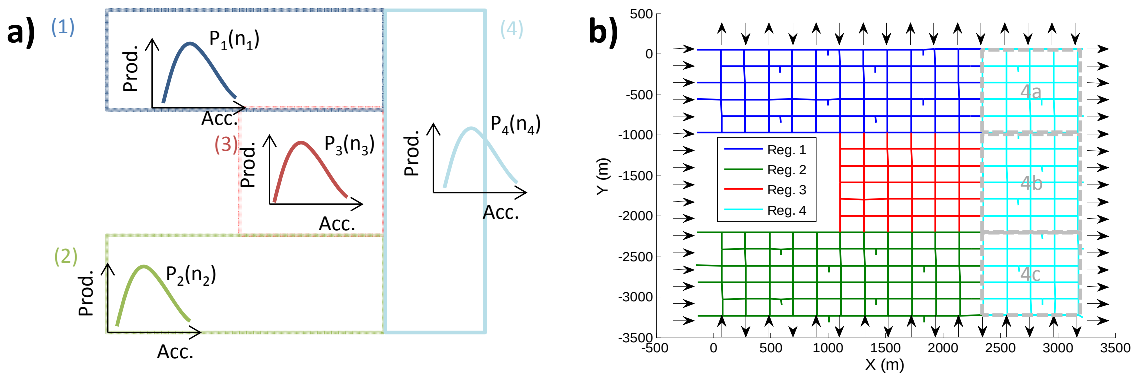

Figure 3: Case study network a. regional representation, b. link-level representation

the algorithm is divided into smaller sections, which define the trip length crossed in different regions (see Figure 4b). Speed on the links is based on the accumulation of the vehicles in the corresponding region; speed information is determined from the MFD relationship and accumulation level. In other words, the average speed in the region extracted from MFD relationship is considered as the representative for the speed of all links in the region. Finally, the procedure takes a regional path decision based on logit discrete choice model and shortest travel times on alternative regional paths. The sampling procedure is repeated until the convergence conditions are satisfied. SNL returns average trip length and regional path ratios that result from the loading procedure.

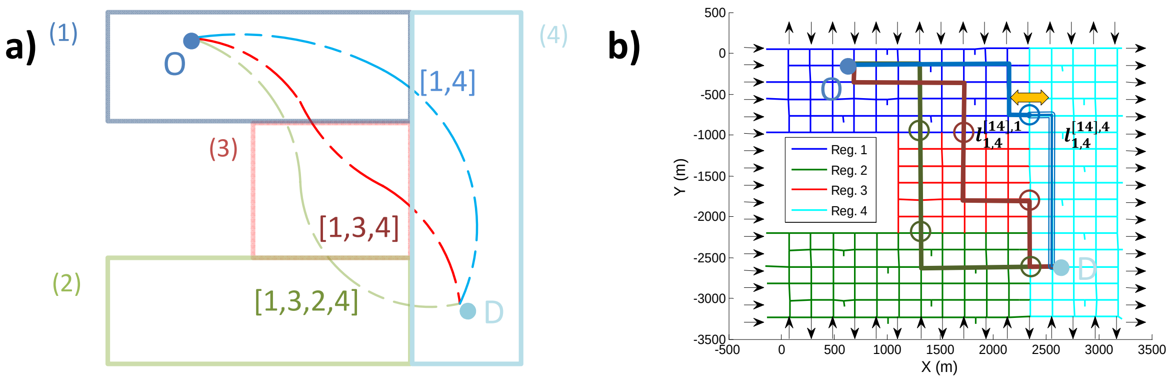

Figure 4: Sampling procedure on a. regional representation, b. link-level representation

Traffic equilibrium models can be formulated as fixed-point problems, where an additional cycle of assignment and loading steps yield the same traffic conditions. The well-known solution heuristics, MSA, is a suitable method in our study considering the characteristics of the problem in hand. MSA has been used in both static and dynamic network equilibrium problems as an incremental assignment type heuristic (Daganzo and Sheffi, 1977, Mahmassani and Peeta, 1993). The method is based on predetermined step sizes along the descent direction. In other words, step size is not determined with respect to the characteristics of the current solution, which requires derivative information. Instead, it is determined a priori. Therefore, the MSA stands as one of the most effective solution heuristics in case the derivative information is difficult to be acquired. Note that the convergence of MSA is not monotonic. This is because of random search direction (auxiliary values produced by stochastic network loading may sometimes point in a direction where objective function increases) and the fixed move size (predetermined step size, $\alpha_m = 1/m$, may overshoot the reduction in the objective function, as it incorporates no information related to the optimal solution neighbourhood). In addition, one can claim that convergence criterion used in MSA is forced to converge due to the nature of step size sequence $\{\alpha_m\}$. However, practical experience indicates reasonable convergence speed and existence of stable solution, before it is forced by the sequence of step size. Regarding size, the number of variables (i.e. speed in each region at each time period) that have to converge is significantly low in this model compared to traditional traffic assignment models. Therefore, a more sophisticated fixed-point algorithm is not expected to improve the results. MSA is an adequate approach to solve the small sized problem in hand.

Further details on the methodological framework will be presented in a journal version of the paper.

3 Model Implementation

This section introduces the results of the DTA model developed in this paper. Figure 5a and b display the evolution of speed and accumulation in the regions that results from the developed traffic assignment model. Results show that assignment model presented in this paper is able to distribute the congestion in an equal manner without details at individual link level. Although the accumulation level differs across the regions, speed values are very close to each other (see Figure 5a and b). Although DUE conditions do not always yield similar traffic conditions throughout the network, in this specific example, we expect to observe similar conditions in the equilibrium state. Unless they by-pass relatively congested links considering the full speed information, vehicles in a grid network are intended to choose a path with the physical length equal to the manhattan distance between origin and destination points. Therefore, in the equilibrium state, where the actual travel times experienced by travelers of the same OD pair departing at the same time are equal and minimal, regions are supposed to have similar average speeds, as there are certain paths in use that cross them.

Figure 6 provides detailed results for the regional OD pair (24), see Figure 4a for regions. Figure 6a displays the route choice parameters; portion of the demand that chooses particular regional

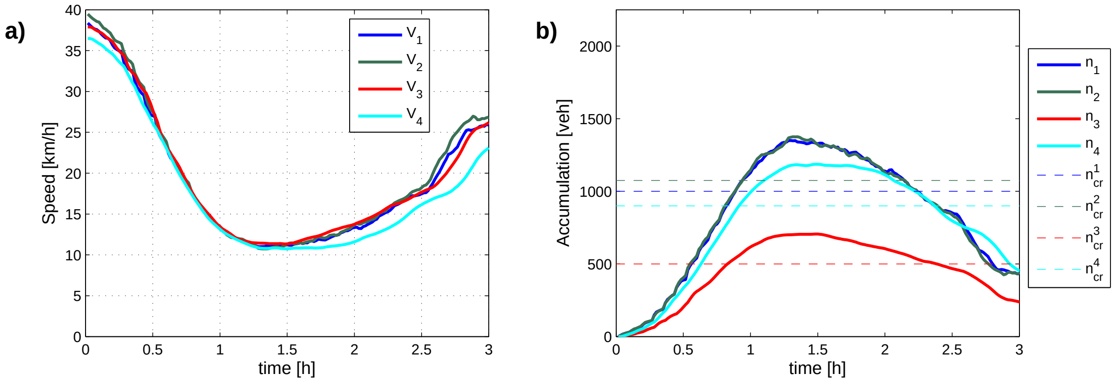

Figure 5: Results from the developed DTA model a. speed evolution, b. accumulation evolution

paths. Note that θ_{od}^{p} represents the portion of the demand that travels between region o and d, using path p. In this context, path enumeration is as follows: path 1 is 2→4, path 2 is 2→3→4, and path 3 is 2→3→1→4 (see Figure 4a for regional path examples). Note that the demand is always assigned to one of these three paths, and the sum of route choice parameters is always equal to 1. Route choice parameters are not subject to oscillatory behavior throughout the simulation time, which indicates consistency with the neighboring time periods. However, this aggregate information is not sufficient to expose the dynamic changes in path assignment.

Figure 6b depicts the route choice parameters for the subset of the OD demand that has multiple options. Note that region 4 is divided into three sub-regions (see Figure 3b). As described in the previous sections, this study deploys a time-dependent shortest path algorithm on a graph where the link speeds are represented by the average speed of the region. Therefore, it is very likely that the shortest path between an origin and destination point has a physical length which is equal to the manhattan distance between the same points. This behavior also affects the route choice decision; travelers have different number of alternative regional paths depending on the exact location of their destination point in region 4. For example, people who travel between region 2 and 4c have no alternative. The only possible regional path that gives manhattan distance solution is (2-4). Travelers between region 2 and 4b have two alternatives; 1→4 and 1→3→4. Likewise, travelers between region 2 and 4a have three alternatives; 1→4, 1→3→4 and 1→3→2→4. Figure 6b presents the portion of the demand that chooses a particular path among a set of alternatives. In other words, θ_{od}^{p*} represents the portion which is not forced to use path p, but prefers path p over other alternatives. Recalculation of route choice parameters allows us to see the actual dynamic change in path decisions throughout the simulation time, especially for the demand assigned to the third path.

Figure 6c displays total average trip length on different paths. Note that l_{od}^{p} represents the total average trip length for vehicles that travel between region o and d, using path p. Because of the underlying route choice phenomenon explained in the previous paragraph, the order of

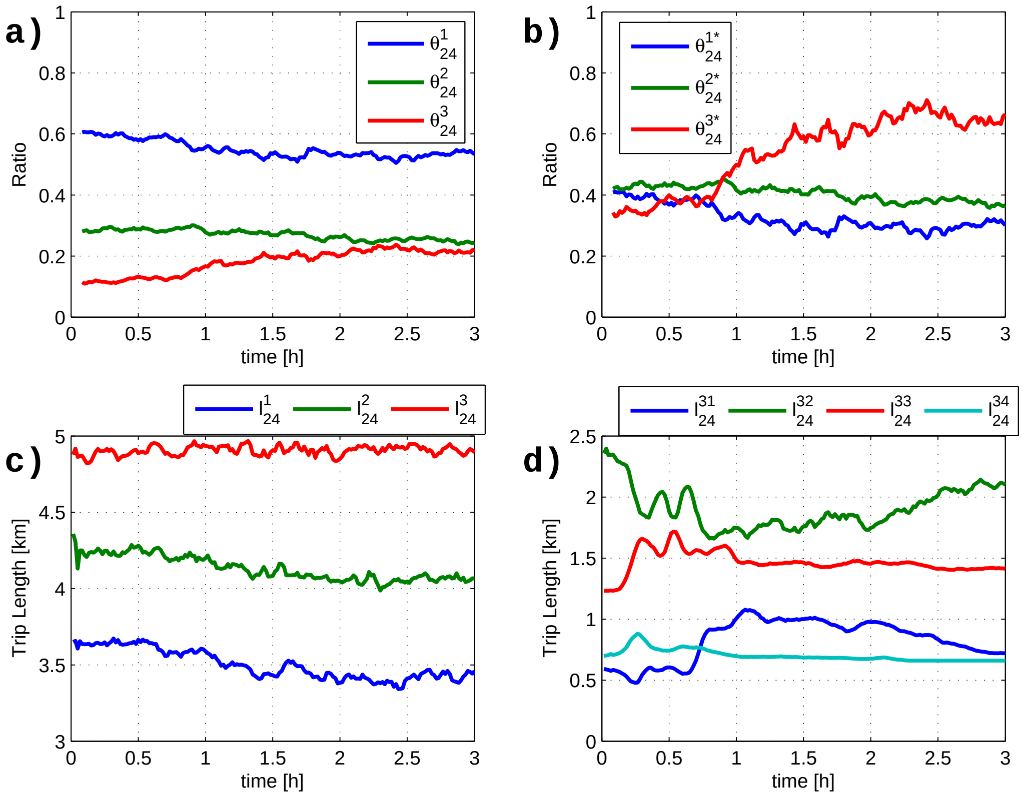

Figure 6: Detailed results for OD pair (24) a. route choice parameters, b. route choice parameters for part of the demand with multiple path options, c. average trip length throughout the path, d. partial trip lengths on a particular path

magnitude of the trip lengths is different; third path has always the longest average trip length, and first path has always the smallest. The total average trip length for the first and second path slightly goes down with time because of the shift of demand to third path. In other words, an increasing number of travelers prefer to use third path to reach their destination in 4a, this leads to a decreasing portion of travelers that have relatively longer trip length among the the vehicles that choose first and second path. Therefore, the total average trip length decreases within this group. Figure 6d displays partial average trip length across different regions on the third path; 1→3→2→4. Note that l^{pr}_{od} represents the total average trip length crossed in region r by the vehicles that travel between region o and d, using path p. The evolution of partial trip lengths shows a quite dynamic behavior, which means travelers change the trip length they cross in different regions according to the traffic conditions in the network. For instance, Figure 5a shows that region 2 has slightly higher speed values in the beginning and end of the simulation compared to other regions. The difference in speeds leads to an increase in the trip length crossed in region 2 in the start and end of the simulation compared to other time periods (see Figure 6d). In addition, region 4 has always the lowest speed values throughout the simulation time, this results in trip length in region 4 almost always at its lower bound (see Figure 6d).

4 Conclusion

In this study, we develop an approximate DTA model which is employed with MFD dynamics, and can establish DUE conditions. The approach presented in this paper has two main components; SNL and MSA. The first part addresses variant trip lengths within and between regions, randomness of the system due to aggregate modeling and perception errors which may become significant in case of the aggregate approach in hand. The second part has been deployed to average auxiliary directions taken from SNL and to establish equilibrium conditions in an iterative manner. Although MSA applications suffer from real-time feasibility issues in case of detailed traffic modeling, this approach, relying on the aggregate traffic modeling defined by MFD relationship, has exhibited reasonable convergence speeds.

Ongoing work attempts to extend this study to a route guidance strategy, where advantages of MFD dynamics and conclusions from this study can be utilized in conjunction. It is obvious that the results of this study cannot be directly employed in a real-world route guidance scheme, but this is possible with some extra effort due to the low computational effort and the decent performance in describing aggregated network dynamics. The assignment procedure developed in this study produces aggregate traffic parameters such as average trip length and regional path choice ratio. However, the exact location of vehicles and path choice decision specific to that location are not explicitly incorporated in the model. In addition, the model does not consider local congestion pockets within the region. Therefore, it could negatively affect the homogeneity of traffic conditions inside the region. A hierarchical control approach could be a possible solution to this problem. Control or route guidance on the lower level could use the full instantaneous information from the links, while control on the upper level takes regional path decision deploying MFD dynamics and considering the evolution of traffic conditions in the network in the future time periods.

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