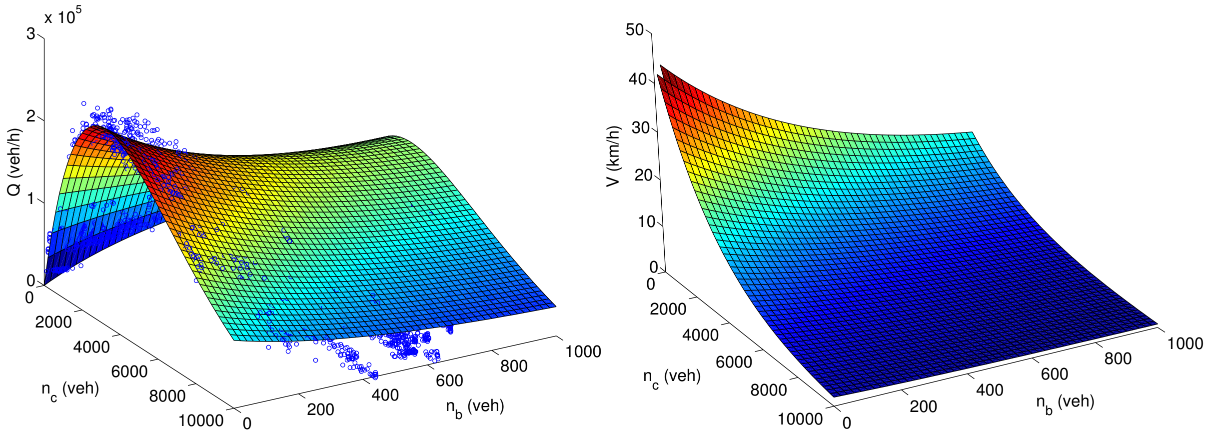

Perimeter flow control of mixed bi-modal urban road networks

Konstantinos Ampountolas

Nan Zheng

Nikolas Geroliminis

École Polytechnique Fédérale de Lausanne (EPFL)

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Konstantinos Ampountolas
School of Engineering
College of Science and Engineering
University of Glasgow
phone: +44-141 330 5343
fax: +44-141 330 4907
Konstantinos.Ampountolas@glasgow.ac.uk

Nan Zheng, Nikolas Geroliminis
Urban Transport Systems Laboratory
School of Architecture, Civil and
Environmental Engineering
École Polytechnique Fédérale de Lausanne
phone: +41-21-693 24 81
fax: +41-21-693 24 79
{Nan.Zheng,Nikolas.Gerolimins}@epfl.ch

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Abstract

In this work we deal with the perimeter flow control problem of bi-modal urban networks by use of a three-dimensional Macroscopic Fundamental Diagram (3D-MFD). The 3D-MFD relates the accumulation of cars and buses and the outflow (or circulating flow) in a bi-modal traffic network. We assume that the impact of each mode on the traffic flow in the network is different, i.e. each bus can not be considered as equivalent to some number of passenger cars. In particular, cars are usually faster than buses (because of the bus stops) but if the percentage of buses in the overall accumulation is high, then the average speed of vehicles certainly differs from the one sustainable without the interference of buses. Thus the maximum throughput (capacity flow) varies with the composition of traffic in the network. For this reason, we introduce the composition of traffic in the network as a parameter that affects the shape of the 3D-MFD. The 3D-MFD is used to describe the aggregated traffic dynamics in the bi-modal urban network. A linear parameter varying model with uncertain parameter the composition of traffic in the network is used as a basis for designing a proportional robust perimeter flow controller. The control gain of the proposed scheme is calculated off-line using convex optimisation and semi-definite programming. To evaluate the proposed scheme, a simulation-based comparison of the robust perimeter flow controller with a pre-timed control plan for an area of the Downtown San Francisco is carried out.

Keywords

Three-dimensional macroscopic fundamental diagram; Bi-modal urban networks; Perimeter flow control; Robust feedback control

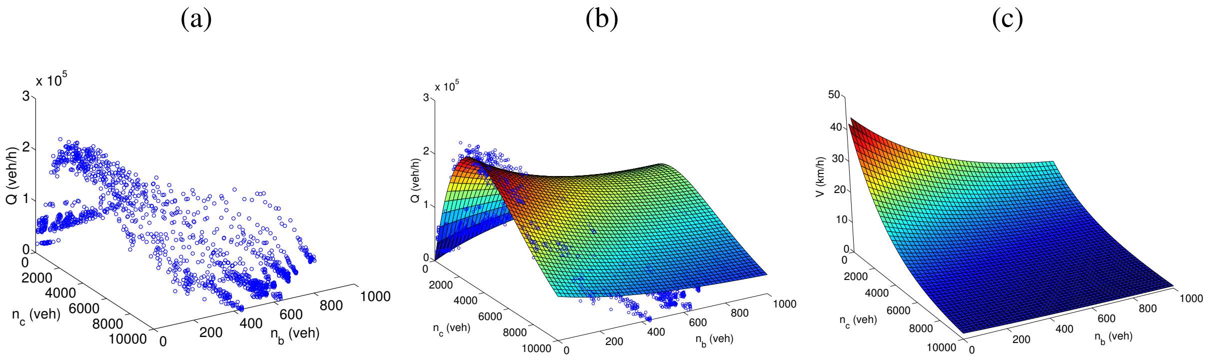
1 Introduction

Urban transportation systems consist of multiple modes sharing and competing for the same road space including pedestrians, non-motorised vehicles, cars, taxis, delivery trucks and more productive modes, such as buses or trams. Realistic modelling and efficient control of multimodal transportation systems remain an important challenge, due to limited understanding about the dynamic interactions of the modes at the network level. In this paper, we focus on bi-modal urban networks consisting of private cars and public transport. It is intuitive that the effect of the public transport stops to pick up and alight passengers during light demand conditions in the network capacity is almost negligible, but for severe congestion and high frequency in time and space of these stops the performance of the system is influenced and interactions should be considered. An aggregated modelling for multimodal systems following the concept of a Macroscopic Fundamental Diagram (MFD) can be a strong alternative if it unveils similar properties as in the single-mode case of vehicular traffic (see Geroliminis and Daganzo (2008), Daganzo and Geroliminis (2008)). Perimeter control strategies based on the concept of MFD for single-mode (car only) networks have been analysed in Haddad and Geroliminis (2012), Keyvan-Ekbatani *et al.* (2012), Geroliminis *et al.* (2013), Haddad *et al.* (2013), Aboudolas and Geroliminis (2013).

Recently, Geroliminis *et al.* (2014) have reported promising modelling results for bi-modal networks that can allow simple perimeter flow control policies to be developed. More specifically, they identified that traffic movements in mixed bi-modal urban networks can be modelled on an aggregate level by use of a generic tool like the MFD. Based on simulation data, the authors have introduced a three-dimensional MFD (3D-MFD) relating the accumulation of cars and buses with the total circulating flow in the network (and the corresponding transformation with the space-mean speed in the network). Moreover, the authors have unveiled the importance of considering passengers rather than vehicles flows by proposed an elegant model to estimate a passenger 3D-MFD. The authors showed that: (i) the network's vehicle throughput decrease monotonically by increasing the number of buses serving in the network, (ii) the space-mean speed decreases monotonically as the number of cars and buses increases, and (iii) the passenger throughput is maximised at a non-zero accumulation of buses.

In this work we put some effort to deal with the perimeter flow control problem in bi-modal urban networks by use of the 3D-MFD. We assume that the impact of each mode on the traffic flow in the network is different, i.e. each bus can not be considered as equivalent to some number of passenger cars. In particular, cars are usually faster than buses (because of the bus stops) but if the percentage of buses in the overall accumulation is high, then the average speed of vehicles certainly differs from the one sustainable without the interference of buses. Thus the maximum

Figure 1: (a) Pairs of bi-modal traffic (composition rate δ); n_c is based on real origin-destination data; n_b is determined by the number of public lines in the network and their operational frequency; (b) The approximated 3D-MFD relating accumulation of cars and buses with output (circulating flow); (c) The approximated 3D-MFD relating accumulation of cars and buses with space-mean speed.



throughput (capacity) varies with the composition of traffic in the network. For this reason, we introduce the composition of traffic in the network as a parameter that affects the shape of the 3D-MFD. We consider a city with an extensive network of public bus lines with a (slow) varying range of service frequencies. The number of public lines and the service frequency can determine the composition of traffic rate in the network, which is assumed slowly time-varying. Two different functions (quadratic and exponential according to Ampountolas *et al.* (2014a)) are used to capture the shape of the 3D-MFD. The 3D-MFD is used to describe the aggregated traffic dynamics in the bi-modal urban network. A Linear Parameter Varying (LPV) model with uncertain parameter the composition of traffic in the network is used as a basis for designing a proportional robust perimeter flow controller. The control gain of the proposed scheme is calculated off-line using convex optimisation and Semi-definite Programming (SDP). In order to evaluate the proposed scheme, a simulation-based comparison of the robust perimeter flow controller with a pre-timed control plan for an area of the Downtown of San Francisco, is carried out.

2 Aggregated dynamics of bi-modal traffic

Consider now a city with an extensive network of public bus lines with a (slow) varying range of service frequencies. The number of public lines and the service frequency can determine the composition of traffic rate δ in the network, which is assumed slowly time-varying. We assume that there exist a 3D-MFD, $O(n(t), \delta(t))$, between total accumulation n, composition rate δ and output O (the total circulating outflow in the network), which describes the behaviour of the system when it evolves slowly with time t (see Figure 1 and Geroliminis *et al.* (2014) for details). Furthermore, we assume that the composition rate δ belongs to a polytopic compact set $\Omega = \{\delta(t) \,|\, \delta_{\text{min}} \le \delta(t) \le \delta_{\text{max}}, t \ge 0\}$ that is state independent, where δ_{min} and δ_{max} are the minimum and maximum composition of traffic in the network. The set Ω can be easily specified for a given network from the number of public transport lines in the network and their operational frequency or it can be directly observed with real-time data.

The dynamics of the bi-modal traffic system can be described by the following ordinary differential equation

$$\frac{dn(t)}{dt} = \beta(t) - O(n(t), \delta(t)) + d(t), \quad t \ge 0 \tag{1}$$

where $\beta(t)$, $d(t)$ are the (controlled) input flow and (uncontrolled) traffic demand to the network at time t, respectively. Both accumulation n and composition of traffic δ can be observed in real-time since vehicle accumulation can be directly obtained with different types of sensors while bus transit operations are equipped with GPS trackers capable of providing locational data at any given time.

Given the existence of a 3D-MFD $O(n(t), \delta)$ with an optimum (critical) accumulation $\hat{n}$ at which maximum flow is reached for different δ (see Figure 1 and Geroliminis *et al.* (2014)), the nonlinear model (1) may be linearised around some set point $(\hat{n}, \hat{\beta}, \hat{d})$. The set point $\hat{n}$ may be analytically determined according to Ampountolas *et al.* (2014a) while set point $\hat{\beta}$ can be derived from the inverse image of the 3D-MFD for given $\hat{n}$. Finally, the set-point $\hat{d}$ is usually determined via historical traffic data of a network. Denoting $\Delta x = x - \hat{x}$ analogously for all variables and assuming first-order Taylor approximation, the linearisation yields

$$\frac{d\Delta\dot{n}(t)}{dt} = \Delta\beta(t) - O'(\hat{n}, \delta)\Delta n(t) + \Delta d(t). \tag{2}$$

The linear system (2) with uncertain parameter $\delta \in \Omega$ approximates the original nonlinear system (1) when we are near the equilibrium point about which the system was linearised. This Linear Parameter Varying (LPV) model will be used as a basis for robust control design in next section.

3 Model uncertainty and robust control

Our basic assumption is that the city has an extensive network of public bus lines with a (slow) varying range of service frequencies. Given any composition of traffic with rate $\delta \in \Omega$ vary slowly with the time, the 3D-MFD in Figure 1 and the LPV system (2), we aim at designing a perimeter control strategy for the bi-modal network, which minimises an upper bound of a worst-case cost criterion $\mathcal{L}(n, \beta)$ given the uncertainty of the composition rate δ and an initial condition. In addressing this problem, there are several possibilities for generating the perimeter flow control input $\beta(t)$. A simplest robust feedback law, which is employed in the sequel, is that of constant state-feedback proportional control, i.e., $\Delta\beta(t) = K\Delta n(t)$, where K is a control gain. In this case, the state of the system n is measurable in real-time while the uncertain parameter $\delta \in \Omega$ is assumed known at design time and the control gain K is indirectly dependent on the uncertainty.

A suitable cost criterion for deriving state-feedback control is given by the infinite horizon quadratic cost

$$\mathcal{L}(n, \beta) = \int_0^\infty \left(||\Delta n(t)||_Q^2 + ||\Delta\beta(t)||_R^2 \right) dt \tag{3}$$

where Q and R are positive weighting factors that can influence the magnitude of the state and control actions, respectively. This cost criterion aims at maintaining the LPV system (2) to operate around the desired steady-state $(\hat{n}, \hat{\beta})$ for given $\delta \in \Omega$, while the system's throughput is maximised. The LPV system (2) with a parameter-varying state transition coefficient and a constant control coefficient that describes the evolution of the system in time can be written in standard form (assuming Δd constant or slowly time-varying, i.e. $\Delta d(t) = 0$)

$$\frac{d\Delta \dot{n}(t)}{dt} = A(\delta)\Delta n(t) + B\Delta\beta(t), \quad t \geq 0 \tag{4}$$

where $A(\delta) = -O'(\hat{n}, \delta)$ and $B = 1$.

A robust linear state feedback controller that expresses the aforementioned objective is given by

$$\beta(t) = \hat{\beta} - K\left[n(t) - \hat{n}\right] \tag{5}$$

where K is a control gain, $(\hat{n}, \hat{\beta})$ is a set-point, and n is the total accumulation in the bi-modal network. To calculate the constant gain K the uncertain parameter $\delta \in \Omega$ is assumed known at design time and $A(\delta)$ is parameterised over the polytopic uncertainty region Ω. In this way, the control gain K is indirectly dependent on the uncertainty. This controller calculates the flow of vehicles β that are allowed to enter the network if the current state of the network $n(t)$ is observed in real-time.

The LPV state-feedback control problem can be formulated as parameter dependent Linear Matrix Inequality (LMI) constraints that can be solved using Semidefinite Programming (SDP)

(Boyd *et al.,* 1994) and efficient interior-point optimisation algorithms (Nesterov and Nemirovskii, 1994). More specifically, the calculation of control gain K that minimises an upper bound of the worst-case infinite horizon quadratic cost (3) subject to the LPV system (4) can be effectuated via solution of the following SDP problem

$$\max_{Y,L} \quad \mathrm{trace}(Y) \tag{6}$$

subject to:

$$\begin{bmatrix} -(A(\delta)Y+BL)^{\tau}-(A(\delta)Y+BL) & Y & L^{\tau} \\ Y & Q^{-1} & 0 \\ L & 0 & R^{-1} \end{bmatrix} \geq 0.$$

where $L = KY$ and $Y = P^{-1}$ (P is a positive definite matrix (in the general case of higher order systems including multi-region and multi-modal heterogeneous networks). This problem may be readily solved by public available software (e.g., CSDP (Borchers, 1999)) and the required computational effort is low (polynomial) even for large-scale problems. Moreover, this computational effort is required only off-line, while on-line (i.e. in real-time) the calculations are limited to the execution of (5) with a given constant control gain K and state measurements $n(t)$.

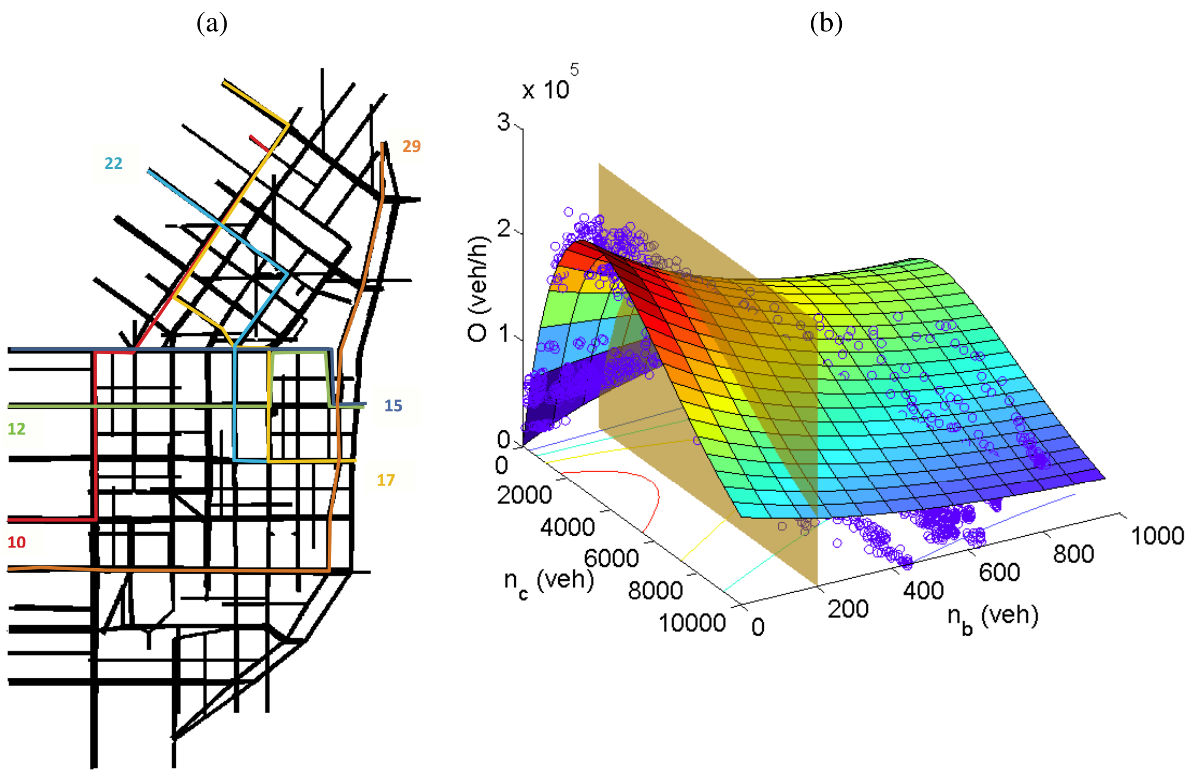
4 Implementation and results

4.1 Design of robust perimeter flow control

The test site is a 2.5 square mile area of Downtown San Francisco that is depicted in Figure 2(a). The implementation of the proposed perimeter strategy to the test site corresponds to the design and application of the robust regulator (5). The controller is activated in real-time at a specific sample interval T (e.g. every 3-5 minutes) and only within specific time windows (e.g. by use of two thresholds n_{act} and n_{stop}), based on the current total accumulation n, to calculate the flow of vehicles β to be allowed to enter the network. The obtained β values (arriving flows) are then used to define the green periods at a number of signalised junctions located at the perimeter of the network. To this end, the arriving flows are equally distributed to the corresponding junctions and converted to an entrance link green stage duration with respect to the saturation flow of the link and the cycle time of the junctions.

For designing the controller the 3D-MFD in Figure 1(b) is used. Figure 2(b) depicts the cross-section (cutting plane) of the 3D surface for a constant accumulation of buses n_b corresponding to

Figure 2: (a) Snapshot of Downtown San Francisco, numbers indicate major public transport lines in the network; (b) The 3D-MFD of Downtown San Francisco relating accumulation of cars and buses with circulating flow; The cross-section of the 3D surface for a constant accumulation of buses $n_b = 200$ demonstrating the typical dependence of flow with the composition of traffic in the network.



a specific number of public transport lines and service frequency in the network (property of the infrastructure), and thus to a slowly time-varying composition of traffic δ. This demonstrates the typical dependence of the flow with the composition of traffic in the network where the maximum flow (optimal operational point) results from analytical formulas according to Ampountolas *et al.* (2014a). In fact, the projection of the 3D surface on the cutting plane $n_b = 200$ in Figure 2(b) provides a typical two-dimensional MFD relating the total accumulation n (where n_b is constant) with the outflow in the network. The shape and characteristics of this two-dimensional MFD for the Downtown San Francisco is similar to that found in previous works (Aboudolas and Geroliminis, 2013). Thus, for different composition of traffic values δ different controllers might be designed. Alternatively, the controller (5) can be designed by solving problem (6) to achieve robust regulation for all $\delta \in \Omega$.

To determine the compact set Ω, simulations have been performed for different demand scenarios to generate various mode compositions with respect to SFMTA real data for the bus frequencies in the public transport lines. The composition of traffic varies from 2% to 15%, i.e. $\Omega =$

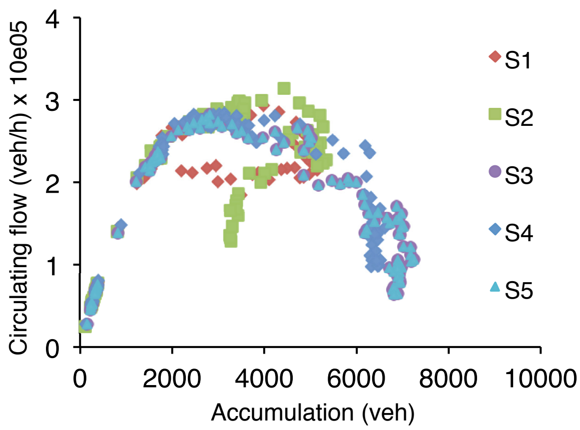
$\{\delta \,|\, 0.02 \le \delta \le 0.15\}$ according to Ampountolas *et al.* (2014a). For the design of the robust controller, the desired accumulation $\hat{n}$ is selected within the optimal range of the 3D-MFD for maximum output with respect to Ω. More specifically, the value $\hat{n} = 2500$ (corresponding nominal arriving flow $\hat{\beta} = 93,540$ veh/h) is selected and the state coefficient $A(\delta)$ in the LPV system (4) is parametrised over Ω. The minimum and maximum permissible entrance flow of mixed traffic are given by $\beta_{\text{min}} = 20,000$ veh/h and $\beta_{\text{max}} = 120,000$ veh/h, respectively. The arriving flows (and operational constrained flows) are equally distributed to 13 signalised junctions located at the perimeter of the network. Finally, the weighting factors Q and R in the cost criterion are set equal to $1/n_{\text{max}}$ ($n_{\text{max}} = 10,000$ veh) and 0.0001, respectively. These values of the parameters above, were found to lead via the solution of problem (6) to control gain $K = 0.0667$ 1/h.

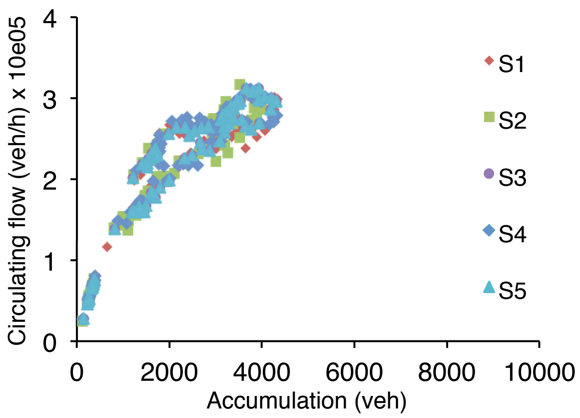
4.2 Results

Figure 3(a) and Figure 3(b) depict the resulting MFD of five scenarios under pre-timed control and perimeter control cases. When perimeter control is applied, the network operates under free flow traffic conditions; under pre-timed control, the network becomes severely congested with states in the congested regime (for almost all scenarios) of the MFD. Moreover, the outflow is maintained to high values around the set point $\hat{\beta}$. We can also observe that the hysteresis formed in the offset period of congestion is reduced significantly, especially for the traffic congested scenarios 2 and 4. The histogram in Figure 3(c) depicts the resulting space-mean speed of each mode in different scenarios. Clearly the proposed perimeter control increases the speed of both modes. In the severe congested scenarios 4 and 5, the speed of cars and buses is improved in average by 60% and 50%, respectively. Additionally, it can be seen that there is a considerable increase in the speed of buses in the less congested scenarios where the space for improvement in the speed of cars is relatively small.

A further analysis of the spatial dimension of traffic congestion in the central avenue (Market ave.) of the network and its upstream links (southeast) can shed more light in the perimeter control actions within the transport public lines. The considered path includes the entire route for public lines 15 and 19 and six other bus lines that overlap part of the path such public lines 5, 11, and 13 (see Figure 2(a)), to investigate the interaction among conflicting public transport lines. To gather the bus trajectories that traverse this path, we simulate buses equipped with GPS-based mobile sensors that reporting their location every 3 seconds. Figure 4 displays the gathered bus trajectories for eight public transport lines (each with different color) during the heart of the rush (11:00 am to 13:00 am), when pre-timed control and perimeter control are applied. In these time-space diagrams, the x-axis reflects the simulation time, while the

Figure 3: Results for five scenarios: (a) and (b) MFDs under pre-timed control and perimeter control cases, respectively. Accumulation at x-axis indicates mixed bi-modal traffic.

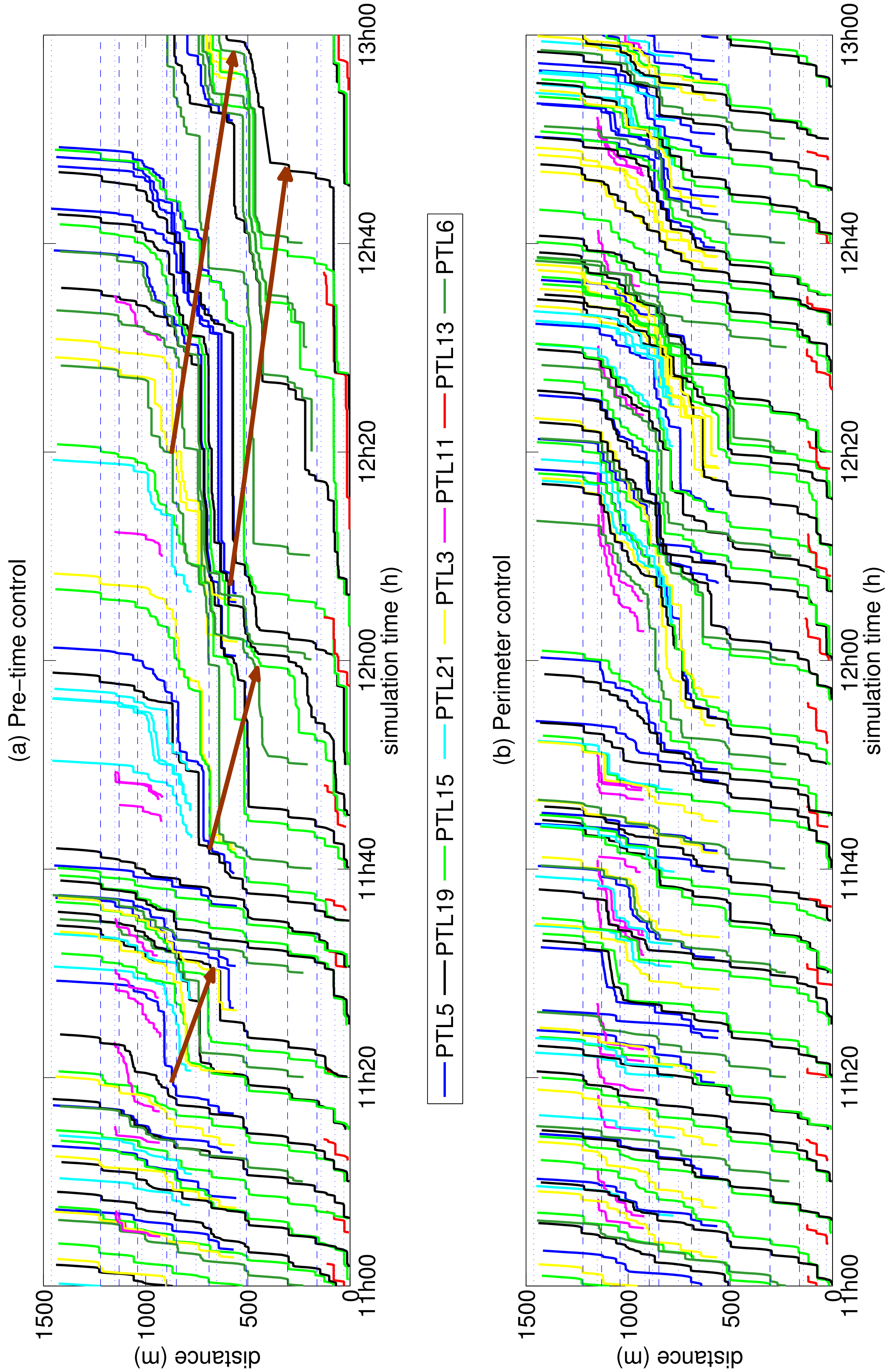




y-axis reflects the one-dimensional distance travelled. Given that the studied network is a grid, the two-dimensional road distance is transformed into one-dimensional by calculating the Manhattan distance (ℓ_1-norm) between the GPS-reported location of a bus and the starting point of the path. The horizontal time distance between consecutive bus trajectories with the same color indicates the headway between two buses servicing the same public transport line. The location of junctions and bus stops are also reported (see caption for details) to allow a better understanding of the stop-and-go phenomena within the public transport lines.

Figure 4 underlines the superiority of perimeter flow control over pre-timed control to maintain public transport lines normal time schedule. Traffic conditions are almost identical for both control cases from 11:00 am to 11:20 am, as time goes on, in the pre-timed control case, buses entering their transport lines (upstream traffic) suffer increasing delays waiting other buses and cars in the centre of the network between 700 m and 1200 m (downstream traffic) to be served. Then traffic condition becomes deteriorated in the center of the network, link queues start spilling back and blocking upstream junctions, thus the entering traffic approximately matches the speed of the downstream traffic. This creates multiple backward moving shockwaves with negative speed that are illustrated with arrows in Figure 4. Clearly when perimeter control is applied the network operates under free-flow traffic conditions and buses are able follow their normal time schedule (with slight travel delays). More specifically, it can be seen that buses only experience delays between 11h50 and 12h30 at the same spatial distance. To further investigate what caused these delays, the traffic conditions in bus line 11 (among others) were carefully analysed. The inspection of different replications eventually shown that the delays are mainly caused by a sudden increase of left turn demand of cars and buses at a specific junction close to the protected network. Note that the existence of such cases can be possible under the perimeter control

Figure 4: Time-space diagram for bus trajectories in several public transport lines in the network during the heart of rush, under pre-timed control and perimeter control cases. Horizontal dotted lines indicate the location of junctions; horizontal dashed lines indicate the location of bus stops.



scheme, since we only control junctions at the perimeter of the network. Extended results for the robust feedback regulators (5) and its comparison with a pre-timed signal plan can be found in Ampountolas *et al.* (2014a,b).

5 Conclusions

In this paper, we addressed the problem of perimeter flow control for bi-modal urban road networks by use of a three-dimensional MFD. We described the dynamics of cars and buses by a Linear Parameter Varying model with uncertain parameter the composition of traffic in the network. We then designed a proportional controller that guarantees robust regulation and stability around a desired set point at the 3D-MFD while the system's throughput is maximised. A key advantage of our approach is that it does not require high computational effort if the current state of the bi-modal network can be observed with loop detector data in real-time. We implemented the proposed controller in a simulation study in Downtown San Francisco. Results showed that the designed robust controller was able to significantly: (i) reduce the overall congestion in the network, (ii) improve the traffic performance of buses, and (iii) avoid queues and gridlock on critical paths of the network.

On-going work considers strategies for controlling two sub-regions of the network where the corresponding composition of bi-modal traffic has different values. This strategy will be able to deal with heterogeneity in congestion levels or bus operation levels. Dynamic lane usage and active Transit Signal Priority (TSP) strategies is another future direction. It is expected that a correct implementation combined with other traffic management strategies (re-distribution of road space, perimeter control and traffic signal optimisation) can significantly improve the mobility of travellers and make public transport more attractive.

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