Exploratory Analysis of Demand Interaction in the Planning of Passenger Railway Service

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Abstract

With the new EU legislature, the track management and the Train Operating Companies (TOCs) have to be separated subjects and hence allowing the private sector to enter the market (also called as liberalization of railways). Up to this point, the market was ruled by one TOC (usually the national carrier) subsidized by local government. Its purpose was to provide accessibility and mobility to the general public. However with a new competition entering the industry, the traditional business model has to be adapted to reflect the demand on profitable lines and at the same time provide the basic accessibility and mobility on lines, that are not profitable (subsidized by government).

In this paper, we analyze the current planning process of the passenger railway service in the context of demand interaction, including references to the fundamental literature on the topic.

Our study shows that the current Train Timetabling Problem (TTP) in the literature does not reflect the actual demand and hence we propose a new problem to be inserted in the planning process: the Ideal Train Timetabling Problem. Such a problem would find the most profitable timetable for each train and would serve as an input to the TTP, in which the feasibility of such timetables would be secured.

Keywords

railway, demand, optimization, timetabling, scheduling, planning

1 Introduction

The time of dominance of one rail operating company (usually national carrier) over the markets in Europe is reaching to an end. With the new EU regulation, the track management and train operating companies (TOC) have to be separated subjects. Thus allowing competition (private sector) to enter the market.

Up to this point, the national carriers were subsidized by local governments and their purpose was to offer the accessibility and mobility to the public (passengers). On the other hand, the goal of the private sector is to generate revenue. In transportation sector, revenue is generated by transporting passengers or goods. The prospective passengers and goods are considered to be the demand to capture, hence TOCs' goal is to maximize the captured demand.

In case of goods, the demand is more flexible and its market is more or less already open for private sector, unlike for passengers. The main driver of goods' demand is cost and in some cases also the trip time. On the other hand, the passenger demand is also sensitive to the time of the departure related to the trip purpose (weekday peak hours for work or school, weekends for leisure, etc.) and others (comfort, perception, etc.).

With the change of the market, the TOCs have to adapt to a new business model and to change their service provided. In this paper, we describe the current way of planning of the passenger railway service, and discuss, how the demand is taken into account in the current planning process. After the analysis, we elaborate on possible changes in the planning process, in a way that new objective is properly taken into account.

2 Railway Planning

Planning a railway operation is a complex task and hence it is partitioned into several problems that are solved sequentially. These problems and the sequence in which they are solved, can be seen on Figure 1.

The main base of providing a railway service is the railway network and as a such, the network design is always the first step in the planning process. However, since the fundamental (or complete) part of the network is usually present, it is not then shown in the planning process. Moreover this decision is not in jurisdiction of TOCs. The TOCs' planning process starts with the line design followed by the timetable design, rolling stock and crew scheduling and train platforming.

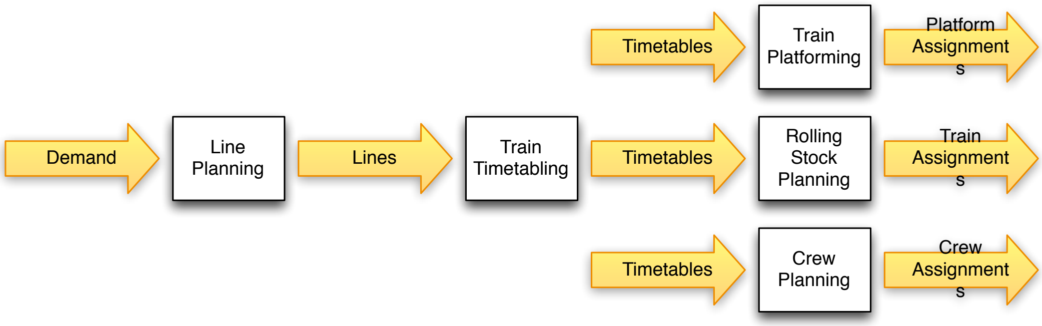

Figure 1: *Planning overview of railway operation*

In the Line Planning Problem, the main input is demand per origin and destination (in form of the OD Matrix) and the global railway infrastructure (network). Based on this input, selection of the most suitable potential train lines, connecting these origins and destinations, is undertaken. Apart of that, frequencies and capacities of the lines are outputted as well.

In the Train Timetabling Problem, as an input serves the pre-created ideal timetables (consisting of the desired start time and the time windows, that give degree of freedom to the problem, in order to find a feasible solution; non-cyclic problem) or frequencies and fixed departures for some of the trains (cyclic problem). The problem then finds the timetables for all of the scheduled trains (result of the Line Planning Problem), such that safety regulations in the railway network are maintained. Two possible timetable settings exist: cyclic and acyclic. The difference between the two is, that trains with the cyclic timetables leave the stations at the beginning of the cycle, *i.e.* if the cycle is one hour, the trains then leave the station every hour in the same minute. The design of this problem suggests, that it serves mainly to the Infrastructure Manager (IM) to secure the safety and the feasibility of the network, whilst maximizing the wishes of TOCs.

The three remaining problems are not directly connected to the demand, they are subject to the created timetables and hence left out of the scope of this study (they are also closer to the day of operation than the two first steps, *i.e.* strategic to operational). Moreover the Train Platforming Problem is in jurisdiction of IM. For more detailed description, including the planning problems, that are not covered in this study, please refer to Caprara *et al.* (2007).

3 Line Planning Problem

The Line Planning Problem (LPP) is the first fundamental step in the railway planning process. The purpose of this problem is to find a set of train lines through the railway network, such that it reflects the passenger demand and that it is also operable by the TOC. Its main restrictions are a global railway network (most often in the form of the preprocessed railway lines to relieve the computational time), the demand along this network and the available rolling stock (capacity, frequency). This problem is considered to be strategic, since building a new railway track (and thus allowing a new additional potential lines to enter the LPP) takes years. This problem is ran by the TOCs.

The line is called a set of trains, that have same route and same stopping stations. The only difference, between the trains in a line, are their respective arrival and departure times. The frequency of a line is an interpretation of the number of trains running on a line in a given period (for instance morning peak hour). Moreover, the line can have been assigned a type, *e.g.* regional, inter-regional, intercity, eurocity, *etc.* The type then defines, which will be the dwelling (stopping) stations: regional trains dwell at every station, inter-regional in the main towns of the regions it passes through, *etc.*

Example of how the result of LPP might look like, can be seen in Figure 2. Note, that the figure also contains the current location of all the trains in the network (marks S, RE, IR, ICN, *etc.*).

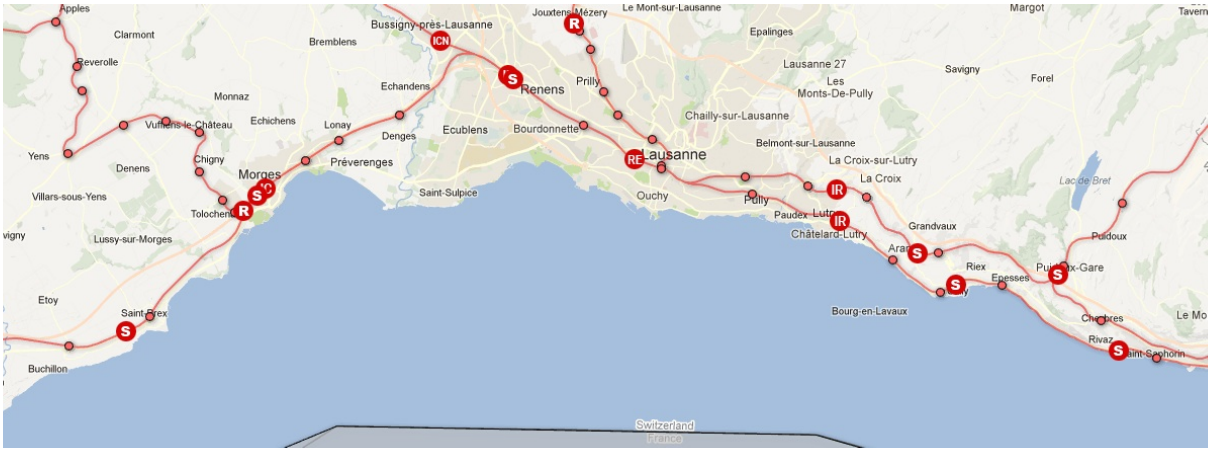

Figure 2: *Example of outcome of LPP (SBB lines in Lausanne area)*[1]

In the LPP, two objectives can be pursued: to maximize the covered demand or (and) to minimize the operational cost. The first objective is maximizing the coverage of direct passengers, this leads to a design of long lines, which are not desirable (propagate delays more easily, need for crew breaks, *etc.*). The second objective has the opposite effect - very short lines, that lead to a high number of transfers for the passengers. Hence the trade-off between the two objectives is a good measure to prevent such consequences.

As mentioned above, the lines are assigned their frequencies and capacities within the model. Again two options are possible: high frequency with small capacity and vice versa. Up to this point, it is on the TOC to pre-assign the combination of the two to every potential line.

[1] source:http://simcity.vasile.ch/sbb/

3.1 Literature

The literature is basically divided into two partitions by passenger based and cost based objective function. One of the first models, that maximizes the direct travelers, can be found in Bussieck *et al.* (1997b). The model is maximizing the amount of direct passengers and uses one binary decision variable (1 – if the line is selected to be in the solution; 0 – otherwise). In order to solve the model efficiently, valid inequalities are introduced. This model is also used by Hooghiemstra *et al.* (1999). The phd thesis Bussieck (1997) extends the methodologies of solving the direct passengers objective and moreover evaluates the minimization of operational costs (and its techniques) as defined by Claessens.

In Claessens and van Dijk (1995) and Claessens *et al.* (1998) different approach is presented: instead of maximizing direct travelers, the minimization of costs is the objective. Moreover additional decision variables on frequency and length of the trains (in terms of the number of carriages) are used. However this leads to a non-linear model and thus linear reformulation is developed instead (one decision variable representing the combination of the above). Another linearizations and additional techniques to solve this model are shown in Zwaneveld (1997), Goossens *et al.* (2001) and Bussieck *et al.* (2004). Goossens *et al.* (2006) also works with the cost optimal model and extends the approach by introducing multiple line types (Goossens (2004) further extends the method). Both of the above types of the model (passenger and cost based) are then presented again in Bussieck *et al.* (1997a).

In Barber *et al.* (2008), another type of the model maximizing passenger coverage is presented. The main difference, comparing to the others, is that the lines are constructed from scratch, instead of using the set of preprocessed lines.

Different kind of model, minimizing the passengers' travel time is presented in Pfetsch and Borndörfer (2005), Schöbel and Scholl (2006) and Borndörfer *et al.* (2007, 2008). The latest paper surveying the published literature on LPP is Schöbel (2012).

3.2 Mathematical Model

The following mathematical model including the description is as used by Caprara *et al.* (2007). The reason is, that in this paper the objective function consists of multiple objectives and is designed to find the trade-off between covering the demand and minimizing the operational cost. The model takes into account only single line type.

Input Parameters Following is the list of parameters used in the model:

$G = (V, E)$	–	undirected graph G representing the railway network
$v \in V$	–	set of stations
$e \in E$	–	set of edges representing the tracks between stations
$p \in P$	–	set of unordered pairs of stations ($p = (p_1, p_2)$) with positive demand
d_p	–	number of passengers, that want to travel between stations p_1 and p_2
E_p	–	set of edges on the shortest path between stations p_1 and p_2
$d_e = \sum_{p:e \in E_p} d_p$	–	the total number of passengers, that want to travel along edge e
$l \in L$	–	set of potential lines (assumed to be known a priori)
E_l	–	set of edges of the line l
$f \in F$	–	set of potential frequencies
$c \in C$	–	set of available capacities
$i \in I$	–	set of indices representing combination of assigned capacity c and frequency f to a line l
k_i	–	operational cost for a combination i (*e.g.* train driver, conductor(s), carriage kilometers

Moreover, the set of stations V can be partitioned into subsets of stations, that are suitable for start and end of the line, and the subset of stations, where it is not possible due to the lack of facilities. It is also assumed, that trains stop at all of the stations in their respective line.

The objective is to select the subset of lines from the potential set L, followed by the assignment of the frequency and the capacity for each of the selected lines. If the TOC desires a cyclic timetable, then it has to be secured, that the desired cycle is divisible by the chosen frequencies.

The demand d_p is usually given per hour and model is mainly solved only for peak hours, as these are the bottlenecks in the network. If we can cover the bottlenecks, then we can also cover the demand for the rest of the day (in terms of the capacity and the frequency). Since the model can handle only single line type only, the demand has to be pre-processed, *i.e.* if the model is run for intercity type and there is a demand between stations s_1 and s_2, where the first station is regional and the second intercity for instance, then the demand starting from s_1 should be transferred to the closest intercity-like station (or another viable approach). Since the passengers seek the shortest traveling time, it is highly possible that they will choose the shortest path through the network.

Decision Variables Following is the list of decision variables used in the model:

$$x_i = \begin{cases} 1 & \text{if and only if line } l_i \text{ is to be operated with a frequency } f_i \text{ and capacity } c_i, \\ 0 & \text{otherwise.} \end{cases}$$

d_{lp} – number of direct passengers traveling on line l between the pair of stations p

Model

$$\begin{aligned}
\max \quad & w_1 \cdot \sum_{l \in L} \sum_{p \in P} d_{lp} - w_2 \cdot \sum_{i \in I} k_i \cdot x_i && && (1) \\
\text{s.t.} \quad & \sum_{i \in I : l_i = l} x_i \leq 1, && \forall l \in L, && (2) \\
& \sum_{i \in I : e \in E_{l_i}} f_i \cdot c_i \cdot x_i \geq d_e, && \forall e \in E, && (3) \\
& \sum_{p \in P : e \in E_p} d_{lp} \leq f_i \cdot c_i \cdot x_i, && \forall l \in L, \forall e \in E, && (4) \\
& \sum_{l \in L : E_p \subset E_l} d_{lp} \leq d_p, && \forall p \in P, && (5) \\
& x_i \in \{0, 1\}, && \forall i \in I, && (6) \\
& d_{lp} \geq 0, && \forall l \in L, \forall p \in P. && (7)
\end{aligned}$$

The multi-objective function **(1)** is optimizing the trade-off between maximization of direct passengers and minimization of the operational costs of the lines. The weights w_1 and w_2 are setting the importance of the 2 respective objectives.

The constraints **(2)** secure that at most one combination of frequency and capacity is assigned to a line. The constraints **(3)** make sure that the capacity provided on each edge is larger than the number of traveling passengers on this edge. The constraints **(4)** specify the relation between the two respective decision variables in a way, that number of passengers traveling on line l between pair of stations p does not exceed the provided total capacity (frequency times capacity of the train set). The constraints **(5)** secure that the total amount of passengers on line l between the pair of stations p does not exceed the total travel demand between these two stations.

4 Train Timetabling Problem

The Train Timetabling Problem (TTP) is the second step in the railway planning process. The objective of this problem is to find a timetable for each train, in such a way that the railway network capacity is not violated, as well as the safety issues. As the main input serves the railway infrastructure and the trains to be scheduled. The problem is tactical, since the train timetables are updated every year (in Europe usually in December, EU directive from 2004 obliges all TOCs in Europe to do so on the same day) and in some countries (France, Great Britain) twice a year. However the timetable development can take more than one year. This problem is ran by the Infrastructure Manager (IM).

The timetable is called an arrival and departure time for each stopping station along the line of a train. The complete timetable is then the set of timetables for each train per given time period. The example of a single timetable of one train, can be seen on Figure 3.

Station	Arrival	Prognosis	Departure	Prognosis	Train	Platform	Occupancy
Genève-Aéroport			16:54		IR 1427	3	1. 2.
Genève	17:01		17:03			4	1. 2.
Nyon	17:16		17:17			2	1. 2.
Morges	17:31		17:32			3	1. 2.
Lausanne	17:42		17:46			3	1. 2.
Vevey	17:59		18:00			3	1. 2.
Montreux	18:05		18:06			3	1. 2.
Aigle	18:16		18:17			1	1. 2.
Bex	18:23		18:24			1	1. 2.
St-Maurice	18:28		18:29			1	1. 2.
Martigny	18:38		18:39			2	1. 2.
Sion	18:53		18:55			2	1. 2.
Sierre/Siders	19:04		19:05			2	1. 2.
Leuk	19:12		19:13			2	1. 2.
Visp	19:22		19:24			6	1. 2.
Brig	19:31					6	

Figure 3: *Example of a single outcome of TTP (SBB InterRegio – Geneva – Brig)*[2]

In general, we can distinguish between two types of timetables: non-cyclic and cyclic. The difference is, that in the cyclic schedules, trains of the same line leave their stations in cycles, *e.g.* if the cycle is one hour, then the trains leave at the same minute every hour.

The main argument of pro-cyclic timetables is the easier memorability for the passengers. Effect of the regular timetables on the demand has been studied in Wardman *et al.* (2004), where a logit model based on the stated preference data is evolved. The results show, that the increase in the demand for regular timetables is very small. This is in line with the fact, that the main drivers for choosing public transit are attributes like travel time, price and reliability.

[2]source:http://sbb.ch

On the other hand, the disadvantages of cyclic timetables are less abstract comparing to passenger demand. The main disadvantage is the higher operation cost. The schedule is almost the same throughout the day, *i.e.* the trains run with same density in and out of the peak hour. The non-peak cost can be reduced by scheduling smaller train sets. However this includes some other operational constraints, when the peak hour starts again (coming from the symmetrical circulation of the rolling stock).

The cyclic schedule favors the passengers that do not look up the departure of the train prior their trip. However the passengers with such behavior are mainly the commuters on highly frequent lines like metro or tram, where few minutes do not play important role. Whereas in the railway service, we can distinguish 2 types of passengers: long distance and commuters. Since the commuters take the train every day, they would know the departure times. On the other hand, the long distance commuters would plan their trip ahead and thus know their departure time prior the travel. Moreover, with the spread of the internet booking, the passengers are more and more aware of their departure times and thus making one of the basic arguments for cyclic timetables invalid.

4.1 Non-Cyclic Timetabling

In the non-cyclic timetabling, there is as an input pre-processed ideal timetable with a possible time windows, in order to be able to shift the timetable, if it is not feasible.

4.1.1 Literature

Most of the models, on the non-cyclic timetabling, in the published literature, formulate the problem either as MILP or ILP. The MILP model uses continues time, whereas the ILP model discretizes the time. Since of the complexity of the problem, many heuristic approaches are considered.

Brannlund *et al.* (1998) use discretized time and solve the problem with lagrangian relaxation of the track capacity constraints. The model is formulated as ILP with a binary decision variable. Caprara *et al.* (2002, 2006), Fischer *et al.* (2008) and Cacchiani *et al.* (2012) also use lagrangian relaxation of the same constraints to solve the problem. In Cacchiani *et al.* (2008), column generation approach is tested. The approach tends to find better bounds than the lagrangian relaxation. In Cacchiani *et al.* (2010a), several ILP re-formulations are tested and compared. In Cacchiani *et al.* (2010b), the ILP formulation is adjusted, in order to be able to schedule extra freight trains, whilst keeping the timetables of the passengers' trains fixed. In Cacchiani *et al.* (2013), dynamic programming, to solve the clique constraints, is used.

In Carey and Lockwood (1995), a heuristic, that considers one train at a time and solves a MILP, based on the already scheduled trains, is introduced. Higgins *et al.* (1997) then show several more heuristics to solve the MILP model.

Oliveira and Smith (2000) and Burdett and Kozan (2010), re-formulate the problem as job-shop scheduling.

Erol (2009) and Harrod (2012), survey different types of models for the TTP.

4.1.2 Mathematical Model

The following mathematical model including the description is taken from Caprara *et al.* (2002, 2007). In the both papers, same model is being presented, whereas in the earlier one in high detail. The model takes into account one-way single line with a starting and ending station and several intermediate stations along the line.

Input Parameters Below is the list of parameters used in the model:

$G = (V, A)$	–	directed acyclic multigraph G
$v \in V$	–	set of nodes
$a \in A$	–	set of arcs
$t \in T$	–	set of trains
$a \in A^t$	–	subset of arcs used by train t
σ	–	source node
τ	–	sink node
p_a	–	profit of arc a
$\delta_t^+(v)$	–	set of arcs in A^t leaving the node v
$\delta_t^-(v)$	–	set of arcs in A^t entering the node v
$\mathbb{C}$	–	family of maximal subsets C of pairwise incompatible arcs

The set of stations $s \in S$ is ordered by the appearance of the stations along the line. A timetable is then defined for each train t by the departure time from the first station f_t, the arrival time to the last station l_t and the arrival and departure times for the intermediate stations $f_t + 1, \ldots, l_t - 1$. Initially, each train t has an ideal timetable with departure time d_{ts} at each station $s \in \{f_t, \dots, l_t - 1\}$ and arrival time a_{ts} at each station $s \in \{f_t + 1, \dots, l_t\}$ assigned. The model then modifies the ideal timetable in order to satisfy the line capacity constraints and the output of the problem is then referred to as the actual timetable.

The nodes v are representing the arrivals and the departures from the stations along the line at a specific times. The arcs a represent the stops at the stations and the trips between the stations. Moreover for each station s except the first one, the nodes associated with an arrival at s at some time are denoted by U^s and for each station s except the last one, the nodes associated with a departure at s at some time are denoted by W^s.

The arc set A is partitioned into arc sets A^t associated with each train t. Arcs in A^t from a node $w \in W^{s-1}$ to a node $u \in U^s$ model train t departing from the station $s - 1$ at the time instant associated with w and arriving at station s at the time instant associated with u. On the top of that, arcs in A^t from a node $u \in U^s$ to a node $w \in W^s$ model train t arriving at station s at the time instant associated with u and departing at the time instant associated with w.

The arcs leaving the source node σ represent the train departures from their first station and the arcs coming in the sink node τ represent the train arrivals at their final station. The graph G then guarantees, that every path from σ to τ using arcs in A^t match a feasible timetable for a train t and vice versa.

The profit p_a associated with arc a is usually represented as follows:

$$\pi_t - \phi_t(v_t) - \gamma_t \mu_t \tag{8}$$

where π_t is the ideal profit of the train t, $\phi_t(\cdot)$ is an user defined nondecreasing function penalizing the train shift v_t at a starting station, μ_t is the sum of differences between the ideal and the actual timetable on the rest of the arcs and γ_t is the is a given nonnegative parameter (*i.e.* the function penalizing the train stretch is assumed to be linear). The profit function is usually the same for the trains of the same type running in the same period of the time. If the result of this function is nonpositive, it is better to cancel the train.

Decision Variables Following is the list of decision variables used in the model:

$$x_a = \begin{cases} 1 & \text{if and only if the path in the solution associated with train } t \text{ contains arc } a, \\ 0 & \text{otherwise.} \end{cases}$$

Model

$$\max \quad \sum_{t\in T}\sum_{a\in A^t} p_a \cdot x_a \tag{9}$$

$$\text{s.t.} \quad \sum_{a\in\delta_t^+(\sigma)} x_a \leq 1, \qquad \forall t \in T, \tag{10}$$

$$\sum_{a\in\delta_t^-(v)} x_a = \sum_{a\in\delta_t^+(v)} x_a, \qquad \forall t \in T, \forall v \in V \setminus \{\sigma, \tau\} \tag{11}$$

$$\sum_{a\in C} x_a \leq 1, \qquad \forall C \in \mathbb{C}, \tag{12}$$

$$x_a \in \{0, 1\}, \qquad \forall a \in A. \tag{13}$$

The objective function **(9)** is maximizing the overall "profit", *i.e.* finding a timetable as close to the desirable timetable as possible.

The constraints **(10)** secure, that at most one schedule per train is selected, *i.e.* the train t leaves the source σ only on one of the possible arcs. Followingly, the constraints **(11)** impose the flow conservation at intermediate nodes. Lastly the clique constraints **(12)** make sure, that the compatibility between selected arcs is satisfied.

4.2 Cyclic Timetabling

In the cyclic timetabling, there is no ideal timetable on the input. That is because of the nature of the problem – the cyclicity. However some of the trains can have a fixed departure time at some of the stations.

4.2.1 Literature

One of the first papers, dealing with cyclic timetables is Serafini and Ukovich (1989). The paper brings up the topic of cyclic scheduling based on Periodic Event Scheduling Problem (PESP). The problem is solved via proposed algorithm.

In Nachtigall and Voget (1996) model for minimization of the waiting times in the railway network, whilst keeping the cyclic timetables (based on PESP), is solved using branch and bound and in Nachtigall (1996) using genetic algorithms. Another algorithm, based on constraint generation, to solve the PESP formulation is presented in Odijk (1996). In Lindner and Zimmermann (2000), branch and bound algorithm is also applied to solve the PESP. In Kroon and Peeters (2003), variable trip times are considered. Peeters (2003) further elaborates on PESP and in Liebchen (2004) implementation of the symmetry in the PESP model is discussed. In Liebchen and Mohring (2002), the PESP attributes are analyzed on the case study of Berlin's underground.

Kroon *et al.* (2007) and Shafia *et al.* (2012), deal with robustness of cyclic timetables. Liebchen and Mohring (2004) propose to integrate network planning, line planning and rolling stock scheduling into the one periodic timetabling model (based on PESP). Caimi *et al.* (2007) introduce flexible PESP – instead of the fixed times of the events, time windows are provided.

4.2.2 Mathematical Model

The following mathematical model including the description is taken from Peeters (2003).

Input Parameters Below is the list of parameters used in the model:

$G = (N, A \cup A^s)$	–	graph G representing the railway network
$n, m \in N$	–	set of nodes
$a \in A$	–	set of regular tracks $a = (n, m)$
$a \in A^s$	–	set of single tracks $a = (n, m) = (m, n)$
$t \in T$	–	set of trains
$n \in N^t \subseteq N$	–	set of nodes visited by train t
$a \in A^t \subseteq A \cup A^s$	–	set of tracks used by train t
$(t, t') \in T_a$	–	set of all pairs of trains (t, t'), that travel along the track a in the same direction, where t' is the faster train
$(t, t') \in T_a^s$	–	set of all pairs of trains (t, t'), that travel along the single track a in the opposite direction, where t departs from n and t' departs from m
$t \in F_n^d, F_n^a$	–	set of all trains t, that have a fixed departure (arrival) at node n
$(t, t') \in S_n$	–	set of all train pairs $(t, t'), t < t'$, for which the departure times are to be synchronized at node n
$(t, t') \in C_n$	–	set of all train pairs $(t, t'), t < t'$, for which turn-around or connection constraint is required from train t to train t' at node n
b	–	cycle of the timetable
h	–	general headway upon departure and arrival at every node
r_a^t	–	time it takes to the train t to traverse the arc a
$\left[\underline{d_n^t}, \overline{d_n^t}\right]$	–	dwell time window of the train t at the node n

$\left[\underline{f_n^t}, \overline{f_n^t}\right]$	–	fixed arrival/departure window of the train t at the node n, in the case of completely fixed arrival/departure $\underline{f_n^t} = \overline{f_n^t}$
$\left[\underline{s_n^{tt'}}, \overline{s_n^{tt'}}\right]$	–	time window for the synchronization of trains t and $t]$ at node n
$\left[\underline{c_n^{tt'}}, \overline{c_n^{tt'}}\right]$	–	time window for the connection or turn around constraint between trains t and t' at node n

The set of nodes N typically consists of all the stations in the network. The regular tracks $a \in A$ are directed. Multiple parallel regular tracks are allowed between the nodes, however in this model maximum of 2 tracks is allowed, otherwise a third index for the track would need to be introduced. The single tracks $a \in A^s$ are not directed. The set of trains T consist of $2f$ trains, since a train line with frequency f yields f trains in each direction.

In the set C_n, if the node n is the destination of the train t, then this train turns into the train t'. Otherwise the pair (t, t') is handled as a connection between the trains.

If there are two trains sharing some parts of their route, then the synchronization can be applied. This means, that if for example both of the trains have frequency one and the cycle is one hour, the synchronization would secure that the trains would leave the shared stations 30 minutes apart and hence secure better coverage for the shared part of the route.

Decision Variables Following is the list of decision variables used in the model:

a_n^t	–	arrival time of train t at node n
d_n^t	–	departure time of train t from node n

In the below model, modulo function is used on a several occasions. Since the timetable is the same in every cycle, it is enough to solve the problem only for one cycle. This is achieved via the modulo function, which is the remainder of the Euclidean division.

Model

$$\max \quad F(a, d) \tag{14}$$

$$\text{s.t.} \quad a_m^t - d_n^t = r_a^t \quad \text{mod } b, \quad \forall t \in T, a \in A^t \tag{15}$$

$$d_n^t - a_n^t \in \left[\underline{d_n^t}, \overline{d_n^t}\right] \quad \text{mod } b, \quad \forall t \in T, \forall n \in N^t, \tag{16}$$

$$d_n^{t'} - d_n^t \in \left[\underline{s_n^{tt'}}, \overline{s_n^{tt'}}\right] \quad \text{mod } b, \quad \forall n \in N, \forall (t, t') \in S_n, \tag{17}$$

$$d_n^{t'} - a_n^t \in \left[\underline{c_n^{tt'}}, \overline{c_n^{tt'}}\right] \quad \text{mod } b, \quad \forall n \in N, \forall (t,t') \in C_n, \tag{18}$$

$$d_n^{t'} - d_n^t \in \left[r_a^t - r_a^{t'} + h, b - h\right] \text{mod } b, \quad \forall a \in A, \forall (t,t') \in T_a, \tag{19}$$

$$a_n^{t'} - d_n^t \in \left[r_a^t + r_a^{t'} + h, b - h\right] \text{mod } b, \quad \forall a \in A^s, \forall (t,t') \in T_a, \tag{20}$$

$$d_n^t \in \left[\underline{f_n^t}, \overline{f_n^t}\right], \quad \forall n \in N, \forall t \in F_n^d, \tag{21}$$

$$a_n^t \in \left[\underline{f_n^t}, \overline{f_n^t}\right], \quad \forall n \in N, \forall t \in F_n^a, \tag{22}$$

$$a_n^t, d_n^t \in \{0, b-1\}, \quad \forall t \in T, \forall n \in N_t. \tag{23}$$

The objective function **(14)** is minimizing the general function F of the timetable. It is up to user to define his/her own objective. Such an objective can be for instance to minimize passenger travel time, maximize timetable robustness, minimize the number of required rolling stock units or to minimize the violation of the initial constraints in case the solution is not feasible.

The constraints **(15)** set the arrival time to the node m based on the departure from the previous node n and the running time on the traversed arc r_a^t. Similarly, constraints **(16)** affect the dwelling time at node n. The constraints **(17)** synchronize the train departures. The constraints **(18)** secure the connection and turn-around between the trains. The constraints **(19)** guarantee no conflict on the track between the nodes and the constraints **(20)** guarantee no conflict in the nodes. The constraints **(21)** and **(22)** model the fixed departures and arrivals. These are not subject to a cycle, hence no modulo operation needed. And lastly the constraints **(23)** show the domain of the decision variables.

5 Additional Demand Enhancing Techniques

Apart of the classical problems (shown on Figure 1), other additional techniques like revenue management, dynamic pricing or discrete choice models can be used to affect the demand. Especially the revenue management, which has been proven effective in the airline industry.

In Ben-Khedher *et al.* (1998), the decision tool RailCap, used by the french national carrier SNCF, is described. The main responsibility of the tool is to adjust the train capacity (by adding new unit to the train, drop empty extra units or open them for reservations on double-unit trains, open an optional train to reservations and assign it an itinerary-compatible fleet type) based on the current reservations, ODs and forecasted demand. The tool is maximizing the expected incremental profit subject to operational constraints (mainly availability of the rolling stock and its routing through the network).

In Chierici *et al.* (2004), model to maximize the demand captured by train is presented. The resulting timetable is cyclic (coming from constraints). The model integrates modal choice model (logit with 3 alternatives - bus, train and car, utility consists of travel time, monetary cost, walking time, average waiting time and comfort). Coefficients are estimated by a revealed preference. Since the MILP is non-convex 2 methods are tested: branch and bound and heuristic approach. Both are tested on a regional network in Italy, with real schedules as input. It is shown that with the current schedule, only 4 % of the population can choose train.

In Lythgoe and Wardman (2002), analysis of the demand for travel from and to airport and a formulation of a discrete choice model are introduced. Whelan and Johnson (2004) are introducing a discrete choice model to decrease the overcrowding on the trains by introducing special ticket costs, when at the same time not reducing the total amount of passengers transported, *i.e.* smoothing the demand along the time horizon. In Cordone and Redaelli (2011), integration of the modal choice model and classic cyclic TTP is presented.

In Li *et al.* (2006), simulation framework based on the dynamic pricing is introduced. In Crevier *et al.* (2012) the aim is to cover the demand with maximizing the profit using different pricing strategies. The model is using preset booked schedules, which are then utilized on an operational level.

Abe *et al.* (2007) describes the revenue management (RM) in the railway industry with case studies of RM around the world. Comparison with the RM in airline industry is elaborated as well. Armstrong and Meissner (2010) shows the overview of RM in the railway industry (both freight and passenger). Wang *et al.* (2012) describes a MILP model for RM. In Bharill and Rangaraj (2008), revenue management for Indian Railways is described.

6 Conclusion

Even though the demand is the driver of any service (not only in the transportation), we can see that in the current railway planning process, it is taken into account only briefly and the main focus, *i.e.* constraint is the railway infrastructure.

In order to adjust the business model, review of the current way of planning of the passenger railway service has been conducted in the above sections. If we have a closer look on the railway planning horizon (Figure 1), we can see that there are two problems directly related to the revenue management of railways: the Line Planning Problem (LPP) and the Train Timetabling Problem (TTP); the others are subject to the results of these two.

The LPP takes as input the passenger demand (in the form of the OD matrix) and the global railway infrastructure (in the form of preprocessed potential lines). The problem then selects the most appropriate lines in order to maximize the coverage of the demand. Apart of that, the problem can also try to minimize the operational cost(s).

The second step is the TTP, where the problem is designed for IM. For the non-cyclic case, as input serve the ideal timetables desired by TOCs (provided with time windows, to give a degree of freedom to IM). The IM then solves the TTP in order to find a feasible solution, such that the railway network capacity and safety is held up to and that the resulting timetables are as close to the wishes of TOCs as possible (maximizing this objective). In the cyclic case, the TOC(s) give the IM the desired frequency of the service along with the list of desired connecting trains. Some of the trains can have fixed departure/arrival. IM then solves the cyclic version of the TTP.

As we can see, there is a gap in the planning horizon: namely the construction of the ideal timetables of the TOCs is missing (in the case of the cyclic problem, this can be addressed as the fixed departure for some trains, *e.g.* the most profitable times). Hence, we are proposing a new problem, that we call the TOC Ideal Train Timetabling Problem (ITTP). The logical location of the problem then would be between the LPP and the TTP (Figure 4).

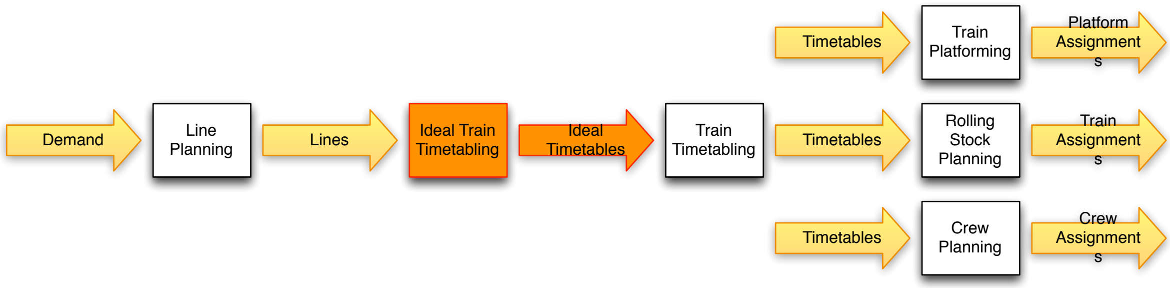

Figure 4: *Proposed planning horizon*

We then propose the following structure of the problem: we consider the lines (outputted by the LPP), the demand along the lines and the cost figures. The demand is in the form of the space-time diagram, where we know the number of potential travelers for each pair of stations (as an OD matrix) per minute. We also consider so-called trip time threshold, the time by which the traveller is willing to postpone his/her trip.

The cost figures include the train driver cost, carriage per kilometer cost, subsidies from the local government (in the form per kilometer per line; profitable lines most likely not subsidized; the purpose of subsidies is to provide accessibility and mobility, e.g. rural and distant areas) and the ticket prices.

The objective function is to maximize the overall profit, i.e., the difference between the revenue (the ticket price and the subsidies, if applicable for the specific line) and the operational cost.

In order to be in line with the TTP (as it is ran by the IM), the time windows (where the scheduled train is still profitable) are provided as well. We then redefine the costs of the timetables as they are passed on the TTP as following: in the current formulation of the TTP, the costs are relative, whereas we propose to use the costs that the TOCs would be willing to pay for the specific time slot to IM (above the "rental" cost of using the infrastructure).

We keep the ITTP as a separate problem, since in the real life the TOCs give the ideal timetables to the IM and then it is IM's responsibility to secure the feasibility of the timetables (in terms of the conflicts among the trains, belonging to either the same TOC or different TOCs).

We believe, that the main reason why the proposed ITTP has not been studied yet is (among others) the lack of detailed data on demand and the non-existence of the liberalization, that have started just recently.

Above the formulation of the ITTP, we would like to conduct an analysis of how much more costly the cyclic timetables are comparing to the non-cyclic timetables. This would be done via running the ITTP and thus getting real profits of the timetables and afterwards run independently both cyclic and non-cyclic TTP and compare their results.

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