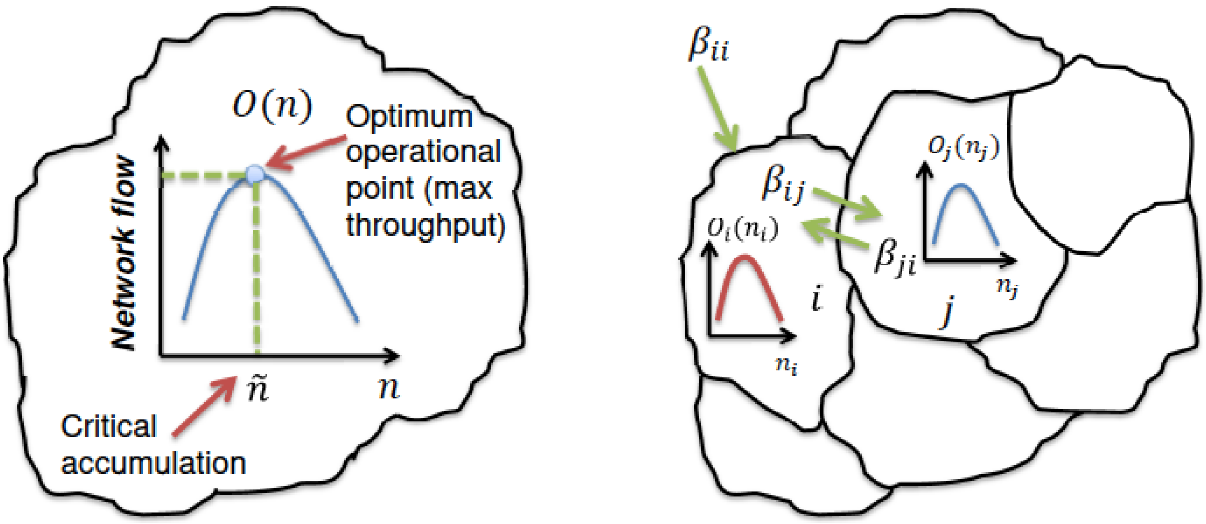

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Abstract

In this paper, we macroscopically describe the traffic dynamics in heterogeneous transportation networks by utilizing the Macroscopic Fundamental Diagram (MFD) for urban networks a widely observed relation between mean flow and density of vehicles. A generic mathematical model for multi-reservoir networks with well-defined MFDs for each reservoir is presented first. Two alternative optimal control methodologies are employed for the design of perimeter flow control strategies that aim at distributing the accumulation in each reservoir as homogeneously as possible, and maintaining the rate of vehicles that are allowed to enter each reservoir around a desired point, while the system's throughput is maximized. Based on the two control methodologies, perimeter control actions may be computed in real-time through a linear multivariable feedback regulator (LQ) and a linear multivariable integral feedback regulator (LQI). To this end, the heterogeneous network of the Downtown of San Francisco is partitioned into three homogeneous reservoirs that exhibit well-defined MFDs. These MFDs are then used to design and compare the proposed multivariable regulators with a pre-timed signal control plan and a single-reservoir perimeter control strategy. The impact of the perimeter control actions to the three reservoirs and the whole network is demonstrated via simulation by the use of the corresponding MFDs.

Keywords

Macroscopic Fundamental Diagram; Heterogeneous Networks; Perimeter Flow Control; Multivariable Feedback Regulators

1 Introduction

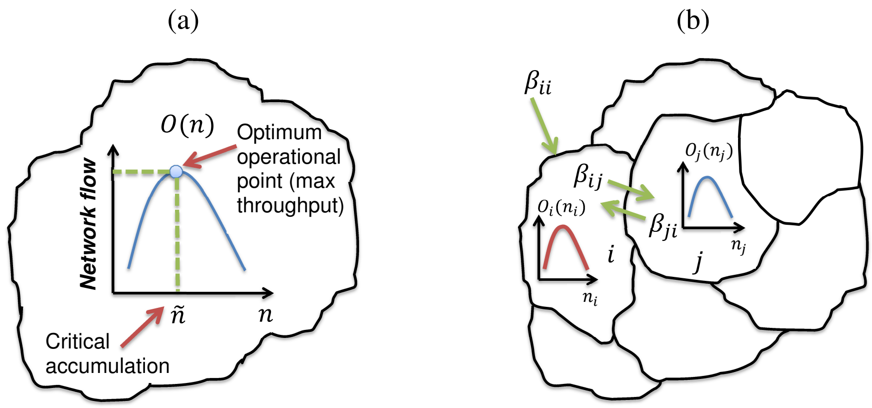
Realistic modeling and efficient control of heterogeneous transportation networks remain a big challenge, due to the high unpredictability of choices of travelers (in terms of route, time and mode of travel), the uncertainty in their reactions to the control, the spatiotemporal propagation of congestion, and the lack of coordinated actions coupled with the limited infrastructure available. With respect to traffic signal control, many methodologies have been developed, but still a major challenge in the coming decade is the deployment of advanced and efficient traffic control strategies in heterogeneous large-scale networks, with particular focus on addressing traffic congestion phenomena. Widely used strategies like SCOOT (Hunt *et al.,* 1982) and SCATS (Lowrie, 1982), although applicable to large-scale networks, are less efficient under oversaturated traffic conditions with long queues and spillbacks. A practicable work to address oversaturated traffic conditions was the more recently developed feedback control strategy TUC (Diakaki *et al.,* 2002, 2003, Kouvelas *et al.,* 2011). TUC attempts to minimize the risk of oversaturation and spillback of link queues by minimizing and balancing the links' relative occupancies. However, this policy might be suboptimal for heterogeneous networks with multiple centers of congestion and heavily directional demand flows.

An alternative to real-time network-wide traffic signal control for urban networks is a hierarchical two-level approach, where at the first level perimeter control between different regions of the network advances the aggregated performance, while at the second level a more detailed control can be applied to smooth traffic movements within these regions (e.g. TUC). The physical tool to advance in a systematic way this research is the *Macroscopic Fundamental Diagram* (MFD) of urban traffic, which provides for some network regions under specific regularity conditions (mainly homogeneity in the spatial distribution of congestion and the network topology), a unimodal, low-scatter relationship between network vehicle accumulations (veh) and network space-mean flow (veh/h), as shown in Figure 1(a).

The idea of an MFD with an optimum accumulation belongs to Godfrey (1969), but the empirical verification of its existence with dynamic features is recent (Geroliminis and Daganzo, 2008). This property is important for modeling purposes as details in individual links are not needed to describe the congestion level of cities and its dynamics. It can also be utilized to introduce simple perimeter control policies to improve mobility in homogeneous networks (Daganzo, 2007, Keyvan-Ekbatani *et al.,* 2012, Geroliminis *et al.,* 2013). The general idea of a perimeter control policy is to *meter* the input flow to the system and to hold vehicles outside the system if necessary.

Despite these recent findings for the existence of MFDs with low scatter, these curves should

Figure 1: A network modeled as (a) single-reservoir system and (b) multi-reservoir system.



not be a universal law. Recent findings (Geroliminis and Sun, 2011, Mazloumian *et al.,* 2010, Daganzo *et al.,* 2011) have identified the spatial distribution of vehicle density in the network as one of the key components that affect the scatter of an MFD and its shape. These findings are of great importance because the concept of an MFD can be applied for heterogeneously loaded networks with multiple centers of congestion, if these networks can be partitioned into a small number of homogeneous reservoirs (regions). The objective is to partition a heterogeneous network into homogeneous reservoirs with small variance of link densities (Ji and Geroliminis, 2012). Single-reservoir perimeter control may enhance an uneven distribution of vehicles in different reservoirs of the network (for example due to asymmetric route choices and origin-destination matrices), and, as a consequence, may invalidate the homogeneity assumption of traffic loads within the reservoirs and degrade the total network throughput.

In this work we put some effort to deal with these important issues of efficiency, heterogeneity and equity in perimeter control. In particular, for a given partition of the network into some homogeneous regions and corresponding MFDs (see Figure 1(b)) with a sweet spot accumulation that maximizes the regional circulating flow (outflow or trip completion rate), we develop perimeter flow control strategies to improve mobility in heterogeneous networks.

2 Dynamics for heterogeneous networks partitioned in reservoirs

Consider a network partitioned in N regions or reservoirs (Figure 1(b)). Denote by $i = 1, \ldots, N$ a reservoir in the system, and n_i the accumulation of vehicles in reservoir i at a given time. We assume that for each reservoir $i = 1, \ldots, N$ there exists an MFD, $O_i(n_i)$, between accumulation

n_i and output O_i, which describes the behavior of the system when it evolves slowly with time. Let $\mathcal{S}_i$ be the set of origin reservoirs which are reachable from reservoir i and $d_i(t)$ be the uncontrolled traffic demand (disturbances) in reservoir i at time t. Note that $d_i(t)$ includes both internal (off-street parking for taxis and pockets for private vehicles) and external (non-controlled) inflows. If i and j are two reservoirs sharing a common boundary, we denote by β_{ji} ($j \neq i$) the rate of vehicles in reservoir j that allowed to enter reservoir i and by β_{ii} the rate of vehicles in the perimeter of the network allowed to enter reservoir i (see Figure 1(b)). The state of each reservoir $i = 1, \ldots, N$ is governed by the following nonlinear conservation equation (Aboudolas and Geroliminis, 2013a,b):

$$\frac{dn_i(t)}{dt} = \sum_{j \in \mathcal{S}_i} \beta_{ji}(t) O_j(n_j(t)) - O_i(n_i(t)) + d_i(t). \tag{1}$$

where $\beta_{ji}(t)$ are the input variables from reservoir j to reservoir i at time t, to be calculated by the perimeter controller.

The overall multi-reservoir dynamics in (1) can be generally viewed as a nonlinear process with input variables $\boldsymbol{\beta}$, state variables $\mathbf{n}$, and disturbances $\mathbf{d}$. The vector $\boldsymbol{\beta}$ includes all controllable reservoir outflows β_{ji} (the inter-transfers between regions of a city). The state vector $\mathbf{n}$ includes all reservoir accumulation of vehicles n_i. The disturbance vector $\mathbf{d}$ consists of all external and internal inflows d_i. The continuous-time nonlinear state system (1) for a city partitioned in N reservoirs may be directly translated in discrete-time and can be rewritten in generalized vector form

$$\mathbf{n}(k+1) = \mathbf{f}\left[\mathbf{n}(k), \boldsymbol{\beta}(k), \mathbf{d}(k), k\right], \; k = 0, 1, \ldots, K-1, \quad \mathbf{n}(0) = \mathbf{n}_0 \quad \text{known} \tag{2}$$

where k is a discrete-time index, K is the finite-time process horizon, and $\mathbf{f}$ is a nonlinear vector function reflecting the right-hand side of (1). This general mathematical description will be used as a basis for perimeter controller design in the subsequent sections.

The basic multi-reservoir problem formulation (2) can be extended to consider a broader class of linear and/or nonlinear constraints (e.g. inequality and/or equality state and control constraints). For example, inequality control constraints may be introduced to prevent overflow phenomena within the reservoirs and to avoid long queues and delays at the perimeter of the network and the boundary of neighborhood reservoirs. These constraints may be brought to the general form

$$\mathbf{h}\left[\mathbf{n}(k), \boldsymbol{\beta}(k), \mathbf{d}(k), k\right] \leq \mathbf{0}, \quad k = 0, 1, \ldots, K-1 \tag{3}$$

where the nonlinear function $\mathbf{h}$ expresses control and state constraints in mathematical terms.

3 Perimeter flow control

3.1 Perimeter flow control objectives

In the case of a single-reservoir system (Figure 1(a)), which exhibit an MFD, a suitable control objective is to minimize the total time that vehicles spend in the system including both time waiting to enter and time traveling in the network. It is known that the corresponding optimal policy is to allow as many vehicles to enter the network as possible without ever allowing the accumulation to reach states in the congested regime (Daganzo, 2007). However, in the case of a multi-reservoir system (Figure 1(b)), such a policy may induce uneven distribution of vehicles in the reservoirs, and, as a consequence, may invalidate the homogeneity assumption of traffic loads within the reservoirs and degrade the total network throughput and efficiency. With these observations at hand, a suitable control objective for a multi-reservoir system aims at: (a) distributing the accumulation of vehicles n_i in each reservoir i as homogeneously as possible over time and the network reservoirs, and (b) maintaining the rate of vehicles β_{ji} that are allowed to enter each reservoir around a set (desired) point $\hat{\beta}_{ji}$ while the system's throughput is maximized. A quadratic criterion that considers this control objective has the general form (Aboudolas and Geroliminis, 2013b)

$$\mathcal{L}(\boldsymbol{\beta}) = \frac{1}{2}\sum_{k=0}^{\infty}\left(||\Delta\mathbf{n}(k)||^2_{\mathbf{Q}} + ||\Delta\boldsymbol{\beta}(k)||^2_{\mathbf{R}}\right) \tag{4}$$

or (Aboudolas and Geroliminis, 2013c)

$$\mathcal{L}(\boldsymbol{\beta}) = \frac{1}{2}\sum_{k=0}^{\infty}\left(||\Delta\mathbf{n}(k)||^2_{\mathbf{Q}} + ||\Delta\boldsymbol{\beta}(k)||^2_{\mathbf{R}} + ||\mathbf{y}(k)||^2_{\mathbf{S}}\right) \tag{5}$$

where **Q**, **R**, and **S** are non-negative definite, diagonal weighting matrices. The infinite time horizon in (4) is taken in order to obtain time-invariant feedback laws as we will see later in section 3.2. The two terms in (4), (5) are responsible for objective (a) and (b) above, respectively, while the third term corresponds to the magnitude of the error signal **y** (see Section 3.2).

Given the cost criteria in (4), (5), the overall multi-reservoir dynamics in (2), and the control and state constraints in (3), optimal control problems (linear or nonlinear) may be formulated to address the perimeter control problem for multi-region and heterogeneous congested cities.

3.2 Multivariable feedback regulators for perimeter flow control

The linear control theory offers a number of methods and theoretical results for feedback regulator design in a systematic and efficient way. Multivariable feedback regulators have been applied in the transport area mainly for coordinated ramp metering (Papageorgiou *et al.,* 1990) and traffic signal control (Diakaki *et al.,* 2002, 2003, Aboudolas *et al.,* 2009). In the sequel, we present two alternative optimal control methods for the design of feedback perimeter flow control strategies for multi-region and heterogeneously loaded networks. The first methodology is a multivariable feedback regulator derived through the formulation of the problem as a Linear-Quadratic (LQ) optimal control problem. The second methodology obtained through the formulation of the problem as a Linear-Quadratic-Integral (LQI) optimal control problem, which provides zero steady-state error under persistent disturbances and eliminates the need of set values $\hat{\beta}_{ji}$.

A necessary condition for application of the linear control theory to a particular process control problem is the availability of a linear mathematical model capable of describing the basic process behaviour. In case of a nonlinear mathematical model (cf. equation (2)), it is possible to linearize the nonlinear model (e.g. around some set values) before the regulator design. In our case, the nonlinear model (2) may be linearized around some set point $\hat{\beta}_{ji}$, $\hat{n}_i$, and $\hat{d}_i$. In our case, this equilibrium point should be close to the critical accumulation $\tilde{n}_i$ for each reservoir $i = 1, \ldots, N$, where the individual reservoirs' throughput is maximized (see Figure 1 and Section 3.1). This leads to a linear state-space model for a multi-reservoir city of arbitrary size, topology, and characteristics which is given by

$$\Delta\mathbf{n}(k+1) = \mathbf{A}\Delta\mathbf{n}(k) + \mathbf{B}\Delta\boldsymbol{\beta}(k) + \Delta\mathbf{d}(k) \tag{6}$$

where $\mathbf{A}$ and $\mathbf{B}$ are the state and control matrices, respectively. A first approach towards feedback perimeter control is based on the linearized model (6), the quadratic cost criterion (4) and the formulation of the perimeter control problem as a Linear-Quadratic (LQ) optimal control problem. The LQ methodology leads to the following control law (Aboudolas and Geroliminis, 2013b)

$$\boldsymbol{\beta}(k) = \hat{\boldsymbol{\beta}} - \mathbf{K}\left[\mathbf{n}(k) - \hat{\mathbf{n}}\right] \tag{7}$$

where $\mathbf{K}$ is a gain matrix and $\hat{\boldsymbol{\beta}}$ is the set point vector with elements $\hat{\beta}_{ji}$.

A further control law that eliminates the need of set values $\hat{\boldsymbol{\beta}}$ may be obtained through the formulation of the perimeter control problem as a Linear-Quadratic-Integral (LQI) optimal control problem based on an augmented linear model of (6) and the augmented cost functional

(5). The augmented discrete-time model can be written as

$$\Delta\tilde{\mathbf{n}}(k+1) = \tilde{\mathbf{A}}\Delta\tilde{\mathbf{n}}(k) + \tilde{\mathbf{B}}\Delta\boldsymbol{\beta}(k) + \tilde{\mathbf{H}}\Delta\mathbf{d}(k) \tag{8}$$

where $\tilde{\mathbf{n}}(k) = [\mathbf{n}(k) \quad \mathbf{y}(k)]^{\tau}$ is the augmented state vector, and $\tilde{\mathbf{A}}, \tilde{\mathbf{B}}, \tilde{\mathbf{H}}$ are the augmented state, control, and demand matrices, respectively. The augmented state vector $\mathbf{y}$ is used as a feedback term in the controller to provide zero steady-state error. The LQI methodology leads to the following control law (Aboudolas and Geroliminis, 2013c)

$$\boldsymbol{\beta}(k) = \boldsymbol{\beta}(k-1) - \mathbf{K}_{\mathrm{p}}\left[\mathbf{n}(k) - \mathbf{n}(k-1)\right] - \mathbf{K}_{\mathrm{I}}\left[\mathbf{n}(k) - \hat{\mathbf{n}}\right] \tag{9}$$

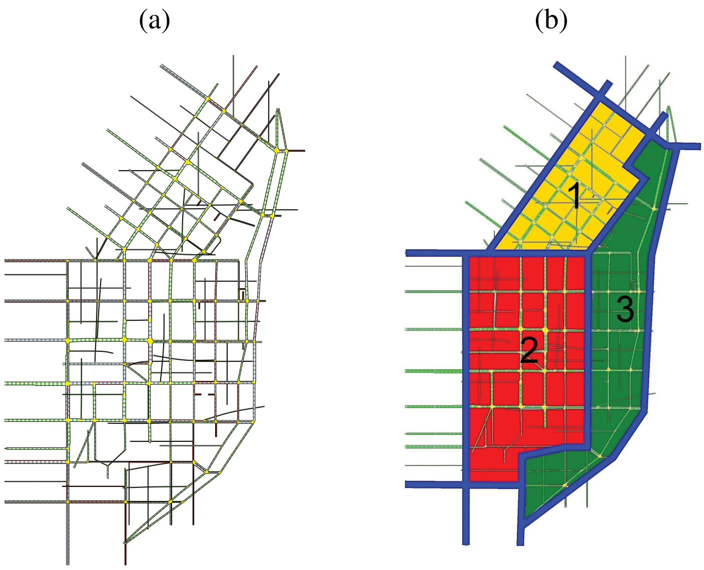
where $\mathbf{K}_{\mathrm{p}}$ and $\mathbf{K}_{\mathrm{I}}$ are gain matrices.

The calculation of $\mathbf{K}$ or $\mathbf{K}_{\mathrm{p}}$ and $\mathbf{K}_{\mathrm{I}}$ via solution of the corresponding discrete-time Riccati equation, is straightforward and the required computational effort is low even for large-scale cities partitioned into many reservoirs. Moreover, this computational effort is required only off-line, while on-line (i.e. in real-time) the calculations are limited to the execution of (7) or (9) with state measurements $\mathbf{n}(k)$. Then the perimeter and boundary controllers resulting from (7) or (9) are forwarded to signalized intersections located on the boundary of neighborhood reservoirs or the perimeter of the test network for application, i.e. by modifying the green duration of the phases, where perimeter and boundary arriving flows are involved.

4 Implementation and results

A simulation in a real urban transportation network further demonstrates the superiority of the proposed feedback perimeter control strategy for $\hat{\mathbf{n}} = 0.85\tilde{\mathbf{n}}$, in effectiveness and robustness compared with the no control case. Figure 3 illustrates a preliminary application of the methodology for a 4hr microscopic simulation of Downtown San Francisco with time dependent conditions in AIMSUN microscopic simulator (Aboudolas and Geroliminis, 2013a,b). Figure 2(a) shows a snapshot of the test area of Downtown San Francisco, while Figure 2(b) shows the result of the partitioning into three homogeneous reservoirs. Figures 3(a), 3(d) displays the unified MFD and the MFDs of the three reservoirs resulting for the considered demand scenario and ten replications, each with different seed in AIMSUN. As a first remark, these figures confirms the existence of an MFD for the test area of Downtown San Francisco as well as for the three reservoirs with moderate scatter across different replications. Nevertheless, there is a clear distinction between congested and uncongested regime for all reservoirs. Note that the accumulation each of the reservoirs reaches the congested regime is very different. The reservoir 1 (blue curve) reaches congestion first and then it propagates in the Reservoirs 3 (red curve)

Figure 2: The test site of Downtown San Francisco: (a) simulation model; (b) partitioning of the network into 3 reservoirs.

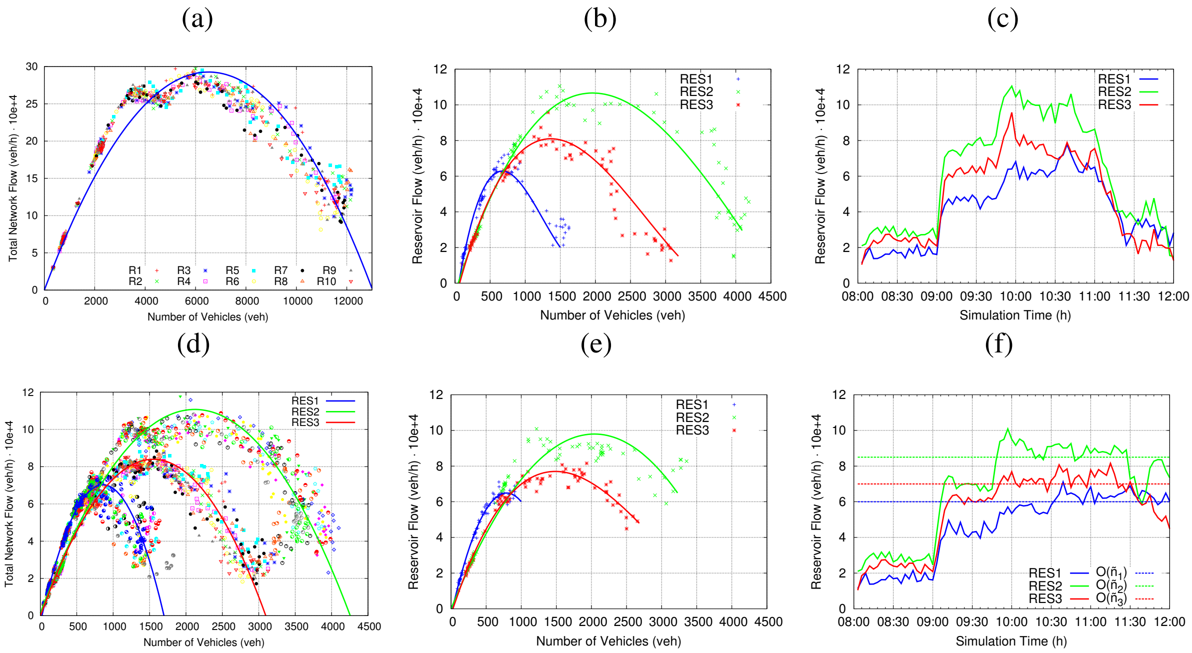


and 2 (green curve). This propagation of congestion would not be observable by looking at the unified MFD in Figure 3(a). Figures 3(b), 3(e) display the MFDs of the three reservoirs resulting for the considered demand scenario and one replication when no control and perimeter control are applied, respectively. Clearly, when perimeter control is applied, the three reservoirs remain semi-congested and only some states observed in the congested regime; under no control, the network becomes severely congested with states in the congested regime of the corresponding MFDs. Finally, Figures 3(c), 3(f) demonstrate that when feedback perimeter control is applied throughput (flow) is maintained at high levels within the three reservoirs, in contrast to the no control case. Extended results for the feedback regulators (7), (9) and their comparison with a pre-timed signal plan and a single-reservoir perimeter control strategy can be found in Aboudolas and Geroliminis (2013a,b,c).

5 Conclusions

In this paper, we addressed the problem of perimeter control for congested networks partitioned in reservoirs. First, by exploiting the properties of the MFD, we described the dynamics of the rush hour in case of multi-region networks that are not uniformly congested. Motivated by the need to distribute the accumulation of vehicles in each reservoir as homogeneously as possible and maintain the rate of vehicles that are allowed to enter each reservoir around a desired point while the system's throughput is maximized, we then stated our control objective. In order to

Figure 3: Demonstration of feedback perimeter control for Downtown San Francisco. (a) Unified MFD for the whole network; (d) MFDs for the 3 regions for a 4hr simulation; (b) MFDs for the 3 regions without control; (e) MFDs for the 3 regions with feedback perimeter control; (c) Region flow over time without control; (f) Reservoir flow over time with feedback perimeter control.



provide solutions that can be implemented in real time, we introduced two control strategies for determining the perimeter and boundary controllers, namely multivariable feedback regulator and integral feedback regulator. A key advantage of our approach is that it does not require high computational effort and future demand data if the state can be observed.

Future work will deal with the comparison of the proposed feedback perimeter control strategies with other approaches (e.g. model-predictive perimeter control (Geroliminis *et al.,* 2013)), their application in real-life conditions and their extensions to multimodal networks with cars and public transport.

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