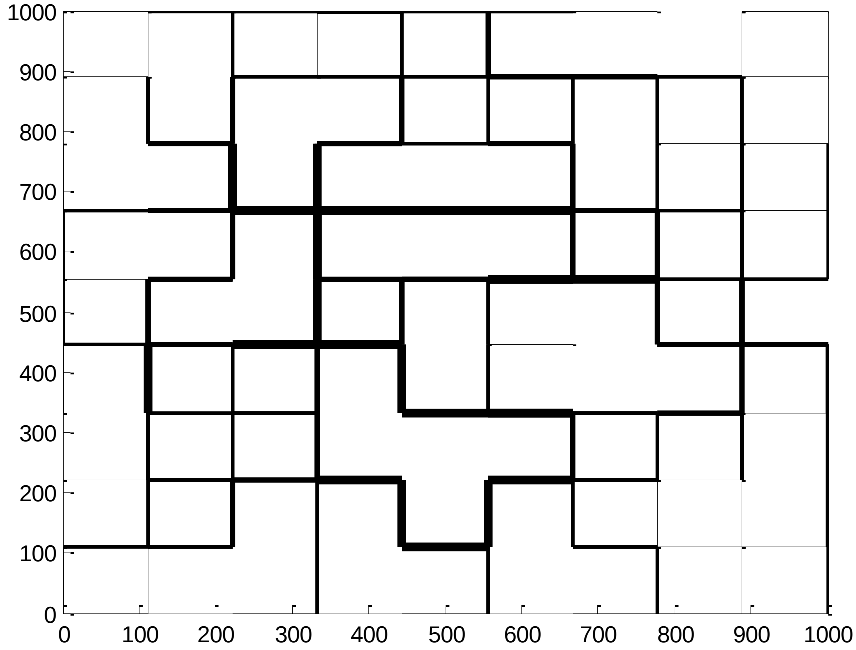

Link removal on a grid street network

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Abstract

This paper explores some properties of a grid street network when accommodating traffic flows under different demand scenarios. The abstract street network considered is a graph defined by nodes and links. The shape of the region is square, therefore the grid is composed of regular squares. Demand is concentrated on the nodes. However, in order to compare different topologies, the total number of trips generated and attracted will be the same independently of the number of nodes (i.e. a model with lesser nodes will concentrate more demand on them). A triangular model, where the trips increase with distance, is proposed.

A static macroscopic traffic assignment model has been developed and applied using the Frank-Wolfe algorithm. The travel time is modeled according to the *Bureau of Public Roads* travel time function, adapted to urban areas. The grid pattern is initially complete (i.e. all the nodes connected according to the grid model). Links are subtracted gradually from the grid in a random manner. For every subtraction, network descriptors are re-calculated. These descriptors are basically related to the density and connectivity of the network (e.g. gamma index). At the same time, for every network configuration, the demand is allocated using the traffic assignment algorithm mentioned above. The traffic performance is then measured using as main inputs the speed and the flows on links, the distance travelled, and the travel time. This iterative process allows us to plot some relationships between the physical characteristics of the network and the traffic performance.

Keywords

Grid – Traffic performance – Connectivity – Static traffic assignment

1. Introduction

This paper explores some properties of an abstract grid street network when accommodating traffic flows under a hypothetic demand scenario. The main focus of this research work is to study the relationship between the physical characteristics of urban patterns and the traffic performance of the city network. This paper is involved in the research field carried out by the SVT-IVT group into the analysis of urbanization models from the traffic perspective.

There is an important field of research that studies how the urban form (location, land use, size and shape of settlements, population density, road network, and urban layout (Stead and Marshall, 2001)) affects the travel behavior. Many of these research works study real cities and how the urban form factors are related to travel characteristics such as the mode choice, the vehicles mile travelled or the travel time (e.g. Frank and Pivo, 1994; Cervero and Gorham,1995; Levinson and Kumar, 1997; Crane and Crepeau, 1998). However, the connection to traffic presented in these studies is still weak. From this traffic perspective, Scott et al. (2006), Xie and Levinson (2009), and Parthasarathi (2011) have influenced the present research. They relate the physical descriptors of urban patterns to the traffic outputs of a simulation experience (Scott et al., 2006; Xie and Levinson, 2009) or real data using regression analysis (Parthasarathi, 2011). Our research work focuses on the study of the grid, as one of the most common patterns in dense urban structures.

The abstract street network considered is a graph defined by nodes and links. The shape of the region is square, therefore the grid is composed of regular squares. Demand is concentrated on the nodes. A triangular model, instead, assigns trips according to the distance between nodes using a bilinear function. A static macroscopic traffic assignment is applied in order to accommodate the demand in the network. The algorithm has been developed from the classical traffic assignment formulations and concepts (Beckmann 1966; Nguyen, 1973; LeBlanc, 1975) using the widely known convex optimization method, the Frank-Wolfe algorithm. The accuracy of this model has been increased by an intersection modeling scheme where every direction (12 for a four way intersection) is represented by a fictitious link (which gives the delay for every approach).

Starting from a complete and regular grid, we have removed random combinations ranging from 0 to 30 % of the links. For each case (defined by network descriptors), we have analyzed certain traffic indicators in order to assess the effect of the link removal. In the following sections, the characteristics of the grid, the methodology employed and the results are presented.

2. Methodology

2.1 Network and demand generation

2.1.1 The grid network

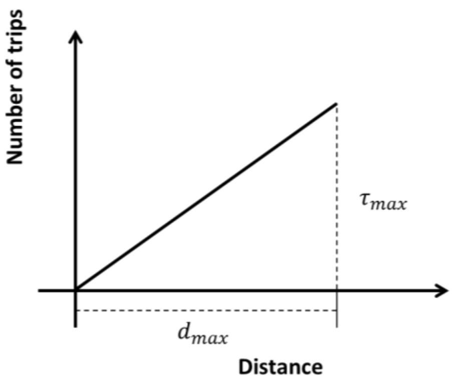
The abstract network used in this paper is a regular grid. Our research focuses on urban patterns; and the grid is one of the most common structures found in these settings. The analysis starts with a regular grid (square blocks) where every node has four connections (except in the perimeter). Every link represents a two-way street, and every node represents a four-way signalized intersection. Every link is 111 meters long (a reasonable block size), and the total area of analysis considered is 1 km^2 (100 nodes). The idea is to emulate reality (in this case a dense urban network) as accurately as possible but with the loss of generalization (Figure 2).

2.1.2 Demand on the network

In order to load the grid with traffic flows, we need to assume some demand models (trip generation and distribution). Our first approach was the most simple possible: every node interchanges the same amount of trips, τ, with the rest. Thus, the OD matrix is n x n, where n is the total number of nodes, and all the elements are equal to τ (except the diagonal that are zeros). For every node in the system there will be the same amount of generated and attracted trips, $\tau \cdot n \cdot (n-1)$.

However, it would be more realistic to consider that the trip demand is somehow affected by the distance between the nodes. Therefore, keeping the simplicity, we analyzed a triangular demand model (Figure 1). It considers that the number of trips increases linearly with the distance.

Figure 1 Triangular demand model.



In urban areas, car trips might tend to increase with the distance for short trip lengths; then, once a certain distance is reached, they decrease with the distance (Ortuzar and Willumsen, 2001). Since the dimensions of our grid are relatively small (1km x 1km), we believe that we fall into the second triangular model (the trips increase with the distance). In our grid, in fact, the maximum distance will be 2 km. We are aware of the limitations of this reasoning, since in European cities, these distances will be covered by public transport, bicycle or walking. However, these overflows could emulate the traffic from the outskirts of the city travelling to the center.

According to the Barcelona City Council (Dades basiques de mobilitat, 2010), in 2010, for example, approximately 4,6 million trip were performed on an average working day by car. The total surface of Barcelona is approximately 100 km^2 but it has very heterogenic urban areas (in terms of density of streets and population). Our abstract network could represent a dense grid neighborhood of Barcelona such as the Eixample. This neighborhood only accounts for a 7,5 % of the total surface of the city but it has the 16,5 % of the population. Considering that fact and estimating 10 effective hours per day, a dense city, network could, roughly, count with a density of 10,000 trips per hour and km^2. That, in our system, corresponds to a uniform demand model where every node interchanges 1 trip with each of the other nodes. Translating this into the triangular model the τ_{max} value corresponds to 3 trips per hour.

2.2 Traffic assignment

2.2.1 Traffic assignment model

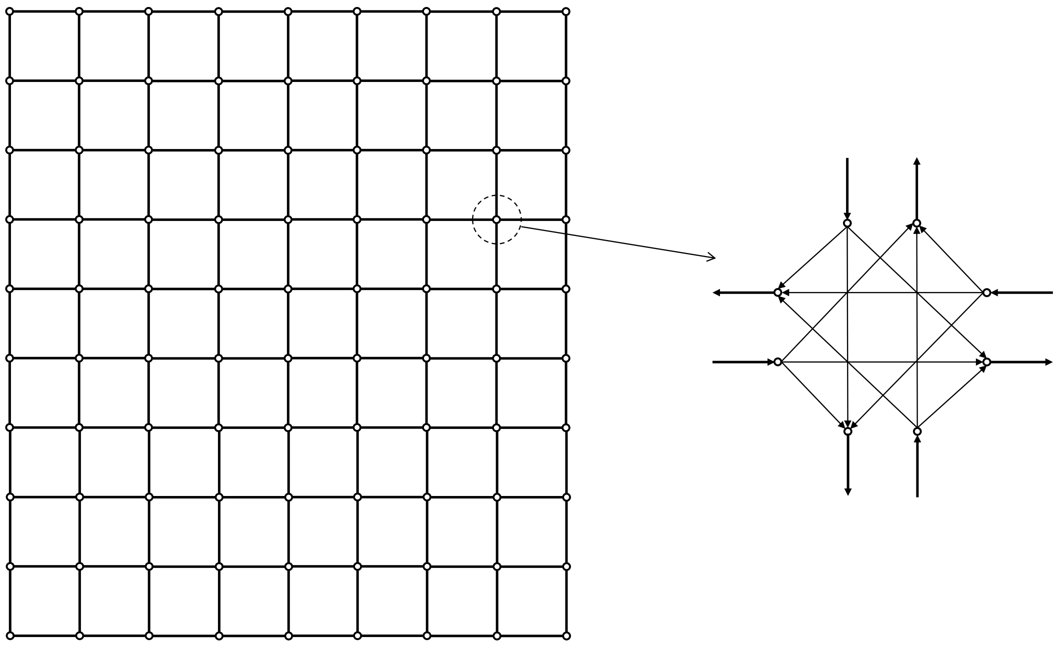
The traffic analysis of our abstract pattern will be very dependent on how we allocate the traffic flows. Because of that, there is a detailed description below of our traffic assignment model. The model is a compromise between the accuracy of the traffic assignment and the complexity of developing and applying the algorithm. It should be noted that even after 50 years of research in the traffic assignment field, still there is a lot of activity devoted to improving the accuracy of these techniques.

Wardrop (1952) first defined the concept of user's equilibrium, which states that drivers will always seek their best route (having complete information of the travel times). Beckmann et al. (1956) set the basis for the formulation of the Traffic Assignment Problem (TAP). In this problem a travel time function is defined for every link in the network. If this function only depends on the volume of that link and it is continuous, differentiable and monotonic, then the problem has a unique equilibrium solution. The problem si then convex optimization. This was first solved by Nguyen et al. (1973) and LeBlank et al. (1975) employing the Frank and Wolfe algorithm (Frank and Wolfe, 1956). This algorithm presents many advantages (e.g. the little use of storage), and it has been modified during the years to solve some of its drawbacks, e.g. slow convergence, bad tailing (Dial, 2006). However, for our purposes, it presents reasonable accuracy level and computational times, hence we employ it in this paper. For review of the formulations of the different traffic assignment problems and the evolutions, see Patriksson (1994) and Florian and Hearn (2001).

This kind of traffic assignment equilibrium problems are called static because they describe a system in steady conditions. When the demand is fixed and travellers have total information of the travel cost it is called deterministic. In addition, if the travel times on the links only depend on the volumes on these links (and the conditions of continuity, differentiability and monotonicity are fulfilled) the demand is separable (Florian and Hearn, 2001) and it becomes a convex optimization as formulated in Beckmann et al. (1956). If the travel time on a link depends on flows from other links (e.g. an intersection with left turns) then the problem becomes asymmetric. Our network presents these kind of intersections since all links have two directions.

In other to overcome the asymmetry, while keeping the simplicity of our traffic assignment technique, we have created 12 dummy links in each intersection that represent the different directions that vehicles can take (Figure 2). Similar solutions are suggested in Branston (1976) and Koutsopoulos and Habbal (1993). Using the same travel time function as in the links, the fictitious distance and capacity emulate the delay that the intersection creates.

Figure 2 The regular 100 nodes grid and the detail of one intersection.



2.2.2 Traffic assignment algorithm

As stated before, the algorithm uses the Frank and Wolfe method to reach the equilibrium state. The process is widely known and utilized and it is described in the vast majority of works about traffic assignment. We have followed LeBlanc (1975) paper for implementation of the algorithm.

It is an iterative process where in every iteration the solution is improved by a certain step length in the direction of the negative gradient function (i.e. towards the minimum). The user's equilibrium problem needs that the following function, *F(v)*, is minimized:

$$Min\, F(v) = Min \int_0^v t(x)dx \qquad (1)$$

Where *t* is the travel time function on a link that depends on the volume, *v*, of that link. The algorithm starts with an initial solution of volumes V_0 assigned considering the initial travel times. Then, with the travel times updated with the initial volumes we allocate traffic (all-or-nothing) finding another volume state, H . The difference between these two states is our local approximation to the maximum change direction. For the next iteration, the initial solution will be updated towards this direction with an optimal step length λ which minimizes the objective function $F(V_{i+1})$:

$$V_{i+1} = V_i + \lambda \cdot (V_i - H_i) \qquad (2)$$

In the case of having a network, if the travel time functions are dependent only on the flows on that link, the problem is separable (as Beckmann (1956) demonstrates). That means that when optimizing every link, the whole network is as well optimized. The flow assignment (all-or-nothing) for each iteration is performed using a shortest path algorithm (Dijkstra). The step length that minimizes $F(V_{i+1})$ is the zero of the function $t(V_i + \lambda \cdot (V_i - H_i)) \cdot (V_i - H_i)$, found applying the fundamental theorem of calculus. We use a Bolzano search to find λ.

Our algorithm considers the intersections as fictitious links. However, for the sake of efficiency, we build two different matrixes to store flows and travel times. One matrix is devoted to the flows on the links and the other to the flows on the nodes. In addition to this, we have modified the Dijkstra algorithm to consider a node cost, instead of directly adding the intersection fictitious links to the general matrix. In this way, we gain simplicity and efficiency. Other works might present more efficient formulation (e.g. Watanabe and Lijiao, 2011) in this concern, but as stated earlier, we have tried to keep a compromise between accuracy and fulfilling the general goals of this paper.

2.2.3 Validation of the algorithm

The algorithm has been validated for the case of not including the penalty created by the intersections. We have used the example presented in (Dial, 2006) where a grid network of 25 nodes is employed. In this case, only four of the nodes act as sources or sinks. The difference between both solutions is below 0.45 %. This error, we assume is a rounding error due to the different number of decimals considered.

2.2.4 Error in the convergence of the algorithm

When running the algorithm in our grid considering both links and nodes, there are some cases where the algorithm does not converge. For certain iterations it is unable to find an optimal step length (λ) in the traffic assignment routine, and it adopts, typically, the value of 0. This error has occurred, approximately, in 5% of the cases. These cases have been removed from the presented results. We observe, that for higher link removal rates, the incidence of this increases. We are currently analyzing the causes of this error and potential solutions. Therefore, the results presented in this paper should be taken as provisional.

2.2.5 Travel time function

The travel time function provides the total travel given the volume in the link and other characteristics of the link, the most important one, the capacity. There are many travel times functions and a lot of research in that field. As a theoretical background one of the first works and most complete can be found in Branston (1976). The most widely used function is the one proposed by the Bureau of Public Roads (1964):

$$T_{ij} = T_{ij}^{0} \cdot \left(1 + a\left(\frac{V_{ij}}{C_{ij}}\right)^{b}\right) \qquad (3)$$

Where, T_{ij} is the travel time on the link, T_{ij}^{0} the free flow travel time on that link, V_{ij} is the volume in the link, C_{ij} is the capacity on the link, and a and b parameters that come from experimental observations. The simplicity, the flexibility and the common use of this function have made it very popular across the world. The values of the parameters a and b are very linked to the definition of the capacity in the link. The initial values set by the Bureau of Public Roads (1964) are a=0.15 and b=4, if the desing capacity is considered. This value is the one that involves a drop by 15 percent on the free flow speed, corresponding to a Level Of Service (LOS) C (Horowitz, 1991). Many authors have proposed formulations for these parameters as well as for the capacity of the link. Some of them consider the ultimate capacity in the link (LOS E). However, considering the abstract nature of our model, we will employ the original parameters of the BPR function, a=0.15 and b=4.

Our urban network is composed by two way streets. According to the DMRB (1999), busy two-lane urban streets can have capacities ranging from 750 to 1410 veh/h. In that calculation, however, intersections are included. Since in our model, the fictitious links represent the intersections, the capacity constraint is given by them. We are going to consider that our urban roads have a design capacity of 1,200 veh/h. The reason for this assumption is that we want to include the possible interactions (typical from a dense urban area) that might decrease capacity (e.g. on-street parking).

2.2.6 Delay at the intersection

There are many different formulations that aim at representing the delay at intersections (e.g. Dion et al., 2004). In our case, the grid is composed by signalized intersections. One fo the most common formulation, used to find the average delay created in these kind of intersections is the one proposed by the HCM (2010). Unfortunately, it does not fulfill the requirements to be applied in the traffic assignment model (i.e. it is not differentiable). In order to overcome that, we have adjusted the BPR formula parameters to fit the HCM-2010 delay expression. In other words, we find the parameters of the BPR formula on a fictitious link that replicates the delay experienced in an intersection approach given the traffic volume and the capacity.

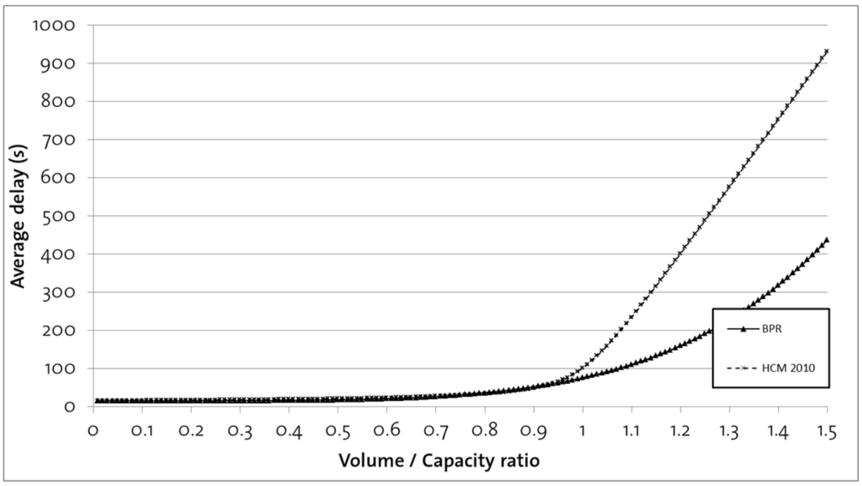
Horowitz (1991) analyses this option but recommends other options, mainly, because of the bad adjustment of the two functions (HCM-1985 and BPR) for oversaturated conditions. We have compared both formulations and found values of a, b, and the distance of the fictitious link that bring both expressions as close as possible. For the sake of simplicity, in this paper we adopt the adjusted BPR formulation although it can bring some inaccuracies.

The analysis period employed in the HCM-2010 expression is 1 hour, the incremental delay factor (k) is 0.5, and the capacity employed has been 510 veh/h, for straight and turn right approaches, and 240 veh/h for the turn left approach. These values have been obtained considering a saturation flow of 1,800 veh/h and 60 s cycle length. We give 17 seconds of effective green to the straight and right approaches, and 8 seconds to the turn left movement. Straight and turn right approaches share the capacity of 510 veh/h, so we consider a capacity of 255 veh/h for every individual approach.

We have adjusted the distance, a, and b, for the straight and right, and the left approaches using the least squares method. In order to give more weight to the adjustment in the range of 0-0.7 volume/capacity ratios, we have followed an iterative process finding the best distance for the 0-0.7 v/c and then, the best a, and b for all the v/c values. However, running the algorithm, we have realized that in some cases, and for some network configurations, the volume/capacity ratios can be very large and the BPR function gives unrealistic delay values. For that reason, we have chosen lower values of a, and b, despite that in oversaturated conditions (as Figure 3 shows) the HCM-2010 formula provides a higher delay value than the BPR. As it will be seen in the results section, even choosing these values, the delay for certain extreme cases is too high. These particular cases have been removed from the dataset in order to not affect the final results.

Figure 3 a) Representation of the delays in an intersection with the HCM (2010) and the BPR formulations; b) values adopted for every approach.

a)



b)

Straight and right approaches:
a=3.8
b=4.8
Distance=104 m

Left approach:
a=3.8
b=4.8
Distance=302.5 m

As soon as we remove links, the intersections might change. We can have the case of a T intersection. In that case, the node goes from 12 direction approaches (in the X intersection case) to 6. Independently of the different signal operative that might be applied, we consider that on average, every approach will have available the double green time, and hence the double capacity. For the case of two links meeting on a node, the intersection disappears since there are not any conflicting approaches.

2.3 Network descriptors

A lot of research has focused on measuring the connectivity of transportation networks. A higher connectivity of an urban road network should result in a bigger choice of routes and therefore the occupancy rate should be more similar throughout the network. Also a well-connected network would probably generate more but shorter trips than a badly connected one. Bell (2000) considers the connectivity of a network as one of the two dimensions of the reliability of a network since the more spare connected a network is, the harder it will be for a traveller to reach his destination if there is a system failure.

Garrison and Marble (1961) as well as Kansky (1963) started using graph-theoretical approaches to describe the connectivity of a network. They developed a set of indices such as the alpha-, beta- and gamma-index that have been used throughout the literature in the original or a slightly adjusted way, e.g. by Rodrigue (2003), Cardillo et al. (2006), Scott et al. (2006), Xie and Levinson (2009), and Parthasarathi (2011). These indicators have been developed for and applied to planar and undirected graphs. A graph is considered to be planar if its edges intersect only at its nodes. Urban road networks are considered to be representable by planar graphs (as long as tunnels and bridges are not taken into consideration). Throughout the literature urban road networks are mostly represented as undirected graphs, which means that all streets are two-way streets.

The alpha-, beta- and gamma-index as developed by Garrison and Marble (1961) for undirected, planar graphs are applied in our study. This means that if a link is removed, both directions are removed. Further work should concentrate on adjusting the indicators so they can also be applied to directed graphs.

The alpha-index is calculated as follows:

$$\alpha = \frac{e-v+1}{2v-5} \qquad (4)$$

Where e is the number of edges (i.e. links) in the graph and v the number of vertices (i.e. vertices). The alpha-index calculates the ratio between the number of circuits in a network and the maximum number of possible circuits in the network. Since the more circuits exist in a network, the more alternative paths are possible between the nodes in a network; the increase of circuits also increases the connectivity (Taffee and Gauthier, 1973). Therefore the alpha-index can be used as a measure of connectivity.

The beta-index is the simplest form of measuring the relationship between edges and vertices in a graph:

$$\beta = \frac{e}{v} \qquad (5)$$

The beta-index is especially useful when comparing different networks with the same amount of vertices. More edges implicate more connectivity. The beta-index decreases when links are removed from an urban road network.

Probably the most commonly employed index when talking about the connectivity of a network is the gamma-index:

$$\gamma = \frac{e}{3(v-2)} \qquad (6)$$

The gamma-index compares the actual amount of edges in a network with the maximum number of possible edges in that same network. In other words, it compares the actual network to the maximally connected network with the same amount of vertices. The range of the gamma-index lies between 0 and 1. The value of the gamma-index is then often expressed as a percentage of connectivity (Taffee and Gauthier, 1973).

Graph theory is also used in many other research fields such as biology, social sciences or technological systems. In these areas especially the statistical features of networks such as the properties of the degree distribution are considered. Since the above-described indicators do not distinguish between the properties of the removed nodes in a network the degree distribution should also be taken into account. Jiang and Claramunt (2004), Lämmer et al. (2006) and Chang et al. (2011) applied indicators such as node degree distribution and clustering coefficients that derive from graph theory as well as from statistics. Another approach used by Jiang (2007) and Chan et al. (2011) to examine urban road networks is the determination of the statistical properties of the nodes. Those are mainly characterized by the degree distribution $p(k_n)$ where the degree k_n of a node is defined as the number of nodes it is directly connected to. The node degree in an urban network also gives information on the connectivity of it. The higher the average node degree of a networks is, the better connected the intersections are to each other. As a general rule, both the average node degree of a network, and the distribution of the node degree in the network should then be considered (the degree distribution can vary for identical average node degrees). If the degree distribution has a large standard deviation quite a few nodes with a much higher degree than the average exist in the network. In our opinion this is good in terms of connectivity whereas it would have a negative impact on traffic flow due to the capacity constraints of big intersections.

2.4 Traffic indicators

Our main research objective is not only to assess how the traffic behaves in certain urban patterns; but to analyze which spatial properties create that behavior. The traffic indicators must provide the notion of traffic behavior and they must be simple enough to be efficiently implemented in our model. According to HCM (2000) to measure system performance one must consider: quantity of service (the amount of vehicles or persons in the system), intensity, duration, and extent of congestion, variability (differences between days), and accessibility (the amount of users that can achieve their travel goals).

Another factor that we must have in mind is which indicators can be extracted from the traffic assignment model. The fact of being a static traffic assignment model, limits, in a certain manner, the possibilities because time dimension and the spatial spread of queues are not considered. This will be definitely studied and improved in the future research. Nonetheless, the robustness (unique solution and convergence guaranteed) and the flexibility of the traffic assignment method, allows us to a complete study of the traffic flows.

The indicators employed to represent the traffic and travel behavior will be:

- **Total travel time:** the total time invested in completing the trips. This is the main indicator of the cost of the system.
- **Total distance travelled:** the distance travelled by drivers is related to travel time and speed, but at the same time, it gives a dimension on the sustainability of the network (i.e. how efficiently the system is able to connect the demand nodes.
- **Average speed:** the average speed on the network provides a view of how congested the network is.
- **Volume to capacity ratio:** same as the speed, will give us an idea of congestion, but as well of how demanded and important within the network the link is.
- **Maximum delay:** we have used this indicator to know the maximum delay incurred at an intersection so we can identify where the most problematic intersection is and how affects the system.

2.5 Link removal

The concept of link removal is widely studied in the literature. It is focused in identifying critical links in systems in order to see the network reliability (e.g. in case of natural disasters) or to plan investment or disinvestment of infrastructures (Scott et al., 2006). In contrast to these studies, the objective of the link removal in this paper is to create modifications of the initial lattice grid to explore less regular and connected patterns. The idea is to remove in an iterative process links from the grid. For every certain percentage of links removed, the spatial and traffic indicators are re-calculated. Every combination of links removed is performed randomly. For that reason the experiment will be repeated several times.

The link removal algorithm extracts a certain number of links randomly. It begins enumerating all the non-zero links of the full grid and then it chooses in every iteration a random link to be removed. In order to not leave any node totally disconnected, a shortest path algorithm is performed to ensure that it exists at least one possible route between all nodes of the grid.

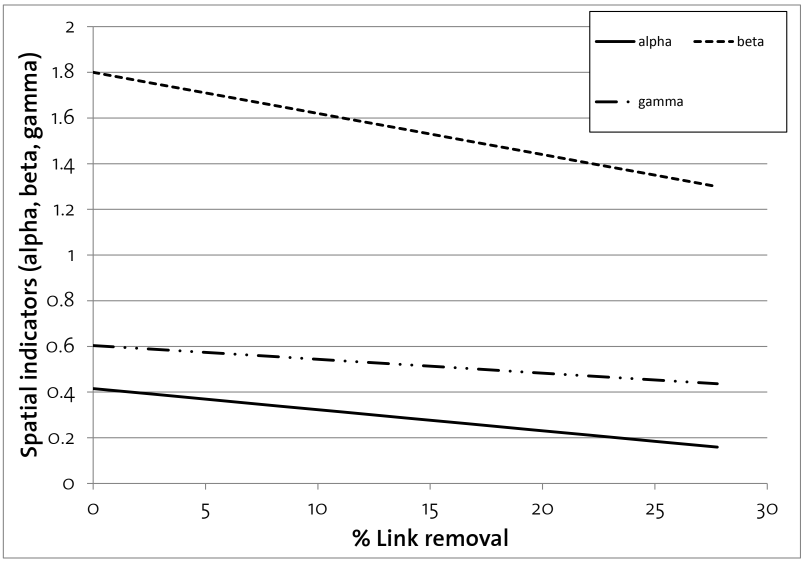
3. Results

In this section we are presenting the results obtained after running our traffic assignment model for certain network configurations. We have started with the complete and regular grid. From this point we have removed random combinations of 5, 10, 15, 20, 25, 30, 35, 40, 45 and 50 full links (i.e. both directions removed). Considering that the initial grid had 180 links, we have removed from 0 to 30 % of the links. Since the removal has been a random process, for every number of links removed the experiment has been repeated 50 times. In this manner, we obtain the individual and average values of the traffic and network indicators. The indicators analyzed (network descriptors and traffic indicators) for each combination have been introduced in the previous sections.

The main purpose of this paper is to understand how urban modifications to the initial complete structure influence the overall traffic performance. We will see, that quasi complete grids (i.e. less than 10% of links removed) cope reasonably well with traffic. In other words, the traffic indicators do not suffer drastic changes if the percentage of links removed is not superior to 10-15% for the specific demand level . The randomness of the analysis might indicate that (as long as the links have the same hierarchy), in a general view, is not as important which links are removed but the number of them. That assumption might be only valid for low link removals since when the percentage is higher we observe more variability among the random combinations.

Let us first analyze the spatial descriptors described previously. We presented three indexes (α,β, and γ) that are generally used to describe connectivity in networks. These indexes are only dependent on the number of links and nodes, hence, they are not affected by the different random combinations of a certain numbers of links removed. Figure 4 shows their relationship with the percentage of link removal.

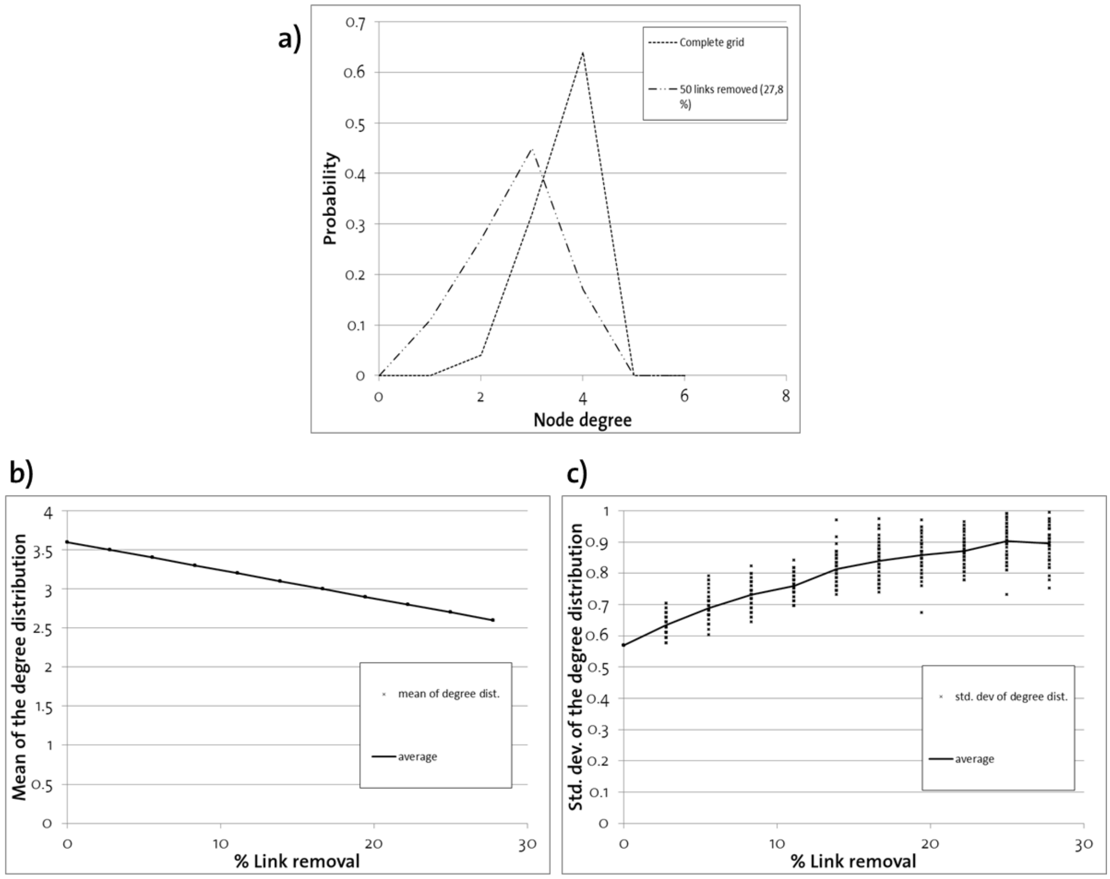
Figure 4 Spatial indicators (α,β, and γ) vs. the percentage of link removal.



The three indicators present a linear relationship with the link removal for our grid model since the number of nodes is not changed in any scenario (see equations (4), (5), (6)). These indicators are inversely proportional to the link removal (i.e. the indicators decrease as the number of removed links increases, indicating lower connectivity).

The last spatial indicator to be considered here, the node degree, depends on each network configuration. For every network case, we can determine which is the frequency distribution of the node degree. Distributions with lower standard deviation typically represent more regular networks since nodes have similar degree (Figure 5a). As soon as we remove links, the network becomes more heterogenic and the distribution flattens (Figure 5a). For that reason, in every analysis we store the mean value and the standard deviation of the degree distribution (Figure 5b and c). The mean is related linearly to the link removal rate and its variation is low (Figure 5b). At the same time, the standard deviation increases, but not drastically (Figure 5c). These results, together with the ones presented above (Figure 5), lead us to consider as spatial indicator only the percentage of link removal. We believe, that this indicator (for the current analysis of the grid network) provides a better understanding of what we are doing. In other words, it can be seen as a rate of how much we deteriorate a complete grid structure.

Figure 5 a) Two degree distributions; b) mean of degree distribution vs. link removal; c) standard deviation of the degree distribution vs. link removal.



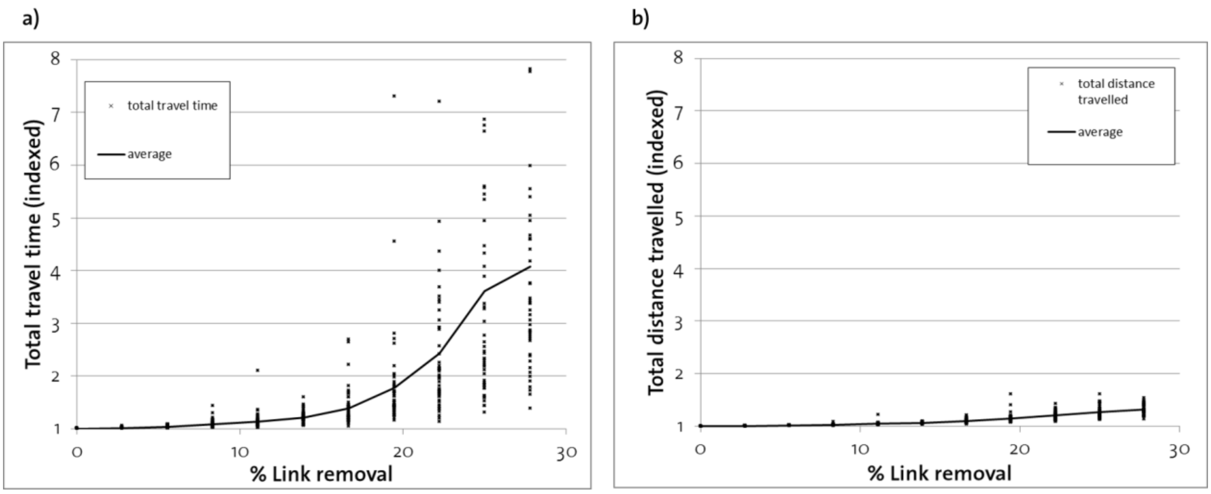
We now analyze the total cost of the system, in this case, the total travel time. Together with the travel time, we include the total distance travel in order to see if they increase in a similar way. In Figure 6, the single and average values for the 50 random cases of each link removal rate are plotted. As it seems logical, both magnitudes increase with the number of links removed since the network loses connectivity and certain routes get more loaded. Notice that such increase in cost does not have that much impact (both in average value and variability) if the percentage of link removal is below 10 %. However, we can see that the total travel time (Figure 6a) increases at a higher rate and with more variability than the total distance travelled (Figure 6b). That is due to the effects of congestion, not due so much to an increase in travel distance (due to the dimensions and properties of the grid), as to an increase in travel times (due to the excessive demand in some links and nodes).

The rapid increase of the average values is caused by the existence of very extreme cases (as it is seen in the following analysis) where travel times are dramatically high. In these cases, the network seems to gridlock. This is due to the specific link removal configuration.

Nevertheless, we are further studying if these results are plausible or they are somewhat dependent on the convergence of the algorithm.

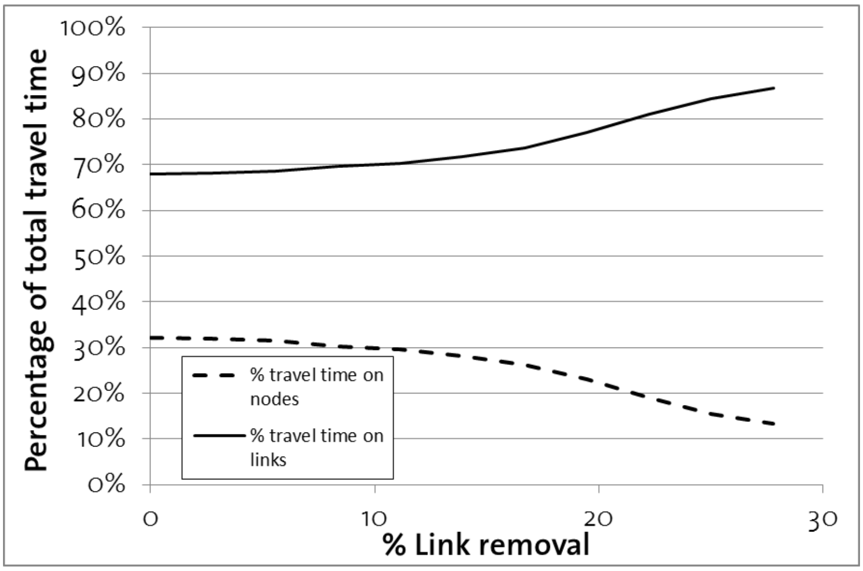
These extreme cases involve very high v/c ratios in the intersection approaches (in some cases close to 4). As we stated earlier, the adjusted BPR function overestimates the delays for v/c higher than 2-2.5. For this reason, we have removed 9 cases from the results dataset that were influencing the average values in an unrealistic way.

Figure 6 a) Total travel time vs. the percentage of link removal; b) total distance travelled vs. the percentage of link removal.



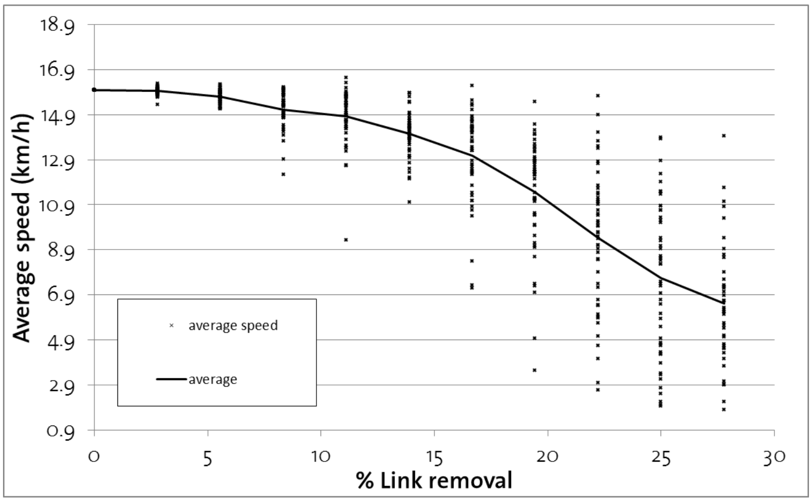
In order to see what triggers congestion (i.e. what it is more restrictive, the capacity on links or nodes), Figure 7 shows the percentage of the travel time that is consumed on the links and in the nodes. We can clearly see how the nodes of the system are the main responsible of the travel time (more than 65 %) and that value increases up to circa 80 % as links are removed.

Figure 7 Percentage of the travel time on links and nodes vs. the percentage of link removal.



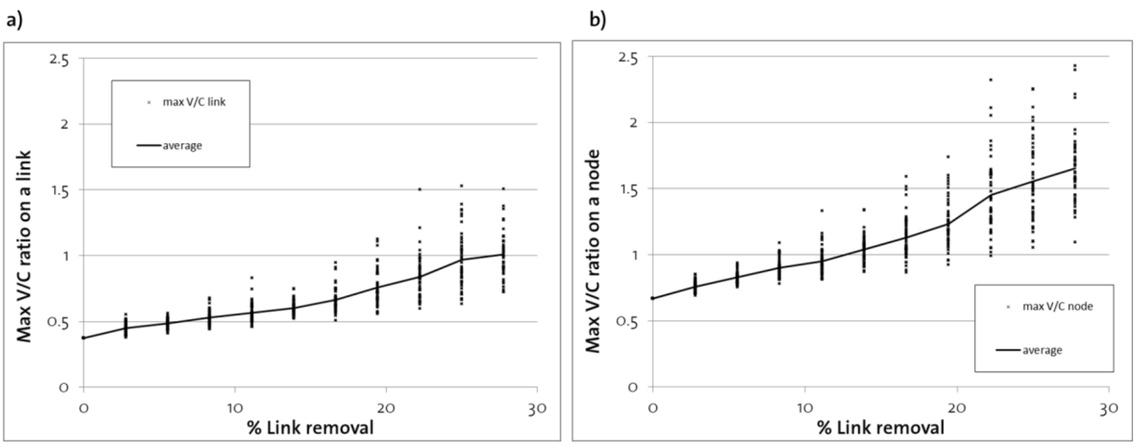
Let us evaluate the average speeds in the network (i.e. the total distance divided by the total travel time). In the complete grid case, as Figure 8 shows, the average speed is close to 17 km/h. That value is approximately one third of the free flow speed considered (50 km/h). However, all the delays (e.g. the intersection delays) are included. Overall, it seems a reasonable value due to the dense nature of our grid. After links are removed, this value is reduced, reaching very low levels in certain combinations (see Figure 8). A surprising fact is that certain combinations of link removal, actually provide a higher average speed than the initial grid (Figure 8). Nevertheless, the total travel time and the distance are higher for these cases. This might be due to the fact that drivers need to navigate the non-complete grid structures. We observe well, as stated earlier, the existence of these network configurations with really high travel times and, hence, very low average speeds (close to 1 km/h).

Figure 8 Average speed in the system vs. the percentage of link removal.



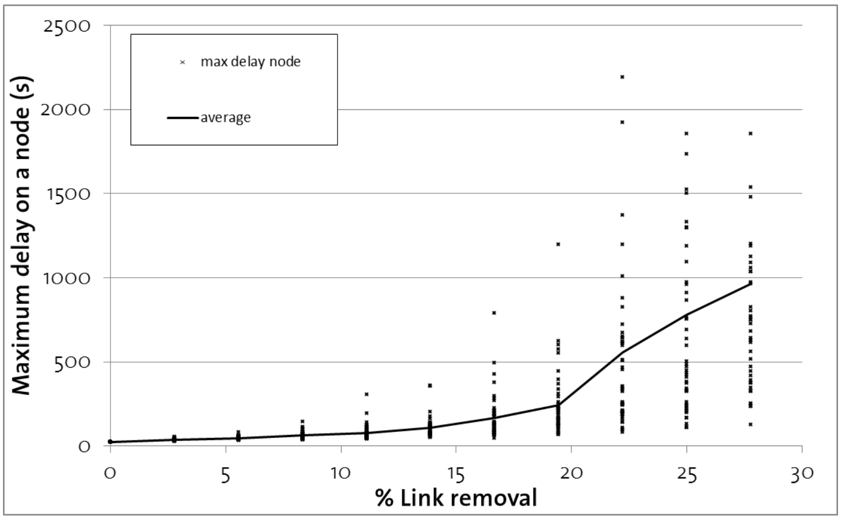
To further analyze the congestion levels in the network we employ the volume/capacity ratio. For every trial, the maximum v/c ratio in links and intersection approaches is collected. As Figure 9 shows, for a percentage of link removal lower than 10 %, the links are not congested because the grid network offers enough possibilities to channelize traffic efficiently for the given demand. For higher removal percentage, the maximum v/c ratio values indicate oversaturated conditions at least in some portions of the network (Figure 9a). This oversaturation is reached earlier in intersection approaches (Figure 9b). This is reasonable as the capacity of the intersection approaches is considerably lower than the one in links.

Figure 9 a) Max v/c ratio on a link vs. link removal; b) max v/c ratio on an intersection (i.e. node) approach vs. Link removal.



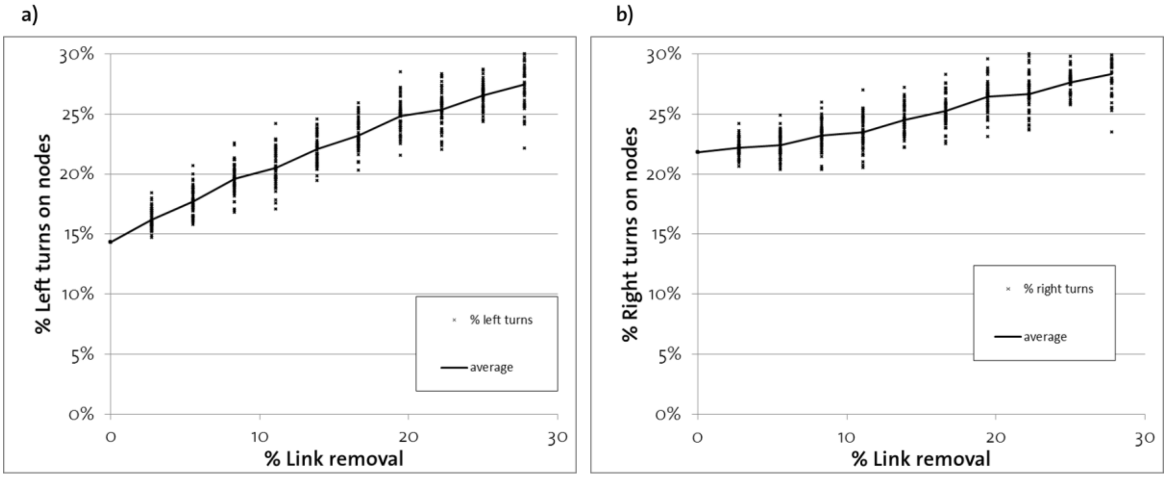
In Figure 10, we plot the maximum delay experienced at an intersection. The first cases (lower link removal) experience reasonable delays. Even when increasing the removal rate, in more congested scenarios, the maximum delays for most of the cases account for 2-3 signal cycles (C=60s) which can be acceptable. However, in the last scenarios, there are also some cases where this delay increases considerably reaching high values (circa 25 minutes). This, evidently, would not happen in real life, the demand would somehow react to such scenarios.

Figure 10 Maximum delay on an intersection approach vs. the percentage of link removal.



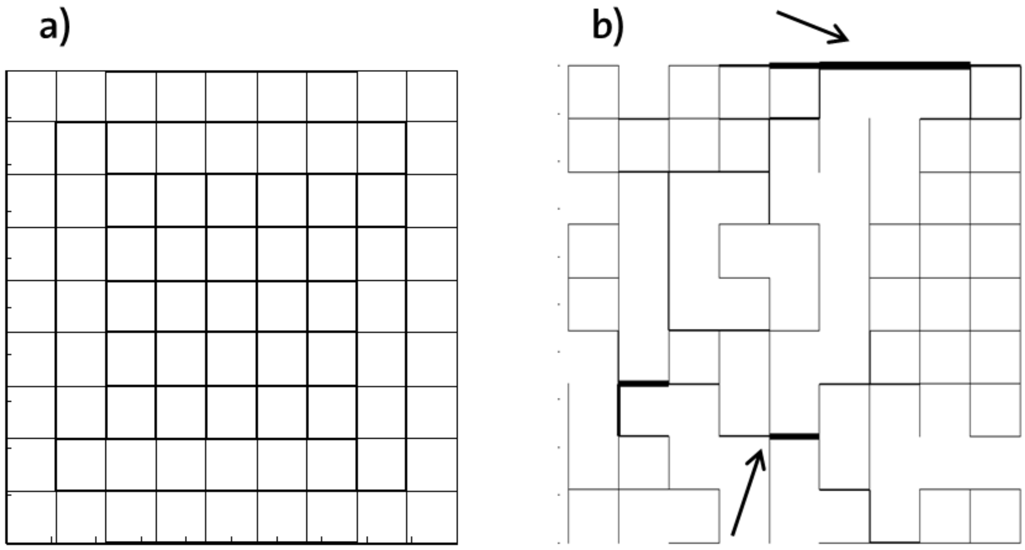
If the intersections are the critical points in our system, the left turns approaches are the weakest point in the intersections. Figure 11 plots the percentage of left and right turns carried out in the intersections for the evaluated scenarios. The left turn represents a slightly higher penalty than the right turn. In the more complete network configurations, the left turns are taken slightly less often than the right turns (15 % vs. 22% of total maneuvers). However, when links are removed, there are less route choices, and more drivers are forced to make left-turns. That fact creates even higher delays.

Figure 11 a) The percentage of left turns in intersections (i.e. nodes) vs. the percentage of link removal; b) the percentage of right turns in intersections vs. the percentage of link removal.



Finally, as an example, in Figure 12 we plot the flows on two network scenarios: the complete grid and a case with 28% of links removed. We can see how the more loaded links correspond to the cases where adjacent links have been removed. The Figure 12b case represents one of the extreme configurations we mentioned earlier. The grid is almost disconnected in two parts, only connected by two links (pointed out in the Figure 12b). These two links will have high flows. In the case of the lower link (Figure 12b) is even more critical because it belongs to a 4 way intersection. The capacity for the intersection approaches is lower, and hence, the delay is higher. It should be noted that in this grid, the signalized intersections are not coordinated. In real urban networks, the coordination between the different signals could help avoiding these critical cases.

Figure 12 Flows on two network scenarios: a) complete grid, and, b) 50 links removed.



4. Conclusions

The main idea of this paper was to explore the traffic response of different urban networks. Starting from a complete grid structure, through a random link removal process, we obtain not as regular and connected patterns (i.e. quasi complete grid patterns). The demand model used is simple and abstract and has as main purpose loading the network effectively. The traffic assignment algorithm is static and considers delay at intersections.

After carrying out this study we can conclude that:

- The overall objective has been achieved since we have developed a methodology (based on a compromise between accuracy and simplicity) in order to cope with all the factors that might influence the traffic performance on an urban network.
- The network descriptors have not been that useful in this particular study. Since they present linear relationships with the percentage of links removed, we have chosen this last indicator to compare traffic magnitudes.
- The quasi complete networks, where less than 10 % of the links have been removed, obtain very similar traffic indicators as the complete grid. In addition to that, the randomness of the link removal does not create much variation between the different combinations explored. This value (10%) is, evidently, a function of the specific demand
- When more than 10% of the links are removed (for the demand scenario considered), the traffic performance might be reduced significantly. In these cases, the traffic performance of the different networks created for the same number of links removed (different random combinations), presents a high variability.
- The intersections are the most critical points in the system. They account for more than two thirds of the total travel time. The delay incurred in the intersections grows notably when the link removal is high. We have observed that for our case, the formulation of the BPR overestimates this delay if v/c ratios are higher than 2.
- The rapid increase of the average values is caused by the existence of very extreme cases where travel times are dramatically high. In these cases, the network seems to gridlock. This is due to the specific link removal configuration. Nevertheless, we are further studying if these results are plausible or they are somewhat dependent on the convergence of the algorithm.
- The algorithm developed performs efficiently in most of the cases. However, for certain iterations it is unable to find an optimal step length (λ) in the traffic assignment routine. This error has occurred, approximately, in 5% of the cases. We are currently analyzing the causes of this error and potential solutions. Therefore, the results presented in this paper should be taken as provisional.

This paper is our first step towards a deeper study of the grid networks' behavior. Future research work should consider the use other traffic assignment algorithms that take into account the time and the spatial dimension (i.e. the propagation of queues). In addition to that, the link removal should be studied in a greater detail. It should be considered the impact of the removal depending on the location of the link. Finally, the directions of the streets should be further analyzed. Different configurations with unidirectional links could be proposed.

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