Dynamic microsimulation of location choices with a quasi-equilibrium auction approach

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Abstract

A method for dynamic simulation of residential location choice modeling at an agent-based level is proposed. The method is based on the bid-auction approach to location choice and assumes that each real estate good is traded in an auction where the best bid defines the price and determines the location of agents. The framework considers a market clearing mechanism that solves a quasi-equilibrium at each period, allowing households to adjust their expectations and bids as a reaction to observed market conditions. The model is implemented and tested for the city of Brussels. Results show that the model is able to reproduce observed price patterns and their evolution in an eight year period, indicating that the spatial distribution of agents forecasted by the model is also accurate.

Keywords

location choice, bid rent, auction, market clearing, real estate

1 Introduction

Interventions on urban systems, such as real estate developments, modifications to the transport system and changes in urban policy, are usually costly to implement and therefore require models to forecast and evaluate their performance and effects in other elements of the system. Land use and transportation models are tools used for this purpose. Among these, microsimulation models are becoming more relevant and attractive due to the possibility of representing individual agents and their complex interactions in a simple, yet robust and flexible, way. Moreover, agent based microsimulation can easily account for the dynamics in the system, something that is hard to achieve in equilibrium models

Location choice is one of the most important processes to model in a land land use model. Location choice and real estate prices have been traditionally modeled under two different paradigms: the choice approach and the bid-auction approach. Under the choice paradigm, households select the location that maximizes their utility, with prices being determined exogenously through a hedonic model. The bid-auction approach assumes that real estate goods are traded in an auction market, where the best bid for a particular location determines both the located household and the price of the dwelling.

The real estate market has two unique characteristics: first, all goods are quasi-unique since, because of their spatial nature, all locations are different and, at the same time, each good must be assigned to one buyer only. Second, demand is inelastic because all agents need to be located somewhere. This means that the market needs to clear because several agents are competing for a finite number of locations and this competition will have an effect on both the prices or goods and the spatial distribution of agents. Market clearing is traditionally achieved in aggregated models by solving an equilibrium: finding the state of the system where all agents can't improve their situation. Finding an equilibrium requires solving a fixed point problem and some unrealistic assumptions like having supply always matching demand or allowing for the relocation of all agents in the system. The market clearing problem has been less visited in the case of agent-based, microsimulation problems and it's either ignored or it's solved at an individual level, assuming that agents are price takers and therefore ignoring market effects.

This paper proposes a method to model location choice and real estate prices simultaneously in a microsimulation context. The method is based on the bid-auction approach and understands both location and prices as a function of the households' preferences. The proposed approach does not require solving for equilibrium, but estimates the maximum bid in each period by simulating the underlying auction process. Given exogenous supply levels, households adjust their preferences (and their willingness to pay) as a reaction to the (observed) market conditions. This adjustment goes in the direction of an equilibrium (although it does not reach it) and avoids over or under-bidding from high and low income households respectively, therefore

producing location probabilities that distribute households into locations that tend to maximize their perceived utility.

The paper is organized as follows: Section 2 describes the main theory behind the bid-auction approach to location choice modeling and explains why we prefer this approach over the choice approach. Section 3 explains the market clearing mechanisms used in equilibrium models and agent-based model. Section 4 proposes a market clearing mechanism for a bid auction location choice model. Section 5 describes the estimation and implementation of the proposed framework for the city of Brussels and shows validation results. Finally, Section 6 concludes the paper and identifies possible further research.

2 The bid approach to location choice

Since Alonso (1964), the real estate market has been understood as an auction market, where households bid their willingness to pay for a particular good (residential unit) which is assigned to the best bidder. This process simultaneously defines the price of the good, understood as the maximum bid in the auction process.

The willingness to pay, from an economic point of view, can be derived from the classical consumer's problem of maximum utility, given income constraints:

$$\max_{x,i} U(x, z_i) \tag{1}$$

$$s.t.\ px + r_i \leq I$$

In the previous problem, the consumer maximizes his utility by choosing a vector of continuous goods (x) and a discrete location (i), described by a set of attributes (z_i). The budget constraint states that the total amount spent in goods (with price p) plus the price of the selected location (r_i) must be smaller that the consumer's available income (I). Solving the problem on x and assuming equality in the budget constraint, the problem can be re-written as

$$\max_{i} V(p, I - r_i, z_i) \tag{2}$$

where V is the indirect utility function, conditional on the the location. Conditional on the level

of maximum utility ($\overline{U}$), the indirect utility can be inverted in the rent variable:

$$r_i = I - V^{-1}(\overline{U}, p, z_i) \tag{3}$$

Under the auction market assumption, the rent variable can be understood as the willingness to pay for a particular location, therefore the bid function B can be expressed as:

$$B_{hi} = I_h - V_h^{-1}(\overline{U}, p, z_i) \tag{4}$$

In the bid function, the index h has been included to take into account heterogeneity in preferences within different households. If we assume bids to be random variables, with an extreme value distributed error term, it is possible to express the probability of a household (h) being the best bidder for a particular location (i) as follows (Ellickson, 1981):

$$P_{h/i} = \frac{\exp(\mu B_{hi})}{\sum_g \exp(\mu B_{gi})} \tag{5}$$

Under the auction market assumption, the price or rent of a good will be the maximum bid. The extreme value distribution assumption allows to express the expected maximum bid for a particular location as the logsum of the bids

$$r_i = \frac{1}{\mu} \ln \left(\sum_g \exp(\mu B_{gi}) \right) \tag{6}$$

The relationship defined by equation (6) is one of the main advantages of the bid approach: prices are explicitly dependent on the preferences of all households involved in the market and, therefore, the price formation process is clear and reacts to market conditions. A hedonic model (usually used to describe prices in a choice approach) estimates prices by assigning a marginal value to each attribute of a location, independent of the involved agents and therefore being less sensitive to market conditions like demand or supply surplus. An analysis of the disadvantages of using hedonic prices and the differences between them and maximum bid prices can be found in Hurtubia *et al.* (2010).

3 Market clearing

Equilibrium models assume that the market clears through the adjustment of prices until every agent (or group of agents) has achieved a state where unilateral decision can't improve their

perceived utility. This means that there is a combination of location choices and equilibrium prices where every agent achieves maximum utility (Fujita, 1989). Achievement of this estate requires the simultaneous adjustment of all locations and utility levels (or prices). Traditionally, equilibrium-based land use model find equilibrium by adjusting this variables until every agent is located somewhere and every location is selected by someone. In the case of a choice approach to location choice (McFadden, 1978) this translates into finding prices such that supply equals demand or more specifically into solving the vector of rents (r) that solves the following system of equations (Anas, 1982):

$$\sum_h H_h P_{i/h}(V(r, P_{i/\overline{h}})) = S_i \quad \forall i \tag{7}$$

where $P_{i/h}$ is the probability of household h choosing location i, this probability depends on the perceived utility (V_{hi}), that depends at the same time on the rent or price of the location (r_i) and the location choices of the rest of the households ($\overline{h}$). The system of equations is a fixed point problem due to this interdependence of the location decisions and to the fact that prices will depend on socioeconomic attributes of the location, that are defined by the location choices. The variables H_h and S_i represent the number of households in group h and the supply in location i respectively.

Similarly, in bid-auction based equilibrium models like MUSSA (Martínez, 1996),clearing the market involves finding the vector of utility levels (U) that solve the following system of equations:

$$\sum_i S_i P_{h/i}(B_{hi,}(U_h, P_{\overline{h}/i})) = S_i \quad \forall h \tag{8}$$

where the best bidder probability for a given location $P_{h/i}$ depends on the bid function or willingness to pay of a households for a location (B_{hi}) which at the same time depends on the utility level achieved by the household (U_h) and the location distribution of the rest of the agents ($\overline{h}$). Notice the analogy between rents in the choice approach and utility levels in the bid approach, both being elements that are adjusted to achieve equilibrium.

If prices are the outcome of an auction process and the market clears, the distribution of households across locations obtained through solving (7) will be the same as the distribution obtained from solving (8) when supply and demand are equilibrated (Martinez, 1992). This also means that the previous equilibrium conditions of (7) and (8) can be achieved only when an absolute equality between supply (the number of location alternatives) and demand (the number of

households) holds, meaning that:

$$\sum_h \sum_i P(i,h) = H = S \tag{9}$$

with H the total number of households and S the total number of locations. This is a very restrictive and unrealistic assumption, specially if an agent based model is to be implemented.

An alternative way to deal with the market clearing is the disaggregated approach proposed in models like ILUTE (Salvini and Miller, 2005) where each transaction (or matching between agent and location) is microsimulated by modeling the interaction between sellers and buyers who negotiate based on their willingness to pay and reservation prices respectively. This approach does not require strong assumptions like the perfect satisfaction of demand, but is extremely expensive in computational terms. It also depends heavily in the choice set formation process that defines which households consider which locations to negotiate prices, which is hard to model because the choice set of decision makers is usually unidentifiable. This motivates the search of alternative ways to clear the market that are compatible with an agent based approach.

4 Quasi equilibrium market clearing

Implementing an equilibrium-based market clearing process is not straight forward to implement in an agent-based framework, because it requires to adjust the location of all agents and prices can only be determined if a supply-demand equilibrium is achieved and bids are adjusted to this. The complexity comes from the fact that equality between demand and supply is usually not guaranteed in a microsimulation (because of an independent supply generation process). This difficulties are addressed and partly overcome in Martínez and Hurtubia (2006), but in an aggregated, quasi-equilibrium context.

We propose a model where, at each period of time, the auction for each good is simulated, therefore obtaining rent levels that reflect the competition between different bidders for the good. The solution is inspired in the utility adjustment of bid-based equilibrium models but does not solve a fixed point problem.

We assume the bid function to be composed of two elements, therefore, for a particular period t:

$$B_{hi}^t = b_h^t + b_{hi}(z_i^t, \beta) \tag{10}$$

where b_h^t is the adjustment component that relates the bid with the utility level of the household and b_{hi}^t is the hedonic part of the bid expressing the value a household h gives to the attributes (z_i) of a location i through a set of parameters β. The functional form of (10) implies the assumption of a quasi-linear underpinning utility function which allows to the additive decomposition and simplifies the interpretation of each element Martínez and Henríquez (2007). We assume the preferences of households remain constant in time, therefore the value of the hedonic part of a bid for a particular pair (b_{hi}) will remain constant in time unless the attributes of the location (z_i^t) change from one period to the next. It is reasonable to expect changes market conditions from one period to the other (population , income levels, available supply, etc.) making the utility term b_h reacts to these changes, therefore having different values in each period.

The adjustment of b_h follows the logic of households changing their expectations given what they observe in the market and, therefore, increasing or decreasing their bids depending on the conditions of the auction. In each auction, if there is a demand surplus, households will try to outbid other households until reaching an expected average outcome of winning auctions that allows to locate "somewhere" (although it does not ensure their location). Similarly, in the presence of supply surplus, households will reduce the level of their bids because they can reach an expected number of winning auction that allows to locate somewhere with smaller bids.

In each period, the knowledge of the state of the market comes from the observed rents from previous periods (r_i^t). We assume that households also observe the available supply (S^t) and know the number of households looking for a location in each period (H^t). However, we assume they don't observe the bids of other households (therefore our system represents a sealed-bid auction). Considering this information, each household estimates the value of b_h^t required to make the expected number of winning auctions equal to one.

$$\sum_i P_{h/i}^t = \sum_{i \in S^t} \frac{\exp\left(\mu(b_h^t + b_{hi}(z_i^t))\right)}{\sum_{g \in H^t} \exp(\mu B_{gi}^{t-1})} = 1 \tag{11}$$

Since households can't observe the bids of other households in t we assume they observe the bids in the previous period ($t-1$). This is equivalent as observing the rents in the previous period since, following (6), the denominator of (11) can also be expressed as:

$$\sum_{g \in H} \exp(\mu B_{gi}^{t-1}) = \exp(\mu r_i^{t-1}) \tag{12}$$

The previous expression implies the assumption of myopic households, that, being unable to

forecast the future equilibrium rents, use the available historic information of past rents as a proxy.

Clearing b_h^t from (11) and assuming that only rents from the previous period can be observed, we obtain:

$$b_h^t = -\ln\left(\sum_{i\in S^t} \exp\mu\left(b_{hi}^t(z_i^t) - r_i^{t-1}\right)\right) \tag{13}$$

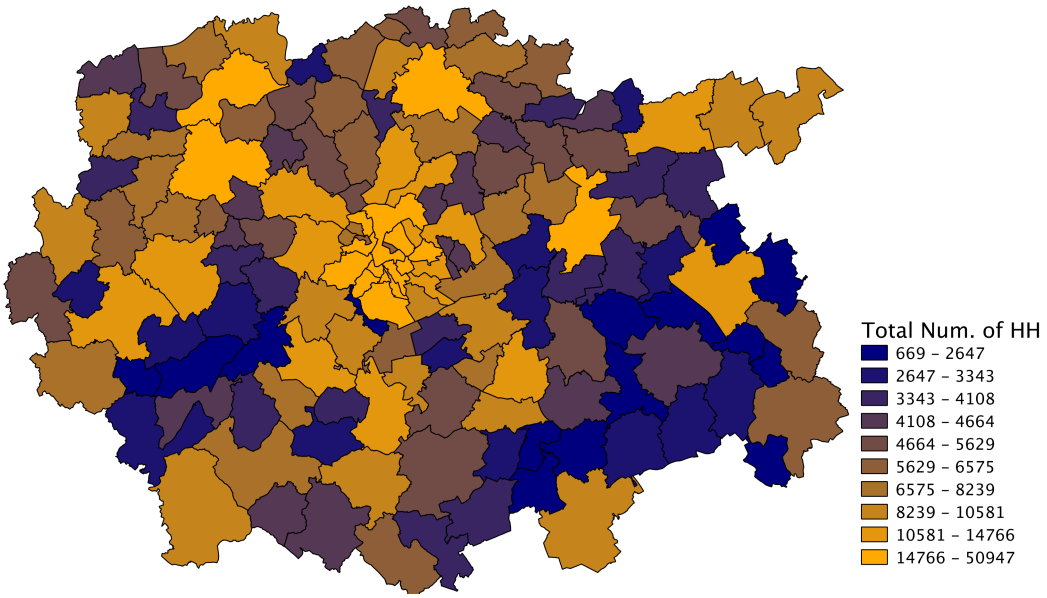
The adjustment of (13) is similar to the one proposed by Martínez and Donoso (2011) with the difference of considering that only the households looking for a dwelling and the available units have an effect in the bid level correction. After the adjustment of $b_h$is calculated it's possible to calculate the location probabilities and rents in t.

5 Case study

The proposed model is implemented for the city of Brussels, where data has been collected in the context of the European research project SustainCity[1]. The main data sources are the 2001 Population Census and a travel survey containing information about location preferences and socioeconomics for a sample of households (MOBEL 2000). Information about average income per zone an average transaction prices of dwellings was also collected from the Belgian Statistical Office (StatBel). The study area considers an extended metropolitan region, including 151 communes (c) that contain a total of 4945 zones (i). Dwelling alternatives (v) are classified in 3 types of houses (fully detached, semi-detached and attached) and 1 type of multi-family units (apartments). The area of study contains a total of 1213169 households, Figure 1 shows the distribution of households across the communes in the are of study. Central communes (the city of Brussels) concentrate the larger amount of located households and are, at the same time, the most dense communes. Outer communes are less dense with the less populated communes located south east and south-west of Brussels city. For the base year a synthetic population is synthesized (for details of the process see Farooq *et al.* (2011)) where individual households are describe in term of their socioeconomic attributes and their location (building type and zone). The marginal distributions of attributes for the synthetic population are consistant with observed distributions coming from the census and other data sources. This consistency allows to estimate models over CENSUS data that can be implemented over the synthetic population.

[1] www.sustaincity.org

Figure 1: Number of households by commune, 2001



5.1 Bid-auction model estimation

A residential auction-based location choice model is estimated using a double equation method that adjusts the choice probabilities to the observed locations of households while simultaneously reproducing observed prices as a function of the expected maximum bid (see Hurtubia *et al.* 2011 and Hurtubia and Bierlaire 2012). Table 1 describes the specification of the linear-in-parameters bid function that was finally estimated while Table 2 shows the estimation results, obtained with the statistical software BIOGEME (Bierlaire, 2003; Bierlaire and Fetiarison, 2009).

All parameters have the expected signs. The scale parameter μhas been fixed to one. Some interesting results are the different effects of a variable when computed at different spatial aggregation levels. For example the presence of industry in is attractive for households with workers at a commune level but is unattractive for households of high income. This can be interpreted as households liking to have job opportunities nearby but being negatively affected by the externalities of industry when located to close. Another interesting result is the agglomeration effect observed for households of high income (levels 4 and 5) compared with the opposite effect that is observed in low-mid income households (levels 2 and 3).

The parameters α, γ and σ are coefficients for the latent auction price model that computes

Table 1: Bid function specification

Parameter	spatial attribute	×	household (hh) attribute
ASC_2	-		income level constant (745-1859 Euros)
ASC_3	-		income level constant (1860-3099 Euros)
ASC_4	-		income level constant (3100-4958 Euros)
ASC_5	-		income level constant (>4959 Euros)
B_educ_zone	% of education jobs in zone i	×	dummy for hh's with children
B_educ_comm	% of education jobs in commune c	×	dummy for hh's with children
B_house1	dummy for isolated house	×	dummy for hh's with more than 2 people
B_house2	dummy for semi-isolated house	×	dummy for hh's with more than 2 people
B_house3	dummy for attached house	×	dummy for hh's with more than 2 people
B_income_23	% of hh's of income level 2 and 3 in zone i	×	dummy for income level 2 or 3
B_income_45	% of hh's of income level 4 and 5 in zone i	×	dummy for income level 4 or 5
B_indu_zone	% of industry jobs in zone i	×	dummy for income level > 3
B_indu_comm	% of industry jobs in commune c	×	dummy for hh's with active workers
B_service_zone	% of service (office and hotel) jobs in i	×	dummy for hh's with active workers
B_shop_comm	% of retail jobs in commune c	×	dummy for hh's with active workers
B_surf_h	surface of dwelling v	×	dummy for multi-person hh's with inc level > 3
B_surf_m	surface of dwelling v	×	dummy for multi-person hh's with inc level = 3
B_trans	public transport $acces_i$ (facilities/km^2)	×	dummy for hh's with 0 cars
B_trans2	0public transport $acces_i$ (facilities/km^2)	×	dummy for hh's with 2 or more cars
B_univ_comm	% of people with university degree in c	×	dummy for hh's having integrants with univ degree

prices p_{vi} as a function of the expected maximum bid for each real estate unit, following:

$$\ln(p_{vi}) = \alpha + \gamma \cdot r_{vi} \tag{14}$$

where r_{vi} is the logsum of all potential bid for a unit of type v located in zone i as describe by equation (6). The price model is applied at the zone level (i) but, given the available data, it is estimated only at the commune level, therefore replacing i for c in estimation mode.

5.2 Simulation results

Simulations are run for a period of 8 years, from 2001 to 2008. The reason to select this period is the availability of validation data regarding transaction prices for houses and apartments at the commune level.

In each period, and following observed population growth rates, new households are generated. Since there is no information about the socioeconomic composition of households after the base year it is assumed that new household follow the same socioeconomic attribute-distribution as previously existing ones. This means that new households are a replication of randomly selected pre-existing households. For simplicity, the modeled scenarios does not consider the

Table 2: Estimation results

Parameter	Value	Std error	t-test
ASC_2	-0.0496	0.21	-0.24*
ASC_3	-0.442	0.224	-1.97
ASC_4	-0.751	0.181	-4.15
ASC_5	-0.96	0.233	-4.13
B_educ_zone	0.269	0.12	2.25
B_educ_comm	0.562	0.528	1.07*
B_house1	0.755	0.0828	9.11
B_house2	0.935	0.0799	11.7
B_house3	1.12	0.0717	15.62
B_income_23	-0.327	0.231	-1.41
B_income_45	1.91	1.08	1.77*
B_indu_zone	-5.36	2.62	-2.04
B_indu_comm	0.247	0.11	2.25
B_service_zone	0.243	0.0542	4.49
B_shop_comm	3.13	0.458	6.84
B_surf_h	0.00916	0.00197	4.66
B_surf_m	0.00642	0.00124	5.16
B_trans	0.739	0.0811	9.12
B_trans2	-0.548	0.0989	-5.55
B_univ_comm	3.11	0.134	23.25
α	1.84	0.708	2.6
γ	0.659	0.0505	13.04
σ	-1.87	0.0182	-102.42

* Parameter not significant at the 95% level

possibility of relocating households.

After new households have been sampled, a random sample of new residential units is generated. The new supply also following the attribute-distribution and spatial distribution of existing units but is constrained to comply with zone-level capacities and is set to satisfy the new demand at an aggregate level. The market clearing mechanism described in 4 takes place and auctions are simulated. After all households are located, zonal and communal attributes are re-computed and prices are calculated for each period following (14). For simplicity, the non residential attributes remain constant in this simulation.

Figure 2 shows the increase in the number of households by commune. The location of new households follows the spatial distribution of the new supply. Since the generation of new supply is random and constrained by zonal capacity, most new households are located in communes that originally presented low density. The simulation does not take into account land use regulations or development constraints and results could be clearly improved by doing so. However, results are consistant with the observed trend of increase in the population in rural areas.

Figure 2: Increase in number of households by commune (%), 2001-2008

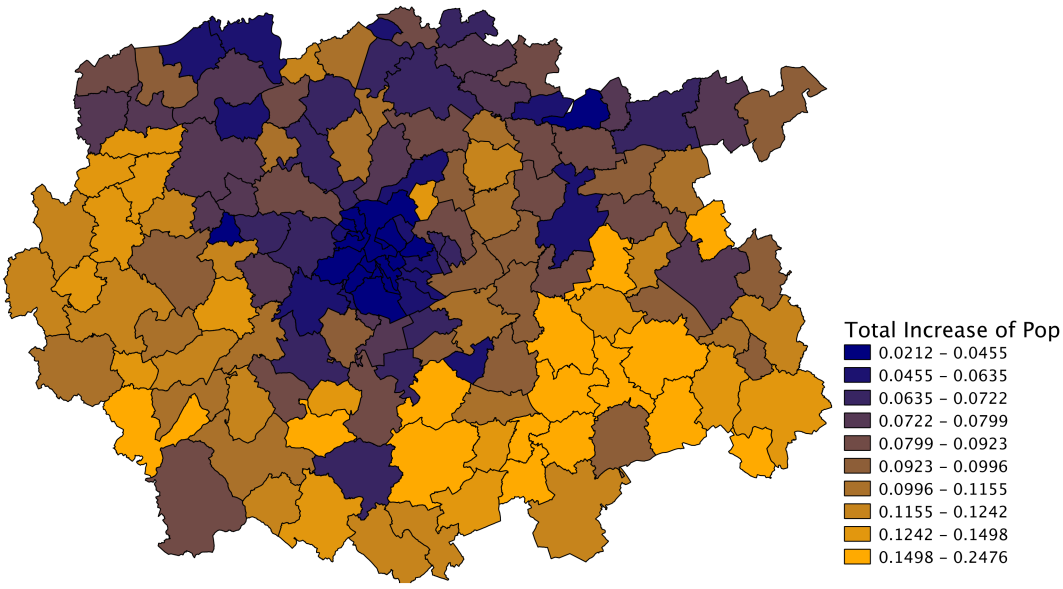
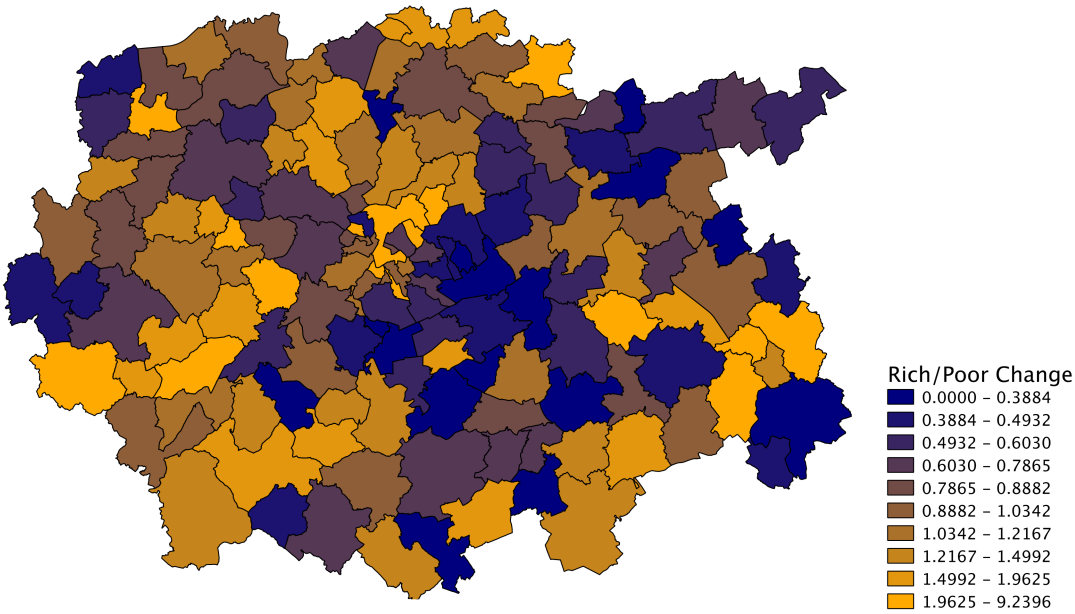


Figure 3 shows the ratio between the increase in the number of rich households (with income higher than 3100 Euros) and the increase in the number of poor households (with income lower than 1860 Euros). A darker color indicates an increase in the proportion of poor households in the commune while a lighter color indicates an increase in the proportion of rich households. The map shows an increase of rich households in the north of the city of Brussels and an increase of poor households in the southern and eastern peri-urban communes of the city. In general the location of new households of high income takes place in rural or suburban regions.

Figure 4 show the evolution of prices between 2001 and 2008. The map shows that price increases are higher in the city of Brussels and, in general, in the northern communes of the area of study. Southeastern communes show a slower increase of prices. In terms of accuracy of the forecast, Figure 5 shows the observed and forecasted prices for years 2001 and 2008, ordered increasingly according to observed prices values for 2008. For year 2001 the model performs reasonably well, with 42% of the forecasted prices having a difference smaller than 10% from observed average prices while 86% have a difference smaller than 25%. It is important to notice that the deviation in forecasted prices is not only explained by differences between observations and predictions, but also by the fact that observed prices consider only the average for houses and apartments by commune, while forecasted prices consider 3 different types of houses and apartments by zone . For year 2008, the forecast is slightly worse, with 38% of the forecasted prices have a difference smaller than 10% from observed average prices while 79% have a dif-

Figure 3: Predicted variation in the rich/poor ratio per commune, 2001-2008



ference smaller than 25%. The most interesting result is the non-uniform increase in observed prices that is correctly followed by forecasted prices, this indicates that the model not only correctly forecasts the general level of prices, but also predicts location choice. This is because the only variables that can change enough to explain the non uniform increase of prices are those related to the location of households: primarily income distribution per zone,and number of people with university degree per zone.

6 Conclusions

The proposed model is able to account for the auctioning process that takes place in each period of a simulation. The advantage of the model lies in the fact that is able to account for changes in the general conditions of the market, like a growth or a reduction of the ratio between demand and available supply. The method is based on a bid approach for location choice modeling and considers a market clearing process where agent adjust their utility levels (and therefore their bids) while adjusting to the market conditions but only being able to observe the state of the market in previous periods.

The framework is implemented for the city of Brussels and the city is simulated between years 2001 and 2008. Results show that including a measurement relationship between the logsums

Figure 4: Predicted increase in price (%) by commune, 2001-2008

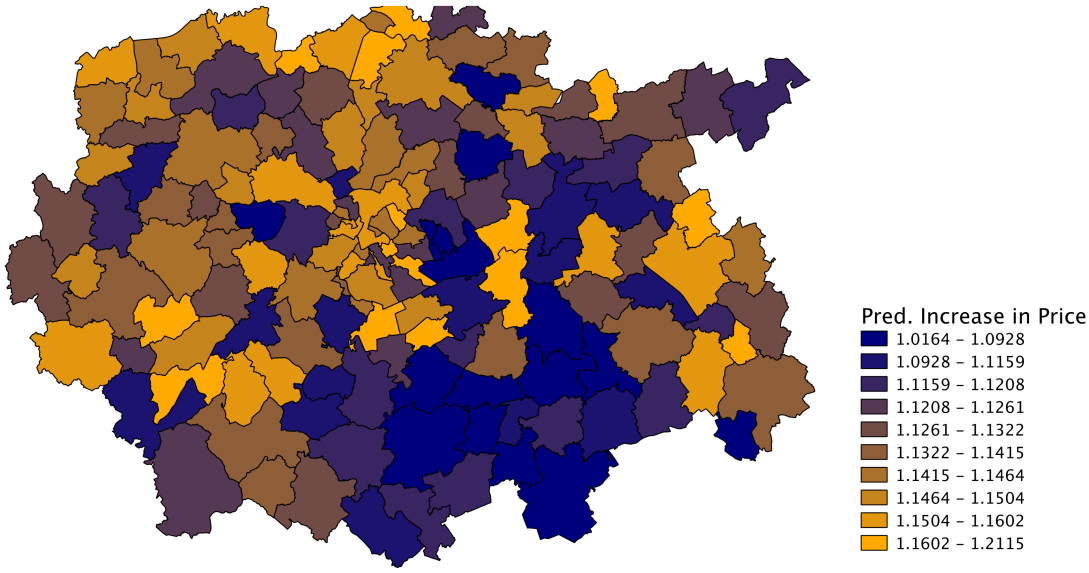
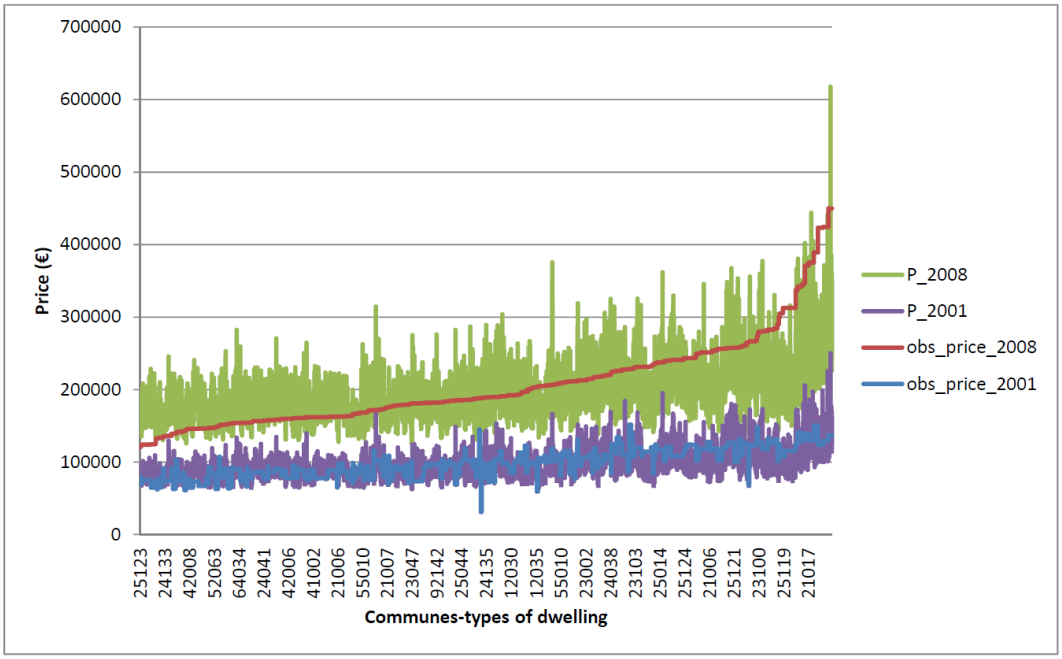


Figure 5: Evolution of observed and forecasted prices, 2001-2008



and the observed prices in the log-likelihood maximization process allows to obtain better estimates of the bid function parameters. The proposed model is able to forecast the prices as a function of the expected maximum bid of the auction process generating results that are consistant with observed trend of prices increasing more for some particular communes. The

differences observed between forecasted and observed prices can be explained by the aggregated nature of the observed price indicator. A more disaggregated indicator should allow for a better estimation and, consequently, a better fit. The fact that only residential location choice dynamics indicates that results could be improved by introducing models for other set of agents and processes.

Future work will improve the quality of the models for generation and location of new supply (randomly generated from control totals in the current version) and will incorporate a non-residential location choice model.

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