An Integrated Transport Network-Computable General Equilibrium Models for Zurich

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1 Introduction

Zurich, the biggest city in Switzerland, will in the next decade face, as many other big cities around the world, substantial changes in transport, housing infrastructure and zoning regulation as well as changes in population and employment. These changes will not only have their impact on the locational decisions of households and firms, but also on wages, housing prices and the environment. These changes are usually studied with the tools of urban economics. Most of these tools are partial models that look at certain aspects of these changes. However, as all these factors are depending on each other, a more comprehensive tool is needed.

A tool that can analyze all the linkages is a computable general equilibrium (CGE) model. CGE models have been used extensively over the past 30 years. They can be used to analyze policies in many economic fields. Trade, environmental, energy and social policy are just a few to name. With CGE models the researcher can look at the effects of policy alternatives compared with the benchmark equilibrium. industrial sector-wise and aggregated prices, outputs, GDP and many other indicators. Most of these models are, however, used for countries or several regions (see for example Fujita et al., 2001) and not at the urban level. Another drawback is the fact that CGE models treat transportation as a normal production sector.

There is a variety of transport models developed by transport engineers that explicitly look at the structure of the public and private transport in a city or region (See for example Ortúzar and Willumsen, 2010).

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There not many urban computable general equilibrium models that link the transportation structure of a city with the economic part of the model. Examples are Anas and Kim (1996) and Anas and Xu (1999). The most obvious reasons for this lacking of integrated urban CGE models are twofold. First, the researcher has to find solutions for two problems: a traffic assignment problem (TAP) and a cge problem. Developing a CGE model might be a tedious task, but this is also true for a TAP.

The second problem when developing an urban general equilibrium model are the lack of model formulations that can be solved by off-the-shelf optimization software.Transportation models are usually solved with heuristic algorithms.[1] In a first step, traffic is assigned to all the arcs, and new travel times over the arcs are calculated. In a next step the traffic assignment is adjusted. This process is repeated until a solution is found.

Ferris et al. (1998) showed in their paper that the Wardropian traffic equilibrium model can be solved efficiently as a mixed complementarity programming problem using off-the-shelf software. In contrast to pure transportation network models, in this formulation this model formulation does not require enumeration of all paths between origin-destination pairs. This paper seemed to never have found its way to the transport engineers.

CGE models can be formulated and solved in different ways (See for example Ginsburgh and Keyzer, 1997). The researcher can write his own algorithm or use off-the-shelf software. This kind of software allows the modeler to concentrate on the model formulation without having to write his own algorithm.[2]

In this paper we present a formulation of an urban general equilibrium model that embeds a closed spatially disaggregate Alonso-Muth-Mills economic model of housing and labor markets[3] within a model of individually-rational route choice on the (congested) traffic network (Wardrop, 1952). The central assumption of the Alonso-Muth-Mills (AMM) model is that households choose residence and employment locations which arbitrage differences various locations within the urban area. This implies that consumers trade locations to work and live on the basis of housing prices, wages, and commuting time (e.g. Anas and Xu, 1999). The transport costs in the AMM Model depend on the distance between the location where the household lives and where he works. In our model there are several ways to travel from one node to another. The distance depends on the route chosen.

In contrast to pure transport models prices and demand for transport, employment and housing

[1]See for example Sheffi (1985), van Vliet (1978),LeBlanc and Mustafa (1979), LeBlanc et al. (1975), Ouorou et al. (2000).

[2]We use GAMS. GAMS is an acronym for General Algebraic Modeling System. It is a high-level modeling system for mathematical programming and optimization and consists of a language compiler and a stable of integrated high-performance solvers. GAMS is tailored for complex, large scale modeling applications, and allows you to build large maintainable models that can be adapted quickly to new situations.

[3]For a description of the AMM model see for example Glaeser (2008, Chapter 2) or Brueckner (1987).

are endogenized. The model has two interacting components: a model which describes the different transport modes, routes and traffic flows given where people live and work, and a sorting model which describes where people choose to live and work. Households can choose between the different transport modes based on a logit demand formulation.

The economic aspects of the model follow the Walrasian-Arrow-Debreu paradigm. Consumers earn money by working, and they allocate their income to housing and consumption. The model has a medium-term perspective by modeling equilibrium sorting of households on a metropolitan road network. The model includes both traffic congestion and colocational externalities among ïňĄrms. Taxes can be applied to both residential and employer locations such that private decisions of households and ïňĄrms produce an optimal pattern of location. Transport costs for commuting are capitalized in housing values (Glazer and Van Dender, 2002) and can lead to wage differentials depending on zone of employment (Darren and Wheaton, 2001, as estimated by).

We will show that this kind of models can be cast in the framework of a mixed complementarity problem and can be solved by off-the-shelve software. We will use the multi commodity formulation of the traffic assignment problem by Ferris et al. (1998) and extend their model by adding the general equilibrium model.

2 Model Description

2.1 Mixed Complementarity Problems

The model for Zurich consists of two sub models: a spatially disaggregate Alonso-Muth-Mills economic model of housing and labor markets and a model of individually-rational route choice on the (congested) traffic network(Wardrop, 1952). Both models are embedded in a single mixed complementarity problem (MCP).

MCPs can be used for expressing a variety of economic models for both markets and games. The word "mixed" in the expression MCP reflects the fact that there may be equations as well as inequalities. Computational evidence suggests that algorithms for solving MCPs are relatively reliable and efficient, particularly for models which are not natural optimization problems. The development of these modeling format was motivated by theoretical and practical developments in algorithms for nonlinear complementarity problems and variational inequalities. The most recent techniques are based on ideas from interior-point algorithms for linear programming (Kojima et al., 1991). A survey of developments in the theory and applications of these methods is provided by Harker and Pang (1990).

A MCP is specified by the lower bounds l, the upper bounds u and the function F (taken from

Ferris and Munson, 2010, p.7):

Given lower bounds $l \in \{\mathbb{R} \cup -\infty\}^n$, upper bounds $u \in \{\mathbb{R} \cup \infty\}^n$ and a function $F : \mathbb{R}^n \to \mathbb{R}^n$ find $z \in \mathbb{R}^n$ such that *precisely* one of the following holds for each $i \in \{1, \ldots, n\}$:

$$F_i(z) = 0 \quad \text{and} \quad l_i \leq z_i \leq u_i$$

$$F_i(z) > 0 \quad \text{and} \quad z_i = l_i$$

$$F_i(z) < 0 \quad \text{and} \quad z_i = u_i.$$

A number of special cases can be formulated as a MCP, including a (non)linear system of equations, a nonlinear (complementarity) problem, and a finite-dimensional system of variational equations.

Often the complementarity problem is just given by the optimality conditions of the original problem. However, and this is the advantage of the MCP formulation, there is no optimization problem corresponding to the complementarity conditions. Examples are the famous transport problem by Dantzig, the Walras equilibrium and the von-Thunen land model. A model formulation of these examples can be found in Ferris and Munson (2010).

Our models can be written as a nonlinear complementarity problem (Ferris and Munson, 2010, p.5): Given a function $F : \mathbb{R}^n \to \mathbb{R}^n$,find $z \in \mathbb{R}^n$ such that $0 \leq z \quad \perp \quad F(z) \geq 0$. The $\perp$ sign signifies that one of the inequalities is satisfied as an equality. This means that either $z_i = 0$ or $F_i(z_i) = 0$ for every i.

2.2 The Transport Sub model

2.2.1 The Transport Problem

The traffic model consists of a public and private transport network. A transport network can be defined as an abstraction of the transport infrastructure system with a number of constraints. It consist of nodes and links with certain characteristics.

The network constraints state that the flows on the arcs never can be negative and should be conservative in every node with the exception of the origin and destination nodes (Steenbrink, 1974, p.22).

Within the transport network one can distinguish between the transport supply and demand side.

The transport problem can now be defined as finding the optimal way to assign the demand to the network. In the transport problem we calculate the fastest times to travel from one node to another for all users of the network either by minimizing the travel time of the individual user (user equilibrium) or the average travel time for all users of the network (social optimum).

The transport problem can be formulated as a multi-commodity flow problem (Steenbrink, 1974, p.22).

The MCP formulation of our network consists of only six complementarity equation groups (Ferris et al., 1998).

In the following i, j, k are indices which will be used to describe the nodes in the network. The indices are also used to describe arcs. Each node has associated with it household groups h (for example high and low skilled). The two different transportation modes are public(with index pb) and private(with index pr).

The first group of equations defines the aggregate flow $F_{i,j}^m$ for mode m on the arc between node i and node j as the sum of the traffic of all households going from i to j. $X_{h,i,j,k}^m$ is the flow of people from household h using arc

$$F_{i,j}^m = \sum_{h,k} X_{h,i,j,k}^m \quad \perp \quad F_{i,j}^m \tag{1}$$

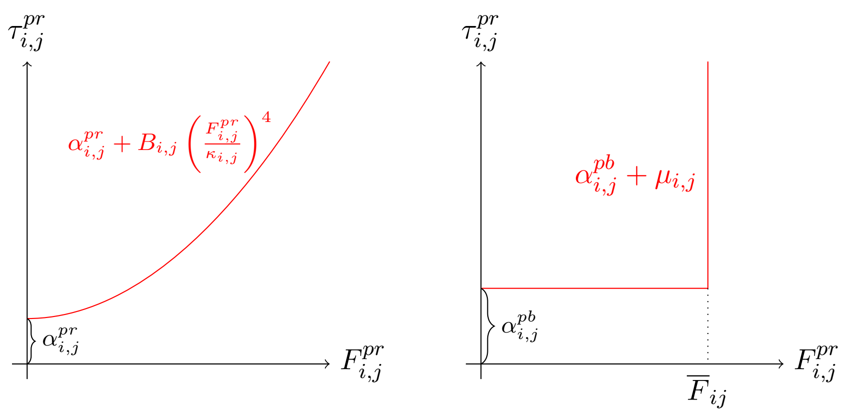
where $F_{i,j}^m \geq 0$ and $X_{h,i,j,k}^m \geq 0$.

The next group of equations define the travel time or costs on an arc. The effect of road capacity on travel times is specified by means of volume-delay functions expressing the travel time (or cost) on a link as a function of the traffic volume. The most popular volume-delay function is the one from the Bureau of Public Roads (1964) and defines the travel time on an arc $\tau_{i,j}^{pr}$ by:

$$\tau_{i,j}^{pr} = \alpha_{i,j}^{pr} + B_{i,j} \left(\frac{F_{i,j}^{pr}}{\kappa_{i,j}} \right)^4 \quad \perp \quad \tau_{i,j}^{pr}, \tag{2}$$

where α is the free-flow time on the arc, the congestion scale factor $(B_{i,j})$ and the reference capacity $(\kappa_{i,j})$. The time costs will increase as the arc gets more congested.[4]

Figure 1 – Volume-delay function private transport (left) and capacity constraint pubic transport (right)



In the case of public transport we assume that the there is a capacity constraint and no congestion. The time for traveling on an arc is given by the third group of equations and is defined as the free

[4]We do not assume that traffic on other links will influence the travel time on a specific arc.

flow time on the arc plus, in case of reaching the capacity constraint, a shadow price $\mu_{i,j}$:

$$\tau_{i,j}^{pb} = \alpha_{i,j}^{pb} + \mu_{i,j} \quad \perp \quad \tau_{i,j}^{pb} \tag{3}$$

The capacity constraint on the public transport arc is given by the next group of equations:

$$\overline{F}_{i,j} \geq F_{i,j}^{pb} \quad \perp \quad \mu_{i,j}. \tag{4}$$

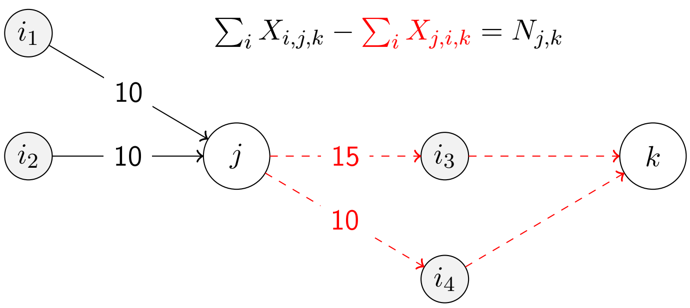
If the capacity is reached, the shadow price of the constraint μ will go to infinity and people will choose another link or mode for traveling.

The fifth group of equations define the flow conservation at node j for the number of people traveling from this node to destination k:

$$\sum_i X_{h,i,j,k}^m - \sum_i X_{h,j,i,k}^m = N_{h,j,k}^m \quad \perp \quad T_{h,j,k}^m \tag{5}$$

where $N_{h,j,k}^m$ is the total flow of people traveling with transport mode m from node j to node k. This number is taken from the origin-destination matrix and equal to 25 in figure 3. This number should be equal to the sum over all passengers traveling with destination k from incoming arcs (in the figure 10 passengers coming from i_1 and i_2; the black arrows) minus the passengers with destination k on the outgoing arcs (these are the dotted arrows pointing to destination node k). Notice that in the example the number of people living at node j and traveling to node k is 5. It is irrelevant how many nodes the traveler passes through when he departs for k from j. The associated complementarity variable is the minimum time from node j to node k.

Figure 2 – Flow balance for node j

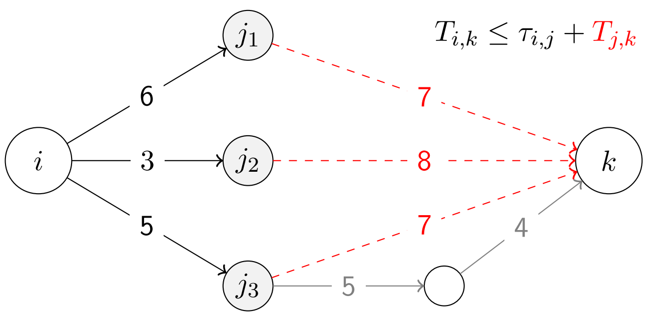


The last equation group reflects the second Wardropian principle. In its original form it states that "the journey times on all the routes actually used are equal, and less than those which would be experienced by a single vehicle on any unused route" (Wardrop, 1952, p. 345). On the left hand side we have the minimal travel time T for household h traveling with mode m from node i to k. This travel time should be less than or equal to the travel time τ on an arc starting from node i to any of the adjacent nodes j, plus the minimal time T from traveling from the adjacent node to the destination node k. In figure 3 we are looking at the minimal time for traveling from node i to node

k. The time for traveling to the adjacent nodes j_1 to j_3 are 6, 3 and 5 minutes. From every adjacent node j the minimal times for traveling to node k are 7, 8 and 7 minutes. This information is coming from the Wardropian equations for these nodes and calculated simultaneously. For our traveler the fastest route is equal to 11 minutes. He travels from node i to node j_2 and then to k.

$$T^m_{h,i,k} \leq \tau^m_{i,j} + T^m_{h,j,k} \quad \text{and} \quad T_{h,k,k} = 0 \quad \perp \quad X^m_{h,i,j}. \tag{6}$$

Figure 3 – The second Wardropian principle for node j



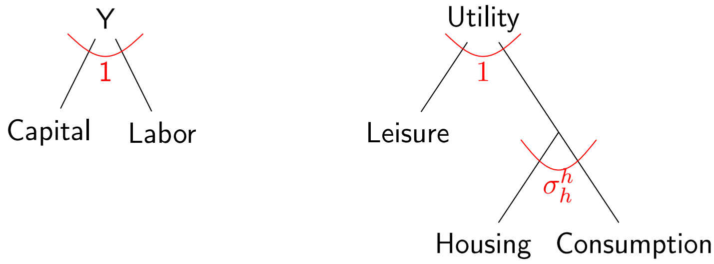
Note that a complete enumeration of all possible routes from node i to node k is not necessary. The information on the fastest routes from the adjacent nodes j to the destination k is given in the corresponding minimum time equations for traveling from j to k. The time minimization equations are associated with the flow on the adjacent arcs $X^m_{h,i,j}$ as complementary variables. This variable is only positive for those adjacent arcs where the traveling time from i to k is minimal. If this is not the case, the flow on that arc will be zero.

2.3 The Economic Sub model

The economic model is formulated as an Arrow-Debreu model with households and firms who maximize their utility and profits respectively. Many of the assumptions on the structure of the model and the share parameters can be easily replaced by more realistic assumptions.

Households are characterized by the place they live (i), the place they work (j), their skill (h) and the transport mode (m) they choose for traveling to work. They maximize their utility level with respect to their income. The utility is given by a nested Constant-Elasticity-of-Substitution function (see equations (7) and figure 4). At the lowest level the household decides on how much it wants to consume ($C_{h,i}$) and on the size of the house it wants to rent ($H_{h,i}$). The substitution elasticity between these two goods is σ^h_h and the value share of house rent is given by θ^h. At the next level it decides between demand for leisure time $L_{h,m,i,j}$ and the aggregate of consumption and housing with substitution elasticity 1 and the share leisure of total consumption (θ^{ls}).

Figure 4 – Production and utility function



$$U_{h,m,i,j} = LS_{h,m,i,j}^{\theta_h^{ls}} \left(\left[\theta^h H_{h,i}^{1-\sigma_h^h} + (1-\theta^h) C_{h,i}^{1-\sigma_h^h} \right]^{\frac{1}{1-\sigma_h^h}} \right)^{1-\theta_h^{ls}} \tag{7}$$

The income of a household working in j with skill h is given by the wage income at j plus the capital income out of the capital ($\overline{K}$) and housing stock ($\overline{H}$). We assume that every household holds the same share of capital with local share share θ^d:

$$INC_{h,j} = PL_{h,j} + \frac{\sum_i (\overline{K}_{h,i} + \overline{H}_{h,i})}{\sum_m n_{h,m}^{tot}} \left[\theta_h^d PK + (1-\theta_h^d) \right] \quad \perp \quad INC_{h,j} \tag{8}$$

$$PC_{h,i} = \left(hvs_h PH_{h,i}^{1-\sigma_h^h} + (1-hvs_h) PC^{1-\sigma_h^h} \right)^{\frac{1}{1-\sigma_h^h}} \quad \perp \quad PC_{h,i} \tag{9}$$

On the production side we have at every node identical firms who use either high- or low-skilled labor and capital as inputs to produce a single output Y (see the left part of figure 4). We assume for simplicity a Cobb-Douglas production function.[5]

Zero profit for production at node j can be formulated as follows:

$$\left(\frac{PL_{s,j}}{\overline{PL}_s} \right)^{1-\theta_s^k} \left(\frac{RK_{s,j}}{\overline{RK}_{s,k}} \right)^{\theta_s^k} > PY_s \quad \perp \quad Y_{s,j} \tag{10}$$

where $PL_{s,j}$ is the wage for labor of skill s and RK_j the rental price at node j. $theta_k$ is the value share of capital. PY_s and $Y_{s,j}$ are the price and the output level of the production sector:

Labor demand at node j is given by:

$$\sum_{m,i} N_{l,m,i,j} = (1-\theta_l^k) \frac{PY_h Y_{l,j}}{PL_{l,j}} \quad \perp \quad PL_{s,j}, \tag{11}$$

where $N_{l,m,i,j}$ is the labor supply from a household with skill s living at node i and working in j who uses m as transport mode.

Capital demand at node j is given by the following equation:

$$\overline{K}_{s,j} = \theta_s^k \frac{PY_s Y_{s,j}}{RK_{s,j}} \quad \perp \quad RK_{s,j} \tag{12}$$

[5] The number of different firms at a node can be easily enlarged and the production technology replaced by another functional form and a more complex nesting structure.

The housing market clearing node i is given by:

$$\overline{H}_{h,i} = \overline{H}^x_{h,i} + \theta_h \left(\sum_{m,j} N_{h,m,i,j} - \overline{N}^x_{h,m,i,j} \right) INC_{h,i} \frac{PC_{h,i}^{\sigma_h^h - 1}}{PH_{h,i}^{\sigma_h^h}} \quad \perp \quad PH_{h,i}, \tag{13}$$

where $\overline{H}_{h,i}$ is the given housing stock and $PH_{h,i}$ is the rent paid by household h at node i. We assume that a certain number of people at i ($\overline{N}^x$ will not move and rent $\overline{H}^x_{h,i}$.

As the capital and housing stock is fixed the capital price index can be written as follows:

$$PK = \frac{\sum_{h,i} PH_{h,i}\overline{H}_{h,i} + \sum_{s,j} RK_{s,j}\overline{K}_{s,j}}{\sum_{h,i} \overline{H}_{h,i} + \sum_{s,j} \overline{K}_{s,j}} \quad \perp \quad PK \tag{14}$$

Leisure supply is calibrated to unity when commute time equals $\overline{t}$:

$$LS_{h,m,i,j} = \frac{t^{max}_{h,m} - T_{h,m,i,j}}{t^{max}_{h,m} - \overline{t}_{h,m}} PC_{h,i} \quad \perp \quad LS_{h,m,i,j} \tag{15}$$

The share of people using a specific transport mode $\theta_{h,m}$ is given by a logit formulation. Note that $\theta_{h,m}$ depends on the the common utility level $U_{h,m}$ of the household group with skill m and using transport mode m and therefore not only on the time costs for traveling.

$$\theta_{h,m} = \frac{\overline{\theta}_{h,m} e^{\lambda_h UH_{h,m}}}{\sum_m \overline{\theta}_{h,m} e^{\lambda_h UH_{h,m}}} \quad \perp \quad \theta_{h,m} \tag{16}$$

The sorting of the households is done by forcing the ratio of the individual household level $U_{h,m,i,j}$ to the benchmark utility level be equal in the equilibrium to common utility level:

$$UH_{h,m} = \frac{U_{h,m,i,j}}{\overline{U}_{h,m,i,j}} \quad \perp \quad U_{h,m,i,j} \tag{17}$$

If one individual household at, for example node r working at node r would attain a higher utility level than the common utility level, households would like to move to live at node k and work at node r using the same transport mode as the household with the higher utility level. This would drive the wages down, and the rental prices and travel costs up. Finally, the common index level and the mode share would adjust in such a way that the individual household utility ratio levels would be equal again.

The number of households with skill h that use transport mode m is given by:

$$\theta_{h,m}\overline{N}^{tot}_h = \sum_{i,j} \left(N_{h,m,i,j} - \overline{N}^x_{h,m,i,j} \right) \quad \perp \quad UH_{h,m}. \tag{18}$$

We assume that a certain number of households ($\overline{N}^x_{h,m,i,j}$) will not change location and work place.

3 Calibration

Ideally we would like to calibrate the model to the observed flows and travel times. As we do not have (yet) this information, we let the transport model sove for this information and use it to calibrate the model.

In the first step we solve for the least-cost flow. We begin with a origin-destination matrix describing where people live and work (assuming that all traffic flows are commuters)[6] We minimize the total travel cost:

$$OBJ = \sum_{h,m,i,j,k} \tau_{i,j} X_{h,i,j,k} \tag{19}$$

with the equations (2), (3), (5) and (4).

This gives us values for the variables $\tau_{i,j}^m$ and $\mu_{i,j}$. These are fixed in the next step. We then compute the flow times. For this we set up a dual LP to determine least cost travel times in the optimal routing assignment:

$$OBJ = \sum_{h,m,i,j} T_{h,m,i,j} \tag{20}$$

subject to equation (6).

In the third step we solve the model with equilibrium traffic flows (Nash equilibrium). This model is stated as a MCP consisting of the equations (2), (3), (5), (4) and (6).

Calibrating the reference wage rate to unity, the value of labor demand at node i equals the number of workers:

$$\overline{LD}_{h,i} = \sum_{m,j} N_{h,m,j,i} \tag{21}$$

where $N_{h,m,j,i}$ is the solution of the Nash Equilibrium.

We calibrate the rental rate of capital to unity as well. The aggregate value of output is based on employment, and the capital value share then defines capital supply in efficiency units:

$$\overline{K}_{s,i} = \frac{\theta_s^k \overline{PL}_s \overline{LD}_{s,i}}{1 - \theta_s^k} \tag{22}$$

where θ_s^k is the capital value share.

The housing stock at node i equals the value of market expenditure times the housing value share. The value of market expenditure net housing equals the value of labor and capital earnings. We can infer this income based on labor supply divided by the labor share of output.

[6] We only had information on where people live and work in Zurich. Therefore, we use a gravity model to find the origin-destination matrix.

The imputed value of labor ($\overline{L}_{h,i}$), capital endowment ($\overline{K}_{h,i}$) and the household stock $\overline{H}_{h,i}$ for every household at node i with skill h is given by:

$$\begin{aligned}\overline{L}_{h,i} &= \frac{\sum_{m,j} \overline{PL}_h N_{h,m,i,j}}{1-\theta_h^k} \\ \overline{K}_{h,i} &= \delta_s \overline{L}_{h,i} \\ \overline{H}_{h,i} &= \frac{\theta^h \overline{L}_{h,i}}{1-\theta_h}\end{aligned}$$

The total number of households with skill h using transport mode m is given by:

$$\overline{N}_{h,m} = \sum_{i,j} N_{h,m,i,j} \tag{23}$$

We assume that the maximum possible commute is twice the longest commute observed in the benchmark traffic flows:

$$t_{h,m}^{max} = max_{h,m,i,j} T_{h,m,i,j} \tag{24}$$

The commuter-weighted average commute time is used to anchor the utility function:

$$\bar{t}_{h,m}^{max} = \sum_{i,j} N_{h,m,i,j} Th, m, i, j \overline{N}_{h,m} \tag{25}$$

4 Data

We used data at the quarter level for Zurich, the biggest city in Switzerland. Quarter specific information can be found in table 1 in the Appendix A.

Zurich has a population of about 370'000 people and almost 350'000 people work in Zurich (Statistik Stadt Zürich, 2007). Zurich has 34 quarters with an average are of 2.7 square kilometers and an average population density of 4'000 persons/km^2.

We have data on travel times for public and private transport from the Cantonal Transport Model for the 302 travel assignment zones. We used this information to calculate the a weighted travel time from the centroids of the quarters to the other quarters. There are 138 arcs (64 bidirectional) and two transport modes.

For the public transport mode we had to assume values for the volume-delay functions (see Appendix).

5 Preliminary Scenarios and Results

The model described above will be extended for the final version of the paper. We will add the possibility to take a look at zoning policies in the city and will improve the data base of the model. In the final version of the paper there will be a more realsitic set of scenarios and in this version

Figure 5 – The 34 quarters (left) and the 64 arcs between the quarters of Zurich (right)

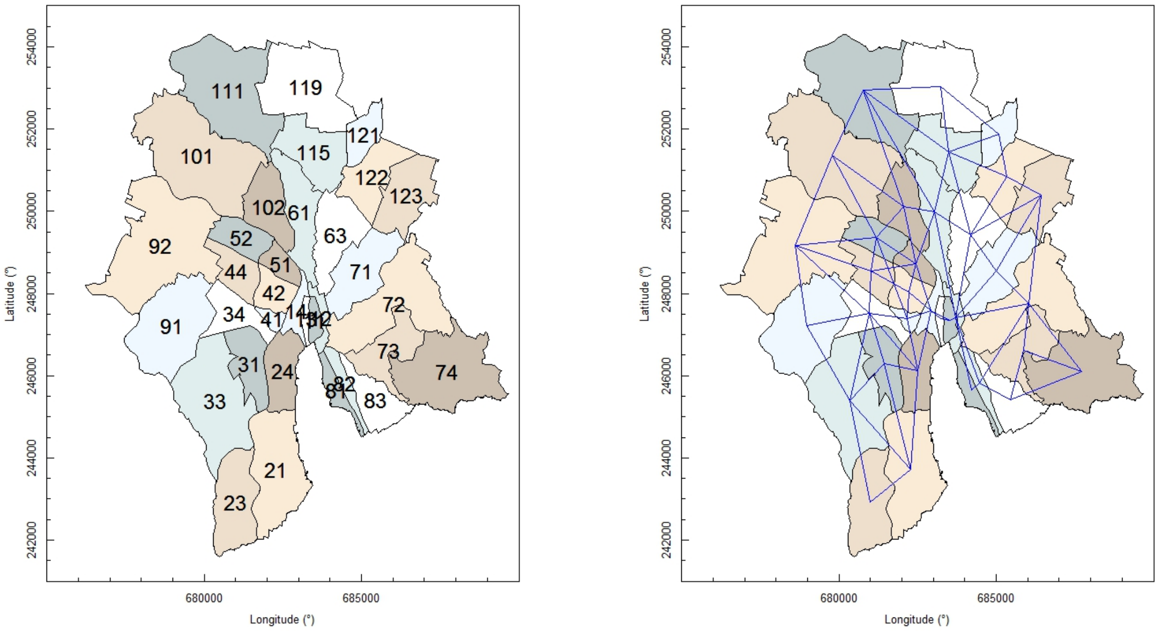
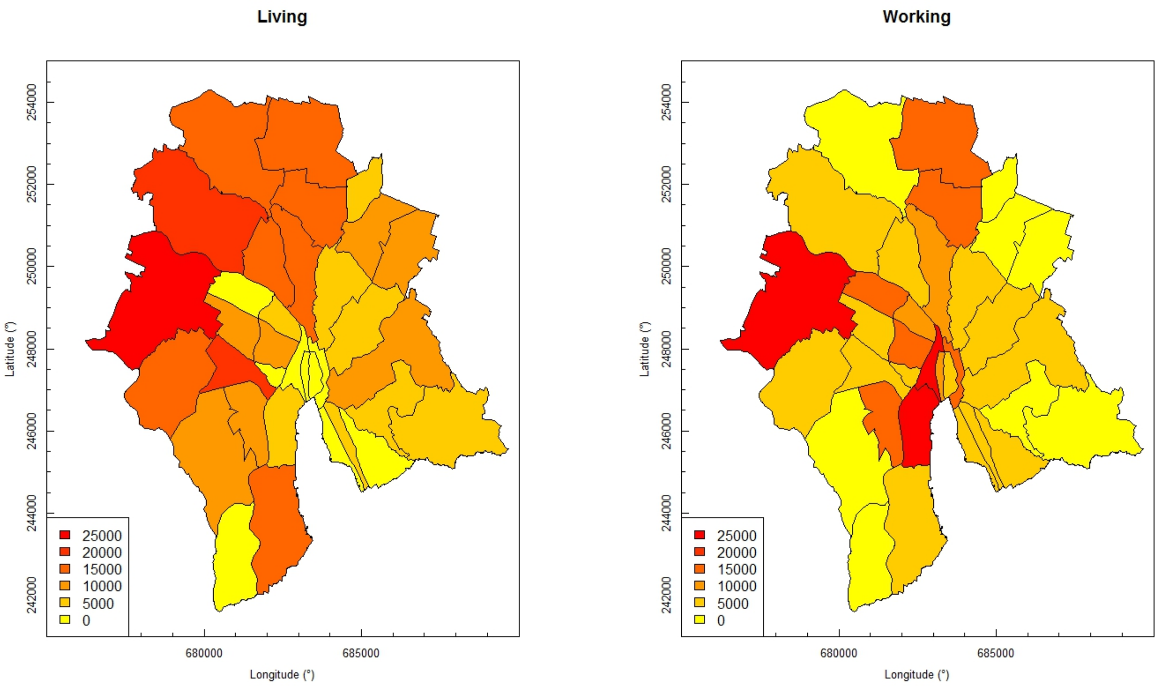


Figure 6 – Distribution of population and employment in Zurich

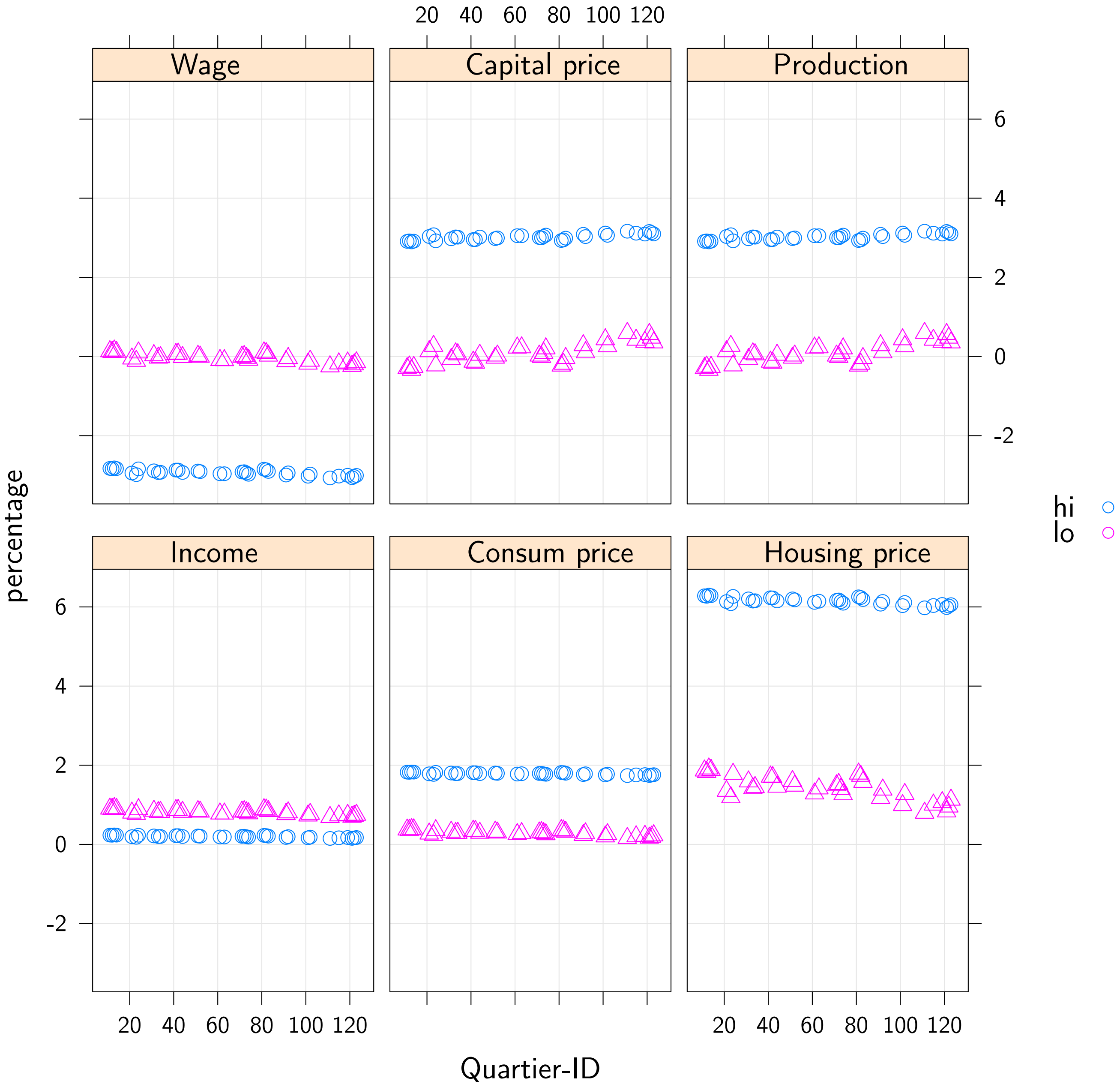


of the paper we will only look at a simple test scenario, where we assume that there is an inflow of high-skilled people of 2% and a part of the people who originally did not have the possibility to change locations for living and working can now change locations. This scenario is only used to show the possiblities of the model sofar.

The following figure shows the results for the main economic indicators. The first row with diagrams show the impact on value added prices and output level. An inflow of high-skilled people will have an effect on the wage of both skill groups in all the quarters of the city (see first diagram on the left). The wage for the high-skilled households falls by almost 3% due to the increased supply of labor. The effect on the wage of the low skilled groups is very small. Note however that for some households it is slightly positive and for others negative. The capital price in the high skilled production sectors increases (diagram in the center of the first row) because it is relatively scarce compared to the input of high skilled labor. The output of the high-skilled sectors increases (diagram on the right). The second row shows the effects on income, price of consumption and household rents. The rent for the high-skilled households goes up by 6%. Note that there is also an increase in the rent for the low-skilled groups.

Impact on traffic (congestion, travel time) will be presented at the conference.

Figure 7 – Effects on economic variables (percentage change)



6 Preliminary Conclusions and further research

The first part of this paper shows that it is possible to cast a general equilibrium model combined with a transport model in a complementarity framework that can be solved very efficiently with off-the-shel solvers. Although we have not tested this formulation with cities with more nodes and modes, we hope that this formulation will also be very efficient for bigger problems. First results with a dummy scenario are promising but have shown that there is still much to do. For this paper we will continue working on the data side by adding more realistic data on households, zoning and transport. On the model side the transport formulation will have to be calibrated to real-world values.

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A Tables

Table 1 – Quarters of Zurich: Number, Name, Population, Employment and Area (Statistik Stadt Zürich, 2007)

ID	Quarter	Population	Empl.Industry	Empl. Services	Area (km^2)
11	Rathaus	3'097	308	7'500	0.38
12	Hochschulen	711	286	14'731	0.56
13	Lindenhof	955	341	10'913	0.23
14	City	845	199	25'831	0.64
21	Wollishofen	15'587	878	4'595	5.75
23	Leimbach	4'944	85	392	2.92
24	Enge	8'367	1'026	25'523	2.4
31	Alt-Wiedikon	15'231	2'347	15'428	1.85
33	Friesenberg	10'342	66	3'770	5.15
34	Sihlfeld	20'314	1'031	7'356	1.64
41	Werd	3'865	1'357	4'901	0.31
42	Langstrasse	10'332	1'037	16'885	1.13
44	Hard	12'508	1'018	3'975	1.46
51	Gewerbeschule	9'735	1'377	9'979	0.73
52	Escher Wyss	2'987	3'050	16'219	1.27
61	Unterstrass	19'959	826	9'497	2.46
63	Oberstrass	9'698	261	5'305	2.64
71	Fluntern	7'379	180	9'202	2.84
72	Hottingen	10'180	607	8'384	5.05
73	Hirslanden	6'904	307	2'472	2.2
74	Witikon	9'958	130	1'308	4.93
81	Seefeld	4'842	1'410	6'759	2.45
82	Mühlebach	5'549	497	6'630	0.63
83	Weinegg	4'816	240	6'748	1.72
91	Albisrieden	17'275	1'268	5'656	4.6
92	Altstetten	28'868	3'883	21'801	7.47
101	Höngg	21'017	548	5'964	6.98
102	Wipkingen	15'392	642	5'593	2.11
111	Affoltern	18'793	722	1'829	6.04
115	Oerlikon	20'318	3'491	11'533	2.67
119	Seebach	20'757	4'541	13'760	4.72
121	Saatlen	6'695	208	807	1.13
122	Schwamendingen-Mitte	10'637	636	1'851	2.23
123	Hirzenbach	11'205	217	897	2.62
	Total	370'062	35'020	293'994	91.91