Route Choice: Models and Challenges

Shlomo Bekhor

Technion – Israel Institute of Technology

Swiss Transport Research Conference

September 1st, 2010

Presentation Topics

- Introduction
- Choice Set Generation
- Model Formulations
- Case Studies
- Accounting for Congestion
- Research Directions

Two types of choice behavior

- **1. Pre–trip choice:** made before starting the trip
  - Continuous service systems (road and pedestrian networks) without unexpected events
- **2. En-route choice:** made during the trip, to adapt to random or unknown events
  - Road systems with unexpected events
  - En-route information
- Route choice models assume either pre-trip or mixed pre-trip/en-route choice behavior
  - Depending on the characteristics of the transportation service they are applied to.

Alternative modeling approaches

- Fuzzy logic (e.g., Lotan and Koutsopoulos, 1993; Lotan, 1997; Henn, 2000; Rilett and Park, 2001; Ridwan, 2004)
- Artificial neural networks (e.g., Yang et al., 1993; Dougherty, 1995; Yamamoto et al., 2002)
- Cognitive psychology (e.g., Nakayama and Kitamura, 2000; Nakayama et al., 2001)
- Random Utility – most common
- **This presentation** – route choice for a single mode (private car)

Route Choice Models: Two-stage Choice Process

1. Choice Set Generation

2. Route Choice Given a Choice Set

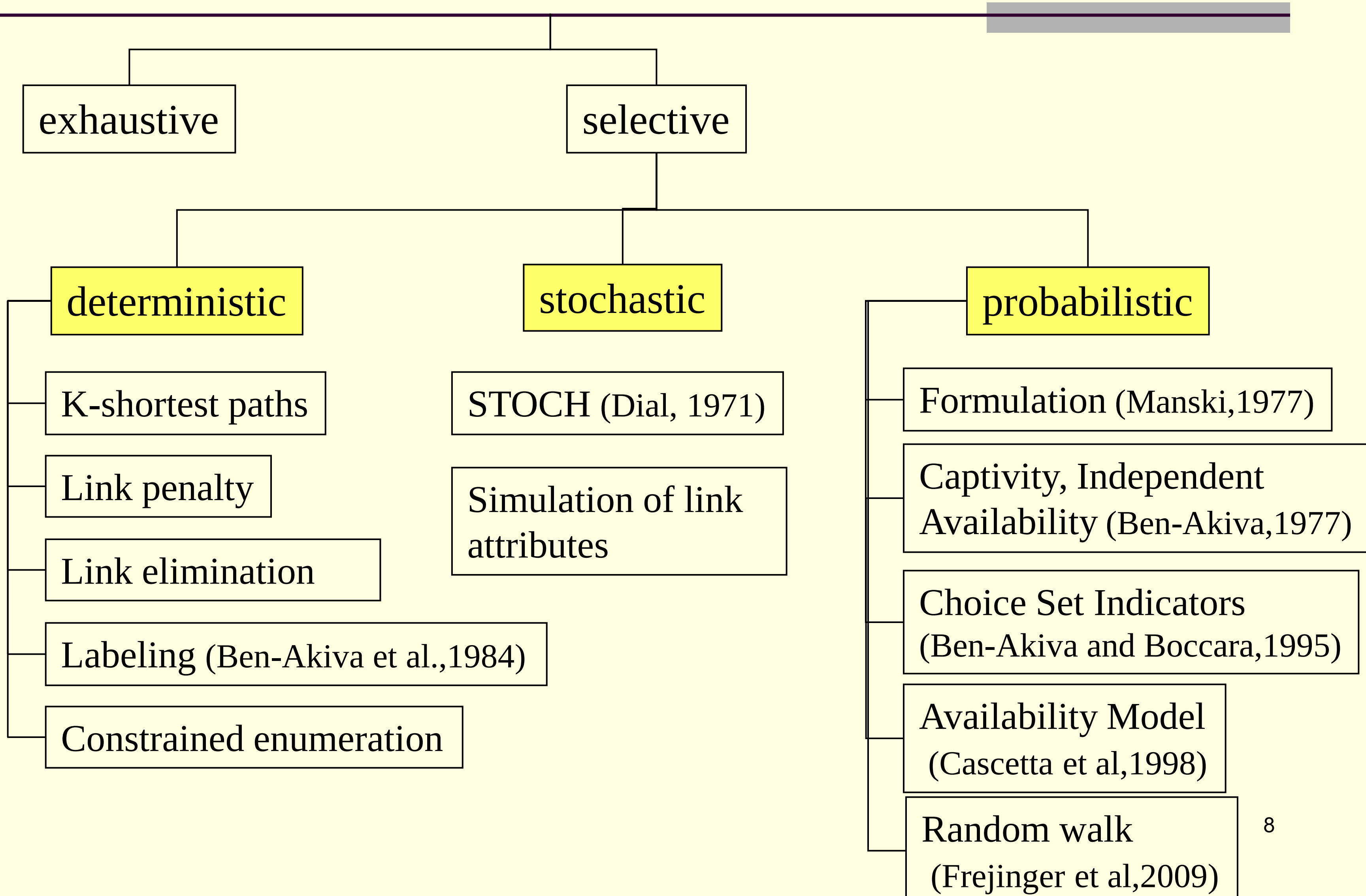
Route Attributes

- Travel Times: Time in Motion, Time at Stop Lights, Delay at Bottlenecks
- Cost: Out-of-Pocket, Long-Term
- Uncertainty or Variance of Travel Time
- Number of Stop Signs or Stop Lights on Route
- Volume of Conflicting Traffic or Pedestrian Movements
- Number of Turns on Route - Ease of Memory, Left Turns Against Traffic, Protected Lefts at Lights
- Street Width, Number of Lanes, Effort Required to Maneuver
- Circuitry of Route
- Safety, Roadway Condition

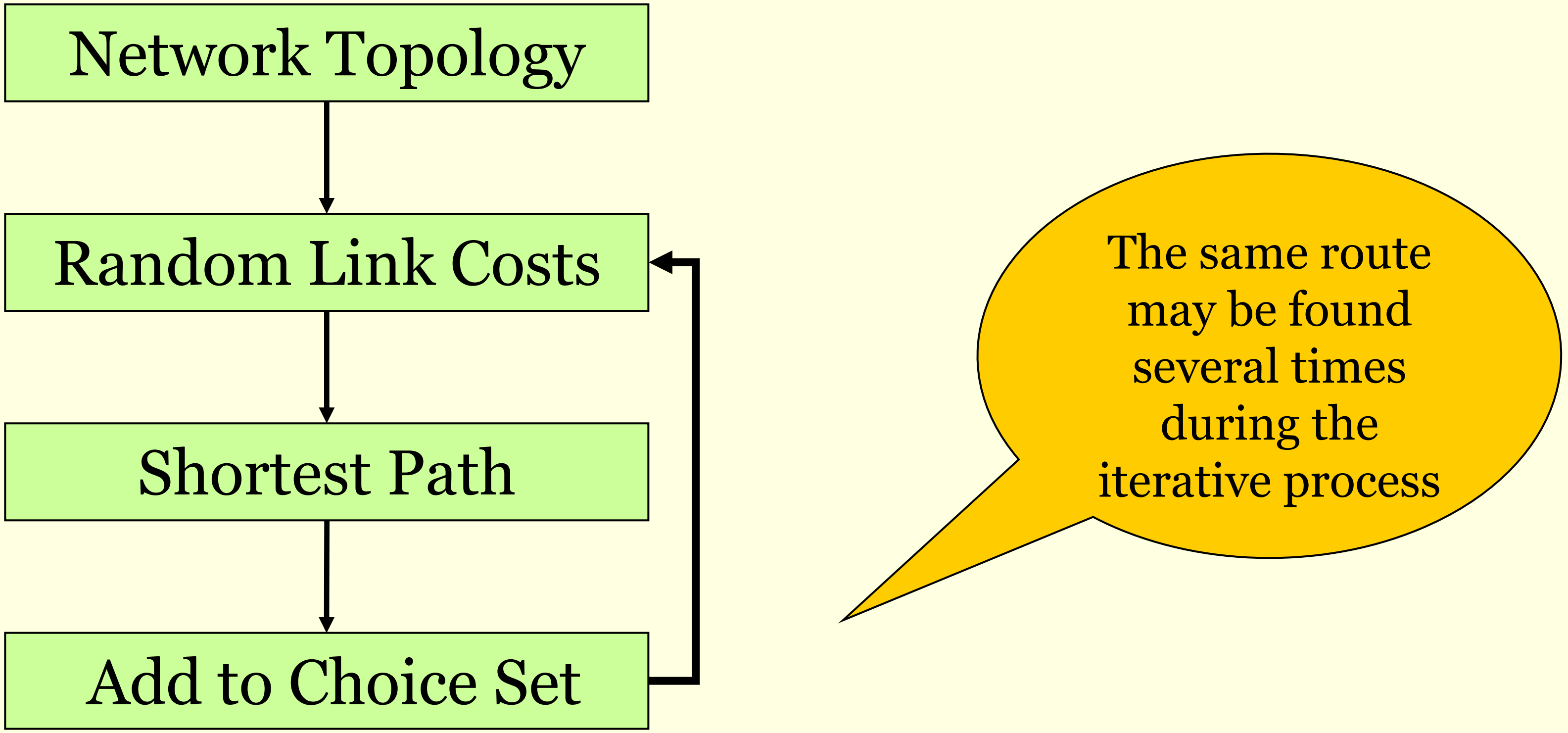
Path Utility Specification

- Link-Based Variables (such as travel time) versus Path-Based Variables (such as scenic route) - Affects Need for Enumeration
- Path-Based Variables (Including the “Path Size” or “Commonality Factor”) Require Enumeration

Choice Set Generation Models

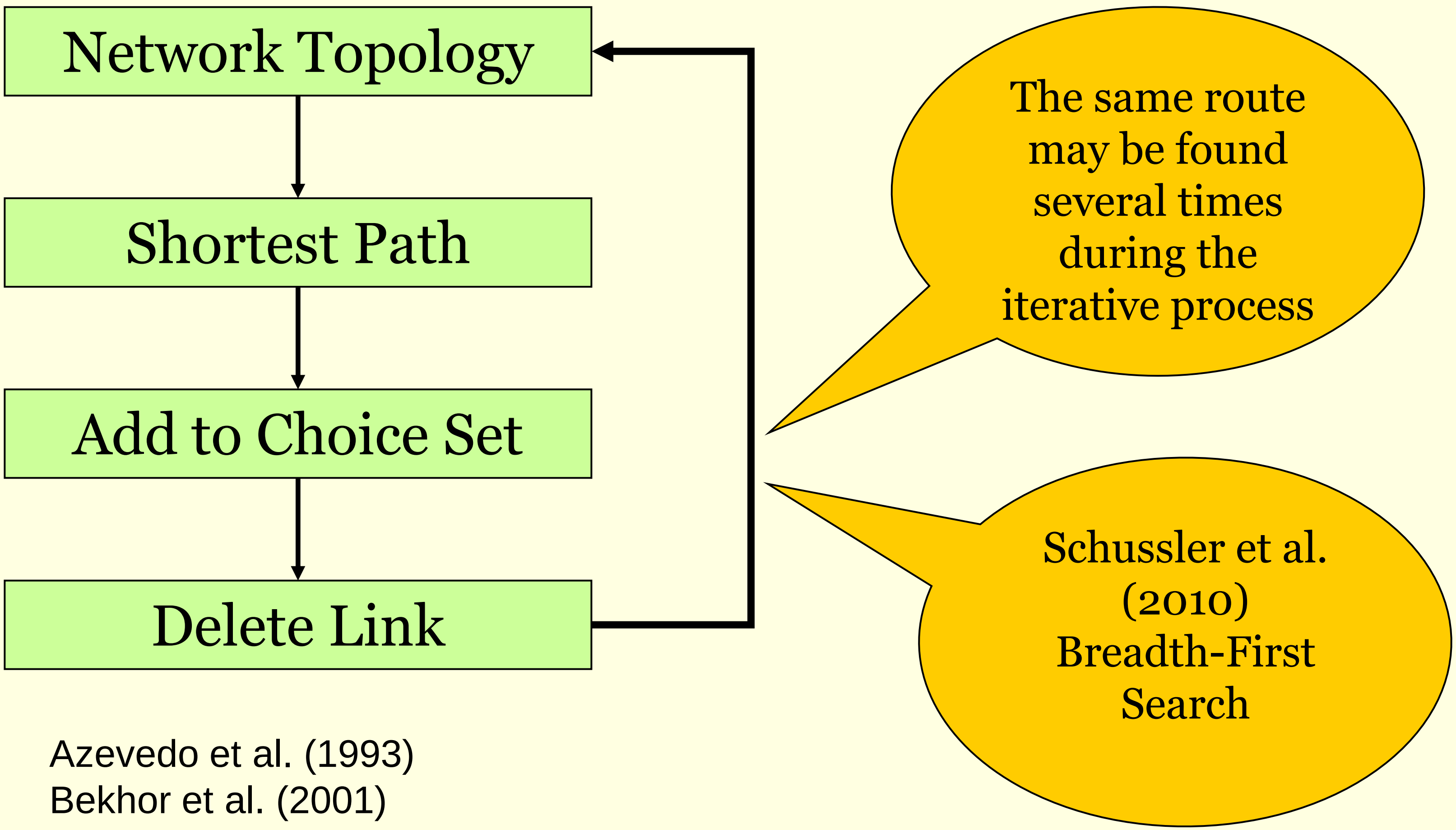


Simulation of Link Attributes



Nielsen (2000)
Bekhor et al. (2001)
Fiorenzo-Catalano and Van der Zijpp (2001)
Bierlaire and Frejinger (2005)
Bovy and Fiorenzo-Catalano (2006)

Link Elimination Method

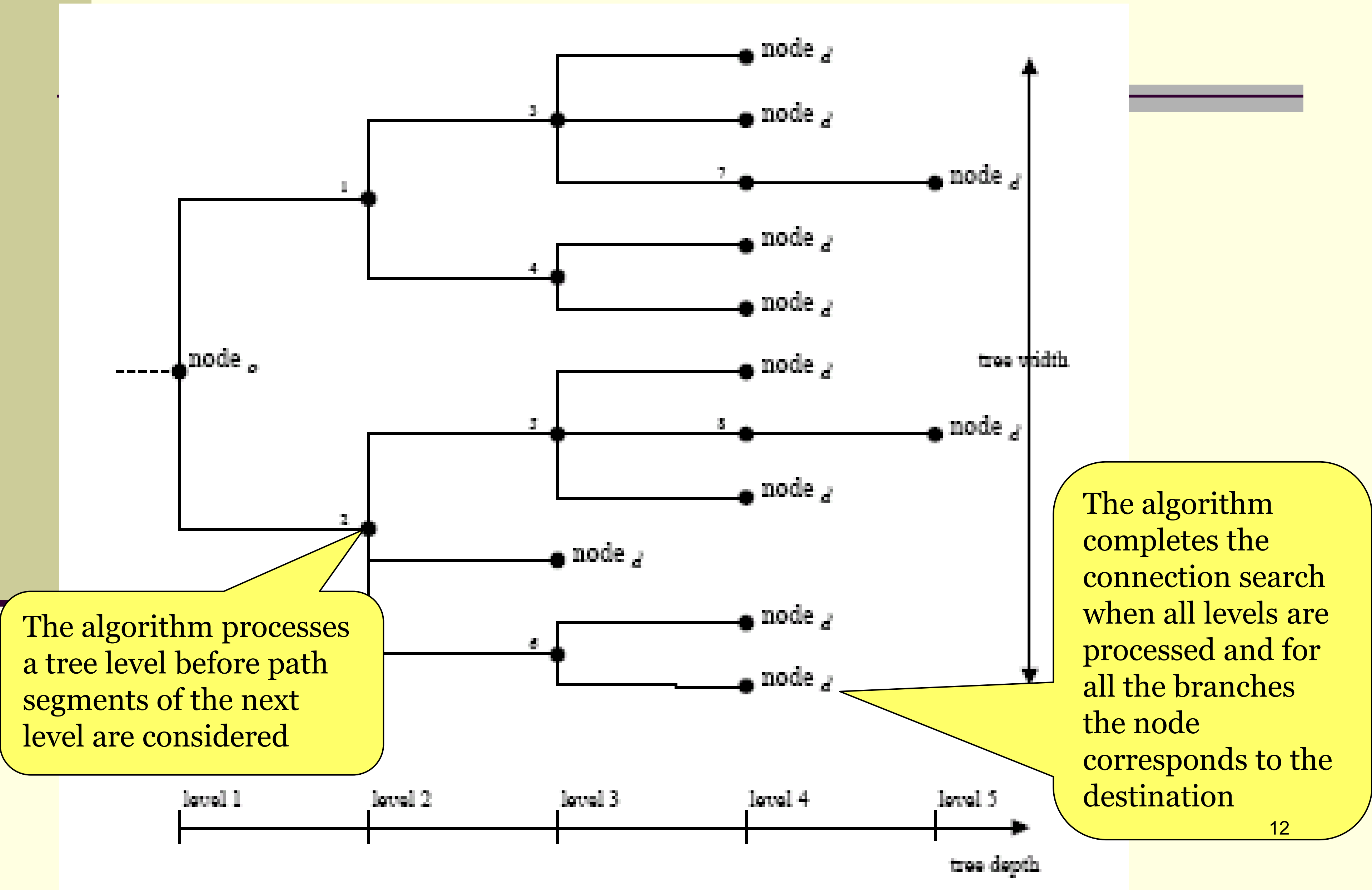


Azevedo et al. (1993)
Bekhor et al. (2001)
Prato and Bekhor (2006)
Frejinger and Bierlaire (2007)

Branch and Bound Method

- A link is inserted to the tree if and only if all the following conditions hold:
  - Directional constraint: (excludes from consideration links that take the driver significantly farther from the destination and closer to the origin)
  - Temporal constraint: (excludes paths with unrealistic travel times)
  - Loop constraint (remove paths with large detours)
  - Similarity constraint (remove high overlapping path segments)
  - Left turn constraint: (maximum number of left-turns per route)

Branch and Bound Tree



Evaluation of Path Generation Algorithms

- Coverage: generated route matches the observed route at a specified threshold (Bovy, 2007):

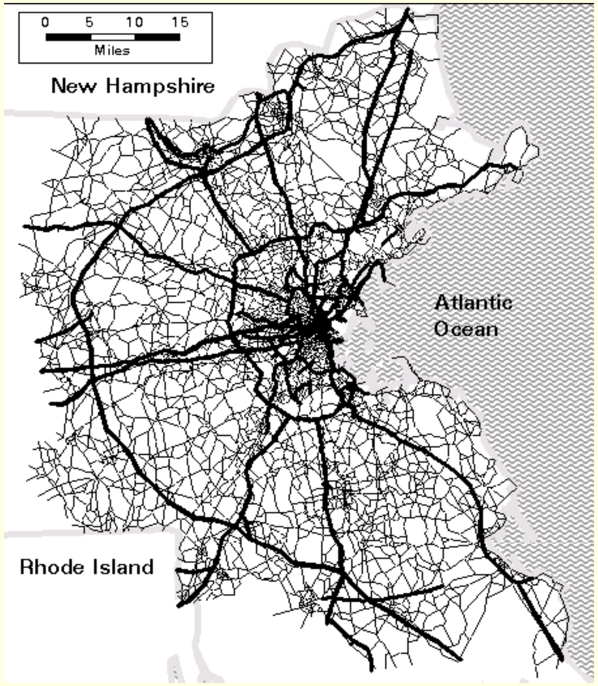
$$C_g = \frac{N_{cov}}{N_{obs}} = \frac{\sum_{n=1}^{N_{obs}} I\left(O_{ng} \geq \delta\right)}{N_{obs}} = \frac{\sum_{n=1}^{N_{obs}} I\left(\frac{L_{ng}}{L_n} \geq \delta\right)}{N_{obs}}$$

- Efficiency index: compares the path generation technique with an ideal algorithm that would replicate link-by-link all the observed routes and would produce for every origin-destination pair the chosen path and an alternative for modeling purposes Bekhor and Prato (2009):

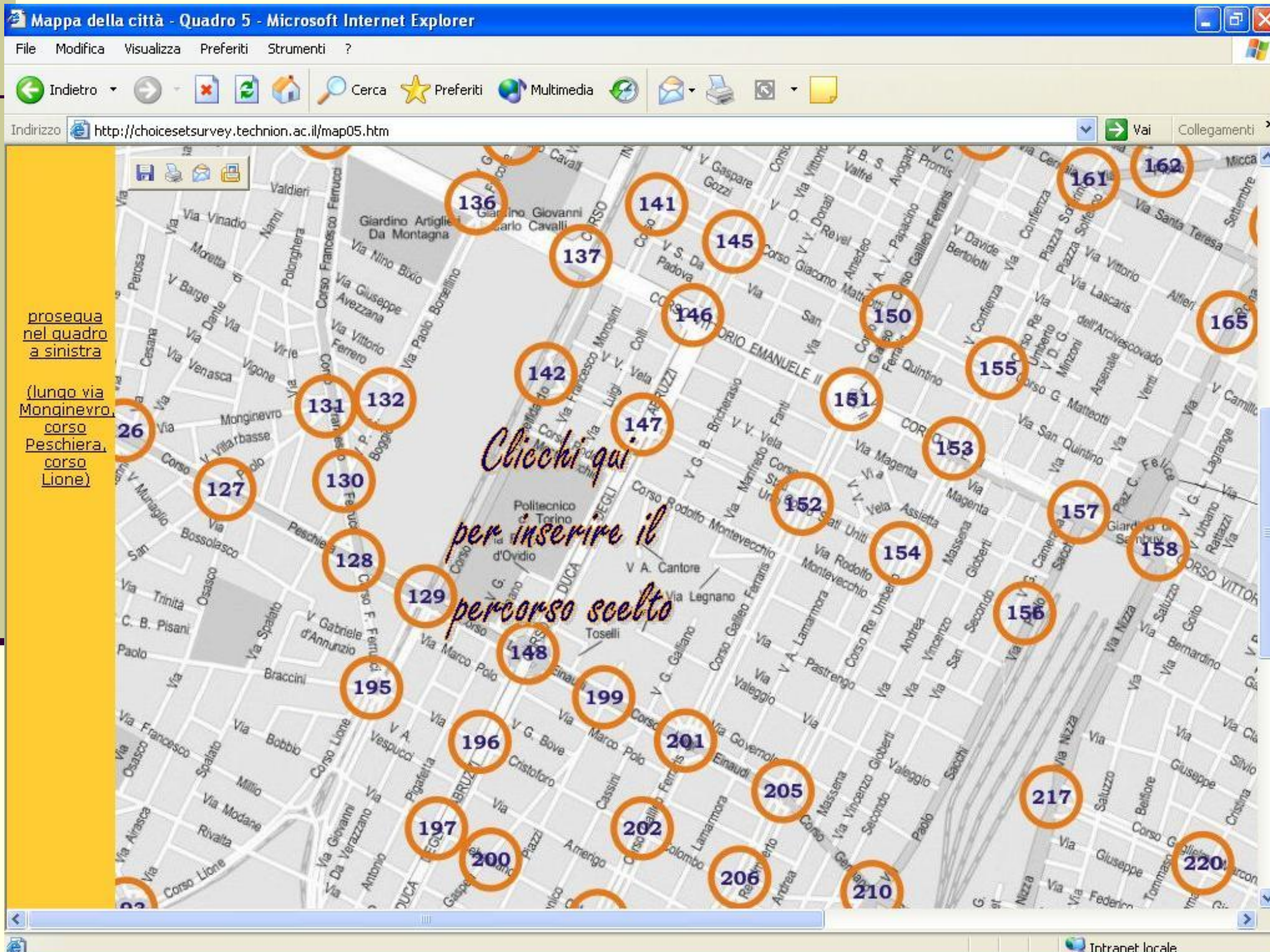
$$EI_G = \frac{1}{N_{obs}} \sum_{n=1}^{N_{obs}} \left\{ \left[I\left(\frac{L_{G,n}}{L_n} \geq \delta\right) + \left(1 - \frac{GR_{gen,n} - R_{rel,n}}{GR_{gen,n}}\right)\right] \Big/ 2 \right\}$$

Case Studies

- Boston (Ramming, 2001)
  - 900 Traffic Zones
  - 12,000 Nodes
  - 20,000 Links
  - 188 Observations
- Turin (Prato, 2005)
  - 182 O-D Pairs
  - 417 Nodes
  - 1,427 Links
  - 236 Observations



Screenshot of the Turin map with nodes



Path generation techniques

Technique	Boston	Turin
Labeling approach	f(h) = min(distance), min(free flow time), min(travel time)	
Link penalty	pf = 1.05, $I_h(L_{i,j})$ = travel time, T = 15 iterations	
Link elimination	remove one link from shortest path, T = 50 iterations	remove one link from shortest path, T = 10 iterations
Simulation	$f(I_h(L_{i,j}))$ are three normal distributions, $I_h(L_{i,j})$ = travel time, T = 16, 32, 48 draws	$f(I_h(L_{i,j}))$ are two truncated normal distributions, $I_h(L_{i,j})$ = travel time, T = 25, 35 draws
Branch and bound	$\Delta_D = 1.10$, $\Delta_T = 1.33$, $\Delta_L = 1.20$, $\Delta_O = 0.80$, $\Delta_{LT} = 7$	$\Delta_D = 1.10$, $\Delta_T = 1.50$, $\Delta_L = 1.20$, $\Delta_O = 0.80$, $\Delta_{LT} = 4$

Coverage for different overlap thresholds

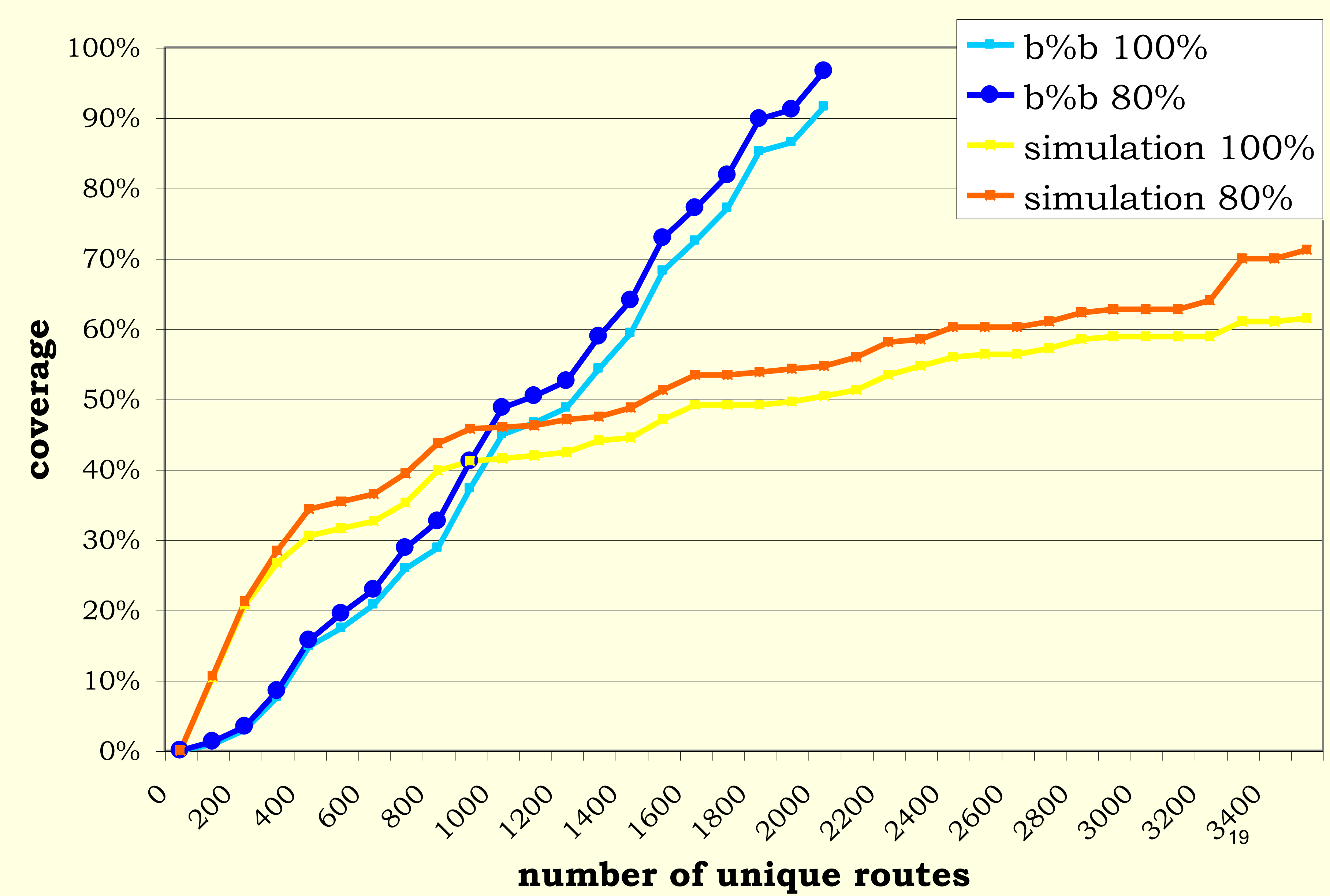
Generation technique	Boston		Turin	
	100%	80%	100%	80%
Labeling approach	39.4	51.6	36.0	39.0
Link elimination	60.1	71.3	58.5	69.9
Link penalty	53.7	73.9	53.8	62.3
Simulation (16 draws)*	43.6	70.7	49.2	54.2
Simulation (32 draws)*	48.9	76.1	61.4	71.2
Simulation (48 draws)	50.0	78.7	-	-
Branch and bound	75.5	96.3	91.1	96.6

* Respectively 25 and 35 draws for the Turin network

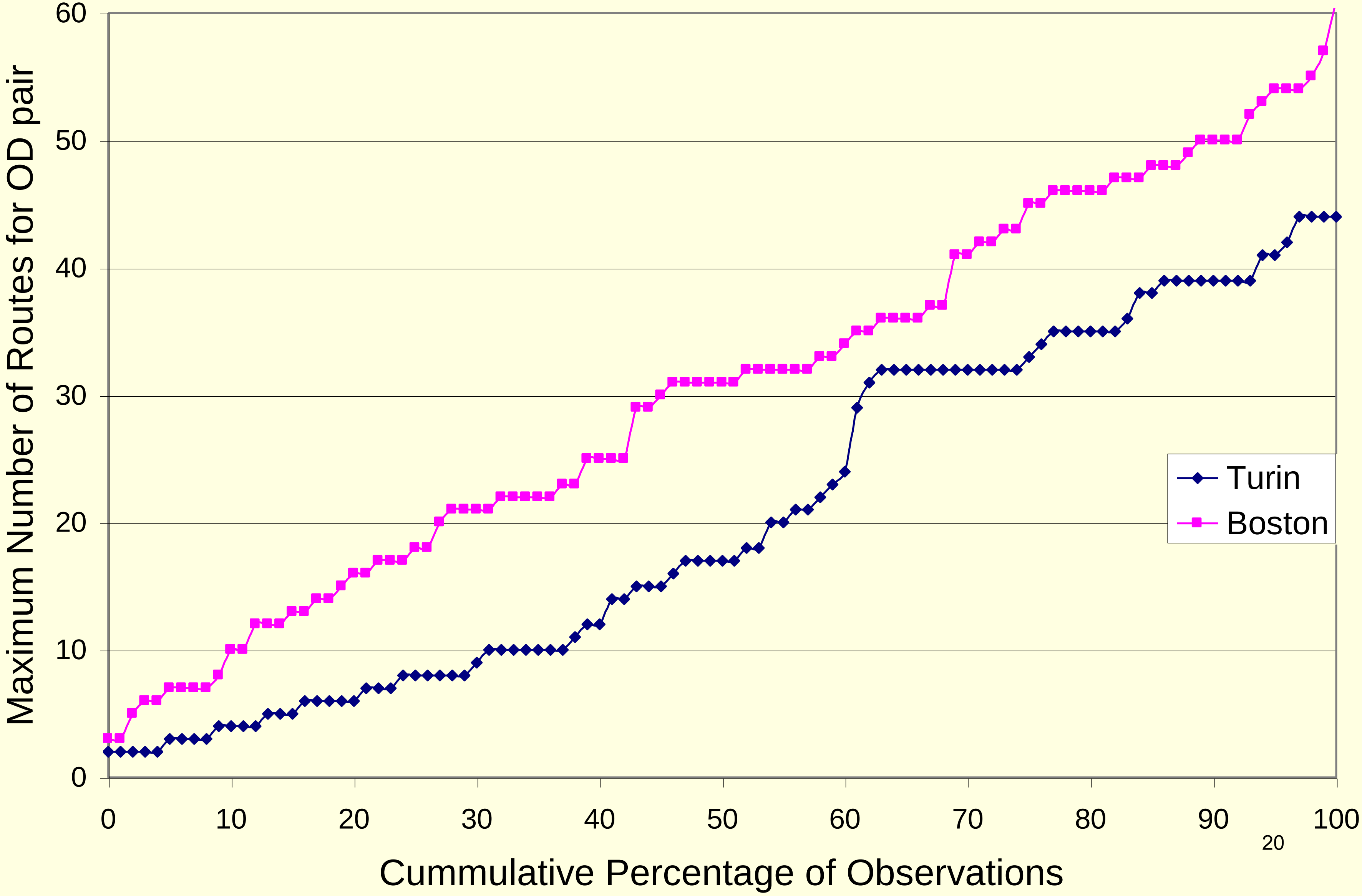
Efficiency Index for the Turin network

path generation technique	**efficiency index (%) for overlap threshold equal to:**		
	100%	**90%**	**80%**
Labeling approach	43.0	43.0	44.5
Link elimination	48.2	48.2	54.0
Link penalty	42.5	42.5	46.8
Simulation (25 draws)	41.2	41.2	43.7
Simulation (35 draws)	36.2	36.4	41.1
Branch and bound	54.5	54.7	57.2

Comparison of Unique Routes Generated



Characteristics of the Choice Set



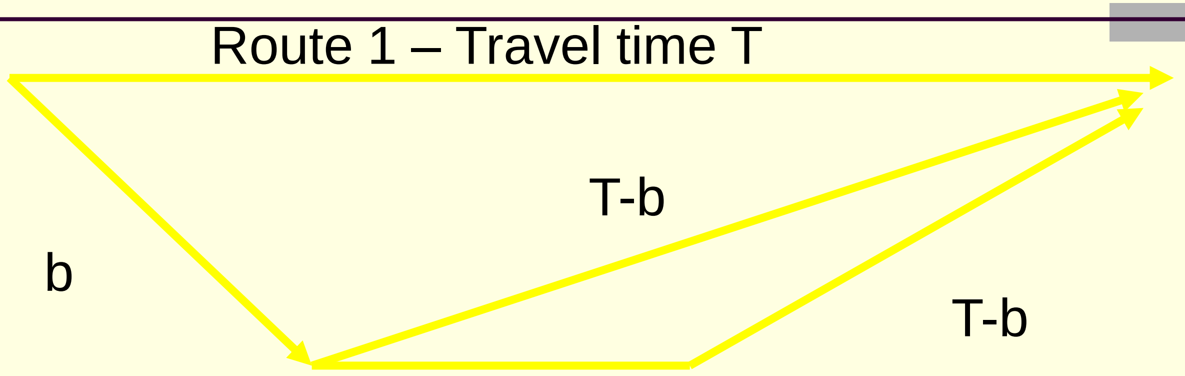
The Sampling Problem

- Stochastic route choice set generation procedures may sample routes with unequal selection probabilities.
- The selection probability of a route depends on the properties of the route itself, such as length or travel time.
- The systematic utility of the routes should be corrected for the unequal selection probabilities.
- Frejinger et al. (2009): Sampling correction using random walk as generation method

Route Choice Model Formulations

- Deterministic choice models
  - Shortest path (generalized cost)
  - Used in most transportation packages
- Probabilistic choice models
  - Multinomial Logit (MNL) – Dial's algorithm
  - Modified Logit (C-Logit, Path-Size Logit)
  - GEV Models (CNL, LNL, PCL)
  - Probit / Logit Kernel

The Overlapping Problem



- Introduces Correlation - Violates IIA - MNL is Unsuitable
- Traditional Example with Three Paths, Two Overlapping
  - As overlap approaches 100 percent (b→ T), expect close to 50/25/25 shares
  - As overlap approaches 0 percent (b→0), expect close to 33/33/33 shares
- MNL Predicts 33/33/33 Shares for Any Value of b

The C-Logit model (Cascetta et al., 1996)

$$P(i) = \frac{\exp \mu\left(V_i - CF_i\right)}{\sum_{j \in C} \exp \mu\left(V_j - CF_j\right)}$$

CF - Commonality Factor - Several possible specifications:

(i) $$CF_i = \ln \sum_{j \in C} \left(\frac{L_{ij}}{\sqrt{L_i L_j}} \right)$$

(ii) $$CF_i = \sum_{a \in \Gamma_i} \left(\frac{L_a}{L_i} \sum_{j \in C} \delta_{aj} \right)$$

Γ_i- Set of links included in route i

The Path-Size Logit model

$$P(i) = \frac{\exp \mu(V_i + \ln PS_i)}{\sum_{j \in C} \exp \mu(V_j + \ln PS_j)}$$

Ben-Akiva and Bierlaire (1998)

$$PS_i = \sum_{a \in \Gamma_i} \frac{L_a}{L_i} \frac{1}{\sum_{j \in C} \delta_{aj}}$$

Ramming (2001)

$$PS_i = \sum_{a \in \Gamma_i} \frac{L_a}{L_i} \frac{1}{\sum_{j \in C} \left(\frac{L_i}{L_j} \right)^{\gamma} \delta_{aj}}$$

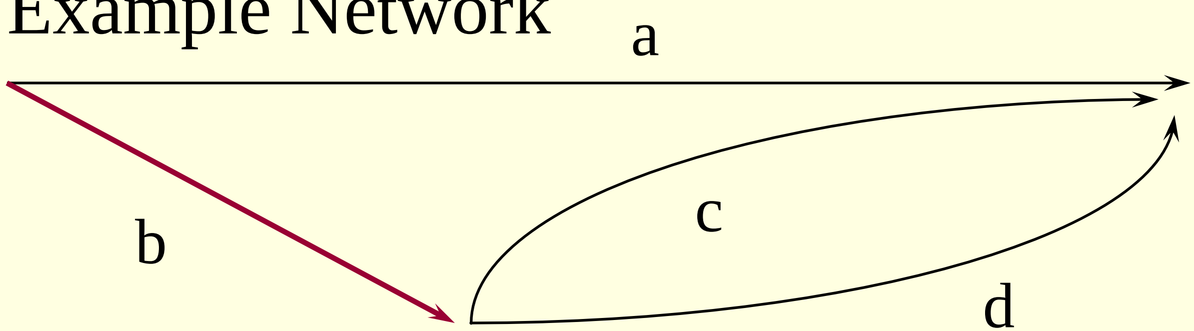
Bovy et al. (2008) – Path Size Correction

$$P(i) = \frac{\exp \mu(V_i + PSC_i)}{\sum_{j \in C} \exp \mu(V_j + PSC_j)}$$

$$PSC_i = -\sum_{a \in \Gamma_i} \frac{L_a}{L_i} \ln \sum_{j \in C} \delta_{aj}$$

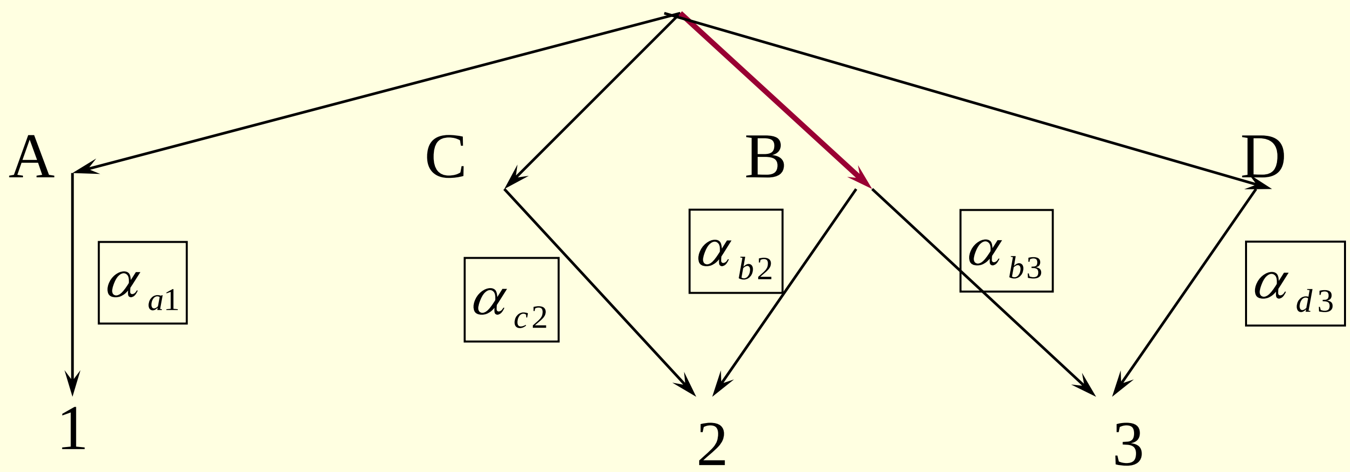
Adapting the CNL to route choice

Example Network



Route 1: Link a
Route 2: Links b-c
Route 3: Links b-d

Model Structure



$$\alpha_{km} = \frac{L_m}{L_k} = \frac{link\ "length"}{route\ "length"}$$

Adapting the Multinomial Probit model to Route Choice Situation

- Problem: define a variance-covariance matrix (Σ)
- Solution: Daganzo (1980), Sheffi and Powell (1982)
  - variances are proportional to the mean travel time
- Example: structured covariance matrix: Yai et al. (1996)

$$\Sigma = \sigma^2 \begin{bmatrix} L_1 & L_{12} & \cdots & L_{1J} \\ L_{12} & L_2 & \cdots & L_{2J} \\ \vdots & \vdots & \ddots & \vdots \\ L_{1J} & L_{2J} & \cdots & L_J \end{bmatrix}$$

Transit route choice (small number of alternatives)

The probability was computed using numerical integration

The Logit Kernel model

$$\mathbf{U} = \boldsymbol{\beta}\mathbf{X} + \mathbf{F}\boldsymbol{\xi} + \boldsymbol{\nu} \qquad\qquad \boldsymbol{\xi} = \mathbf{T}\boldsymbol{\zeta}$$

$$\mathbf{cov(U)} = \mathbf{F}\mathbf{T}\mathbf{T}^{T}\mathbf{F}^{T} + (\mathbf{g}/\boldsymbol{\mu}^{2})\mathbf{I}$$

β - (K*1) vector of unknown parameters

X - (J*K) matrix of explanatory variables

F - (J*M) factor loadings matrix

T - (M*M) lower triangular matrix of unknown parameters

ζ - (M*1) vector of unknown factors

ν - (J*1) vector of i.i.d. Gumbel variables

Logit Kernel Probability Calculation

If the factors ζ are known:

$$\Lambda\left(i\middle|\zeta\right)=\frac{\exp\left(\mu\left(X_i\beta+F_iT\zeta\right)\right)}{\sum_j \exp\left(\mu\left(X_j\beta+F_jT\zeta\right)\right)}$$

Since the factors are unknown, the unconditional probability is given by:

$$P(i)=\int_\xi \Lambda(i|\zeta)\prod\phi(\zeta)d\zeta$$

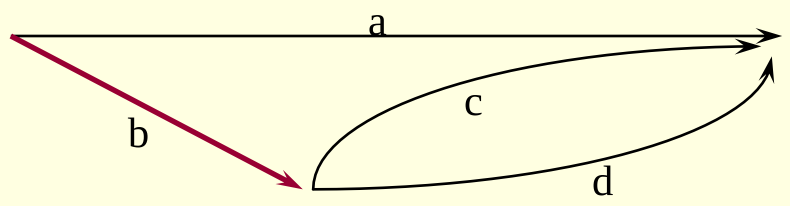
This probability function can be estimated by simulation:

$$P(i)=\frac{1}{D}\sum_{d=1}^{D}\Lambda(i|\zeta^d)$$

Adaptation of LK to route choice situation (as in Probit)

- Link specific factors are iid Normal
- Variance proportional to the link "length"
- The T matrix is the link factors variance matrix (diagonal matrix)
- Bekhor et al. (2002): The F matrix is the link-path incidence matrix
- Frejinger and Bierlaire (2009): Subnetwork approach

Adapting the LK for route choice



Route 1: Link a
Route 2: Links b-c
Route 3: Links b-d

F matrix (J*M)

$$F = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

T matrix (M*M)

$$T = \begin{bmatrix} \sigma_a & 0 & 0 & 0 \\ 0 & \sigma_b & 0 & 0 \\ 0 & 0 & \sigma_c & 0 \\ 0 & 0 & 0 & \sigma_d \end{bmatrix} = \alpha \begin{bmatrix} \sqrt{t_a} & 0 & 0 & 0 \\ 0 & \sqrt{t_b} & 0 & 0 \\ 0 & 0 & \sqrt{t_c} & 0 \\ 0 & 0 & 0 & \sqrt{t_d} \end{bmatrix}$$

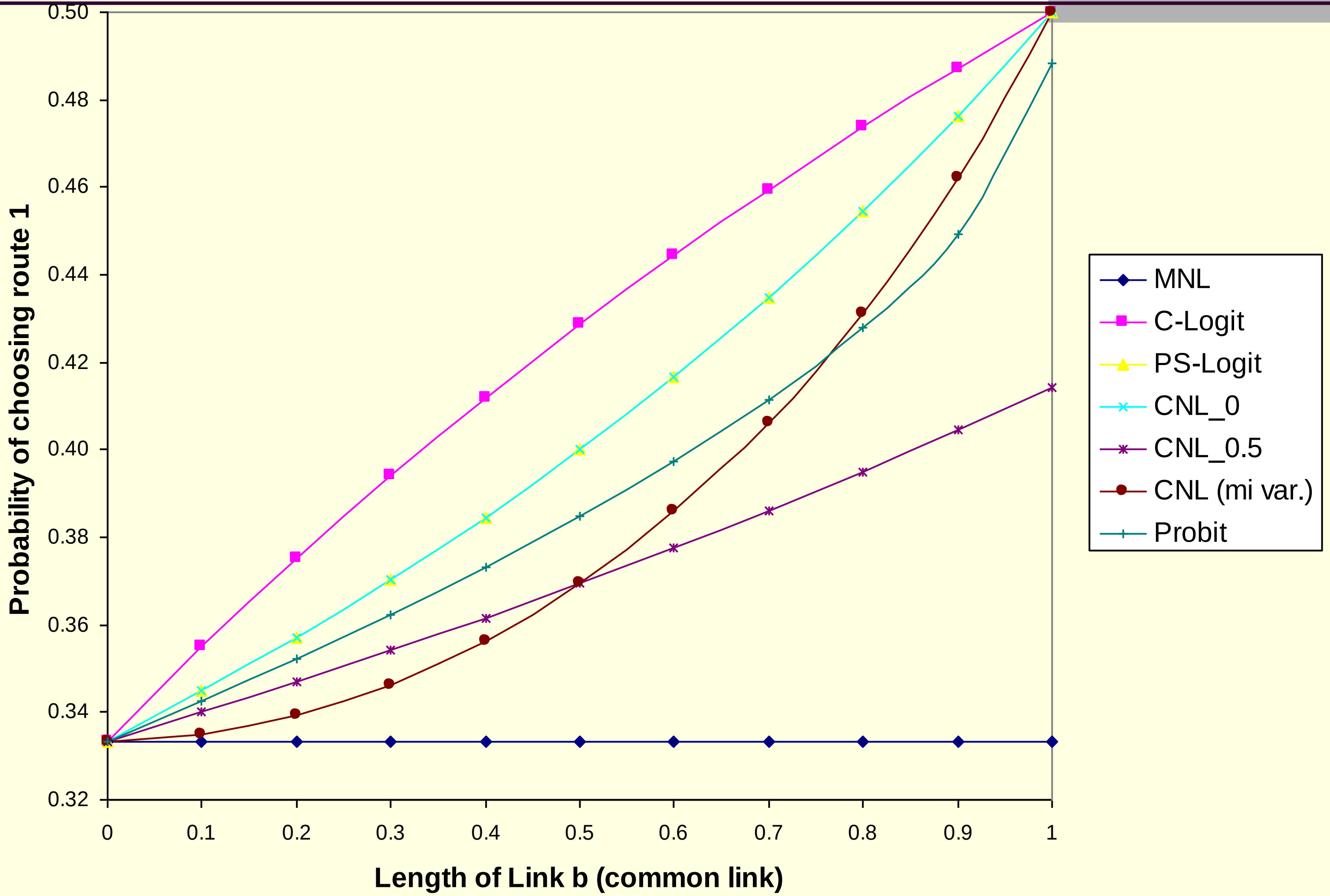
Utility vector

$$U_i = \begin{bmatrix} \beta X_1 + \sigma_a \zeta_1 + \nu_1 \\ \beta X_2 + \sigma_b \zeta_2 + \sigma_c \zeta_3 + \nu_2 \\ \beta X_3 + \sigma_b \zeta_2 + \sigma_d \zeta_4 + \nu_3 \end{bmatrix}$$

Covariance matrix

$$FTT^T F^T = \begin{bmatrix} \sigma_a^2 & 0 & 0 \\ 0 & \sigma_b^2 + \sigma_c^2 & \sigma_b^2 \\ 0 & \sigma_b^2 & \sigma_b^2 + \sigma_d^2 \end{bmatrix}$$

Example 1: Red Bus – Blue Bus Network



Integration of Latent Variables

Latent variables: structural equations

- Latent variables as function of travelers' characteristics
- L latent variables, M explanatory variables

$$X_{ln}^{*} = S_{ln}\gamma_{l} + \omega_{ln} \quad \omega_{ln} \sim N\left(0, \sigma_{\omega_l}\right)$$

Latent variables: measurement equations

- Indicators as functions of latent variables
- R indicators, L latent variables

$$I_{rn} = X_{ln}^{*}\alpha_{r} + \upsilon_{rn} \quad \upsilon_{rn} \sim N\left(0, \sigma_{\upsilon_r}\right)$$

Integrated Model

Route choice: structural equations

- Utilities as functions of observable and unobservable variables
- J alternatives, K observable variables, L latent variables

$$U_{jn} = Z_{jn}\beta_{obs} + X^{*}_{jn}\beta_{lat} + \varepsilon_{jn} \qquad \varepsilon_{jn} \sim Gumbel$$

Route choice: measurement equations

- Choice of alternative i as function of utilities
- J alternatives, N observations

$$y_{in} = \{1 \text{ if } U_{in} \geq U_{jn} \ \forall j \neq i, 0 \text{ otherwise}\}$$

Integrated Model

Case study assumptions

- Latent variables are orthogonal (null covariances in Sw)
- Indicators are independent (estimate variances in Su)
- Choice model is a Path Size Correction Logit

Choice probability

$$P\left(y_n, I_n \middle| Z_n, S_n, \alpha, \beta, \gamma\right) =$$

$$\int_{X_n^*} P\left(y_n \middle| X_n^*, Z_n, \beta\right) g\left(I_n \middle| X_n^*, \alpha\right) f\left(X_n^* \middle| S_n, \gamma\right) dX_n^*$$

Integrated Model

Simulated choice probability

$$\tilde{P}\left(y_n, I_n \middle| Z_n, S_n, \alpha, \beta, \gamma\right) =$$

$$\frac{1}{D}\sum_{d=1}^{D} \frac{\exp\left(Z_{in}\beta_{obs} + X_{in}^{*d}\beta_{lat} + PSC_i\beta_{PSC}\right)}{\sum_j \exp\left(Z_{jn}\beta_{obs} + X_{jn}^{*d}\beta_{lat} + PSC_j\beta_{PSC}\right)} g\left(I_n \middle| X_n^{*d}, \alpha\right)$$

Sequential estimation

- SEM estimator for the latent variables model
- Maximum simulated likelihood for the route choice model

$$X_{ln}^{*d} = S_{ln}\gamma_l + \omega_{ln}^d = S_{ln}\gamma_l + \sigma_{\omega_l}\tilde{\omega}_{ln} \quad \textit{where} \quad \tilde{\omega}_{ln} \approx N(0,1)$$

Measurement equations of the latent variable model

MEM			HAB			TSAV		
Variable	est.	t-stat.	Variable	est.	t-stat	Variable	est.	t-stat.
Memroute	1.000	-	Smrtwork	1.000	-	Esttime	1.000	-
Memhome	0.838	5.74	Smrtshop	0.975	2.46	Useint	3.022	1.99
Memmind	0.712	5.35	Distshop	1.382	2.84	Shortcut	3.573	2.00
Memlayout	0.690	4.96	Sameshop	1.668	2.86	Drvnotl	3.268	2.00
Memway	1.398	7.56	Tendsdch	-0.962	-2.05	Tndesttm	1.759	1.90
Mempark	1.372	7.62	Tendchsg	-1.266	-2.87	Tndestds	1.772	1.88

FAM			SPAB		
Variable	est.	t-stat.	Variable	est.	t-stat.
Dscfamrt	1.000	-	Buymap	1.000	-
Dscrthom	0.612	5.61	Tendmap	0.408	2.06
Evalroute	0.650	5.33	Disttown	2.695	2.72
Navhome	0.150	2.59	Drvlandm	0.625	2.18
Drvmain	-0.116	-2.17	Drvscen	-0.446	-2.26

Structural equations of the latent variable model

	MEM		HAB		FAM		SPAB		TSAV	
Variable	est.	t-stat.	est.	t-stat.	est.	t-stat.	est.	t-stat.	est.	t-stat.
Male	0.228	1.93	-0.080	-0.92	0.260	2.28	0.367	4.32	0.262	2.73
Age<35	0.198	2.83	-0.220	-2.22	-	-	-	-	0.245	2.25
Age>55	-0.134	-1.73	0.233	2.04	-	-	-	-	-0.325	-2.60
Education	0.292	2.37	0.214	2.35	-	-	0.649	7.38	0.274	2.77
Single	-	-	-0.316	-2.63	-	-	-	-	-0.216	-1.66
Children	0.311	2.37	-	-	0.263	2.16	-	-	-	-
Family	-	-	-	-	-	-	-0.075	-2.03	-	-
Stops	-0.360	-2.63	0.287	2.96	-0.360	-2.71	-0.206	-2.12	0.236	2.23
City Resid.	-	-	0.256	2.84	0.231	1.74	-0.150	-1.70	0.250	1.74
Constant	-0.215	-1.87	-0.131	-1.48	-0.293	-2.33	-0.501	-3.46	-0.476	-2.88

Structural equations of the latent variable model

Interpretation

- Males have higher mnemonic capability and better spatial abilities and time saving skills
- Young respondents have better memory and time saving skills
- Educated respondents have higher abilities in terms of memory, spatial orientation, time saving, and also habitual behavior
- Singles have less habitual behavior and lower time saving skills, while having children relates to higher memory and familiarity
- Residents in the city relate with familiar and routine behavior

Model Estimation Results

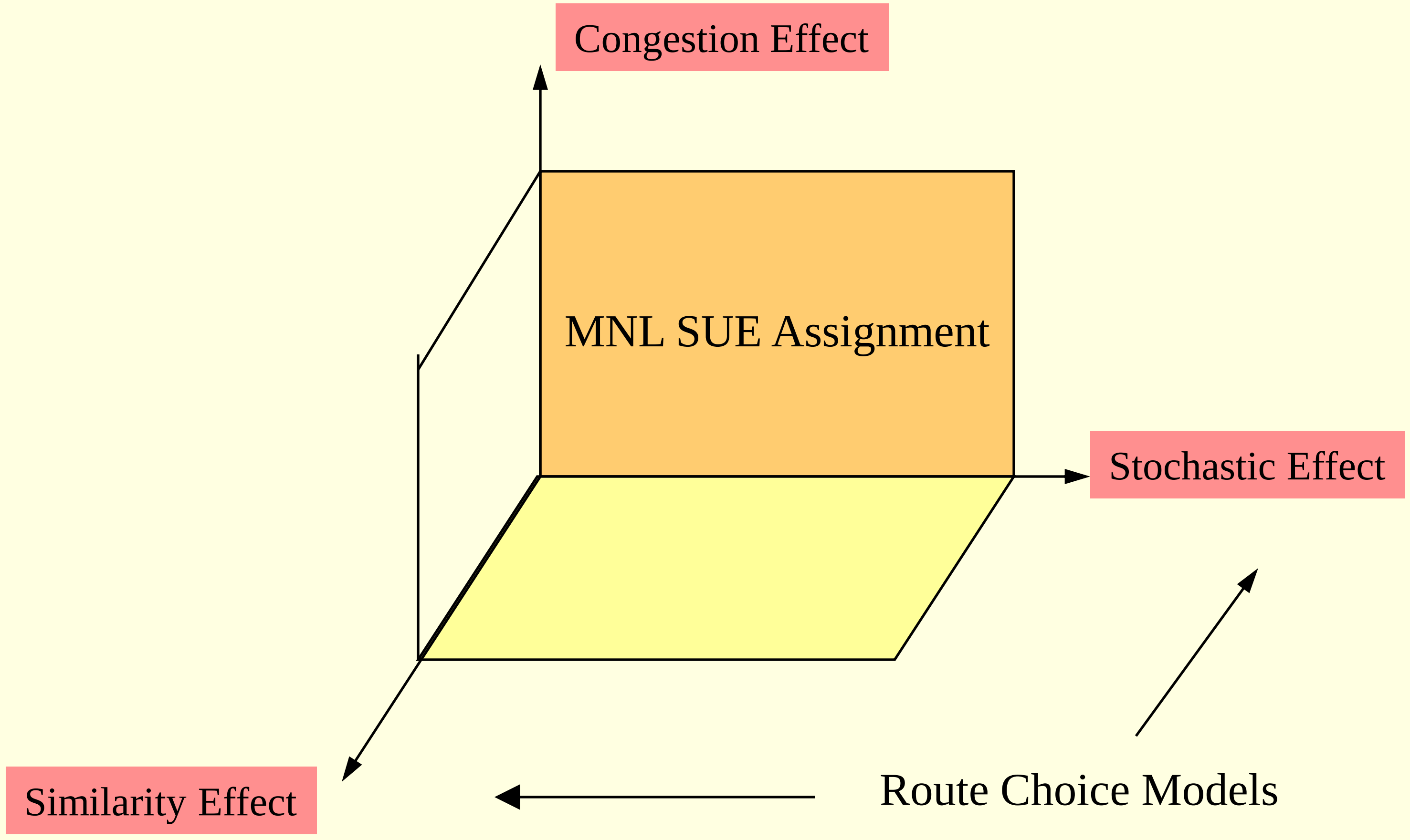
Variables	w/o latent variables Estimate	t-stat.	with latent variables estimate	t-stat.
Dist	-0.680	-4.59	-0.674	-4.47
Time	-0.346	-6.84	-0.338	-6.62
Delay %	-0.458	-3.92	-0.451	-3.85
Time on major roads %	0.530	3.74	0.471	3.26
Path Size Correction	0.681	3.35	0.649	3.17
Memory	-	-	1.408	3.49
Habit	-	-	-0.639	-2.93
Familiarity	-	-	-1.231	-3.25
Spatial ability	-	-	0.416	2.50
Time saving skill	-	-	0.368	1.77
Estimated parameters	5		10	
Null log-likelihood	-1298.38		-1298.38	
Final log-likelihood	-1062.81		-1001.21	
Adjusted rho-bar squared	0.178		0.221	

Route Choice Model

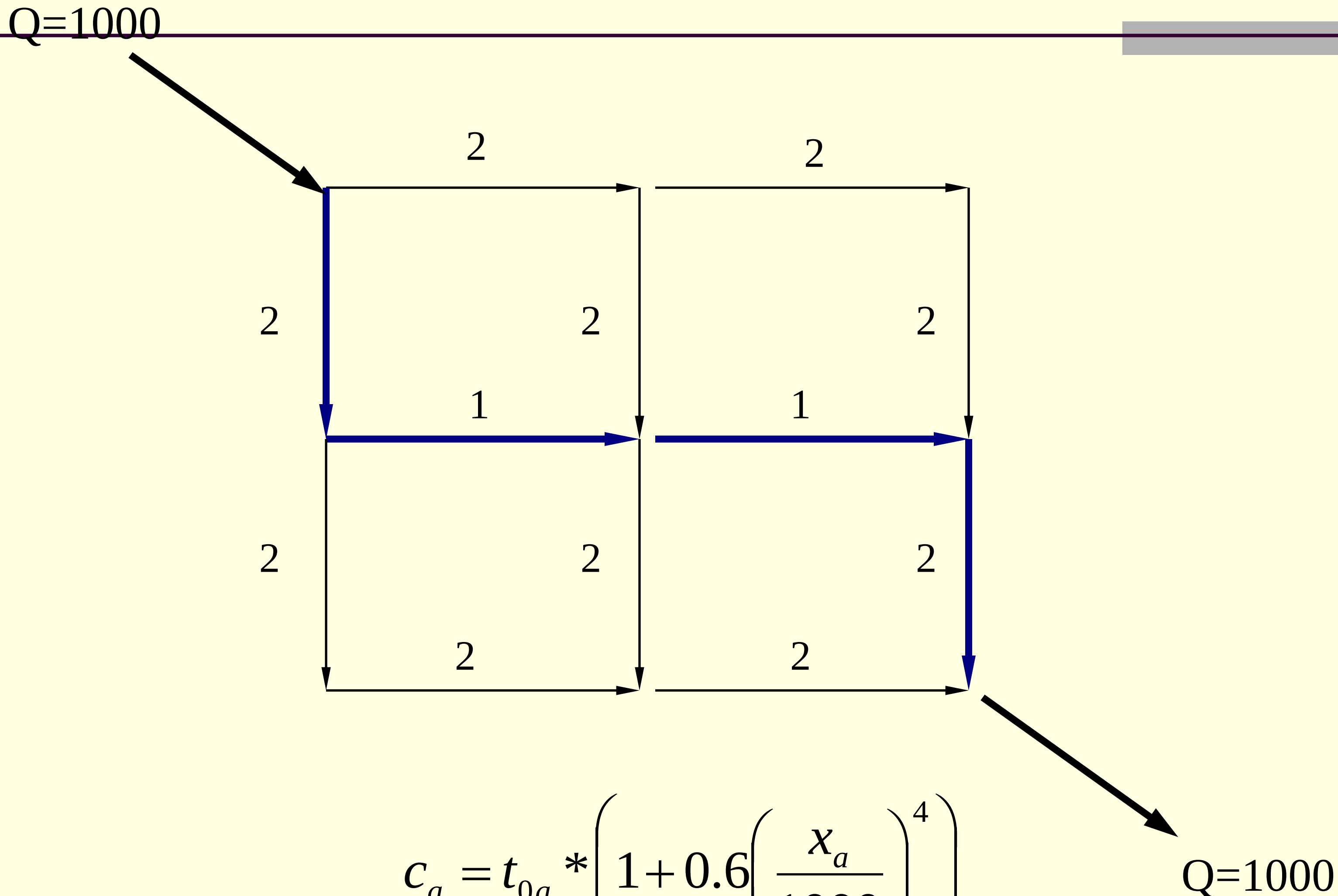
Interpretation

- Expected and logical effects of distance, travel time, congestion and attractiveness of major roads
- Mnemonic, spatial and time saving abilities seem have a positive effect: most likely reflect that individuals tend to look for better alternatives and use them since they remember them
- Habit and familiarity appear to have a negative effect: most likely reflects that individuals do not tend to search for better alternative routes even if their choice is not optimal

Route Choice and Equilibrium models



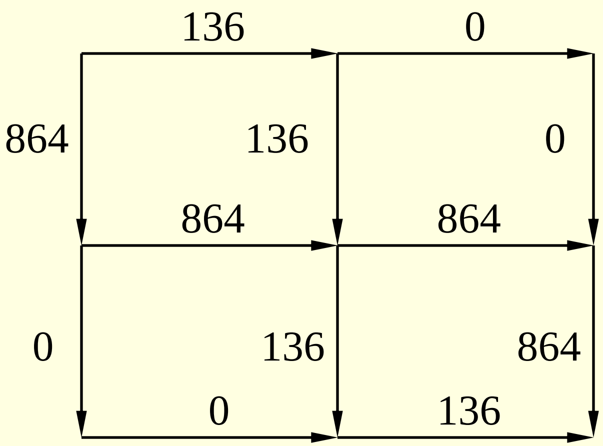
Example 2: Grid Network



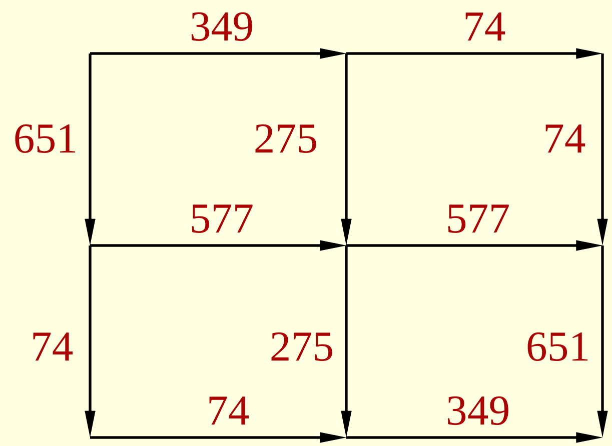
$$c_a = t_{0a} * \left(1 + 0.6 \left(\frac{x_a}{1000} \right)^4 \right)$$

Example 2: Grid Network

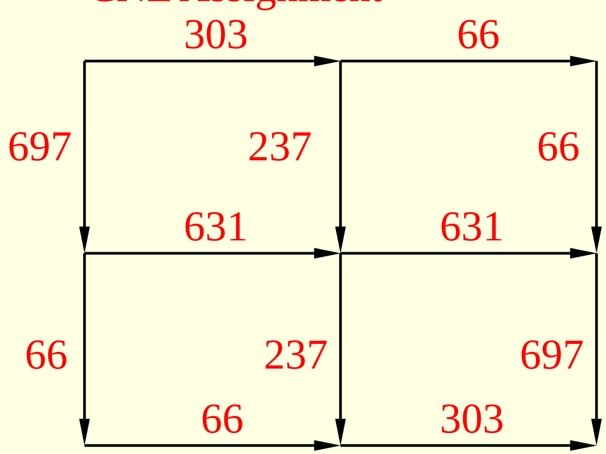
Deterministic UE Assignment



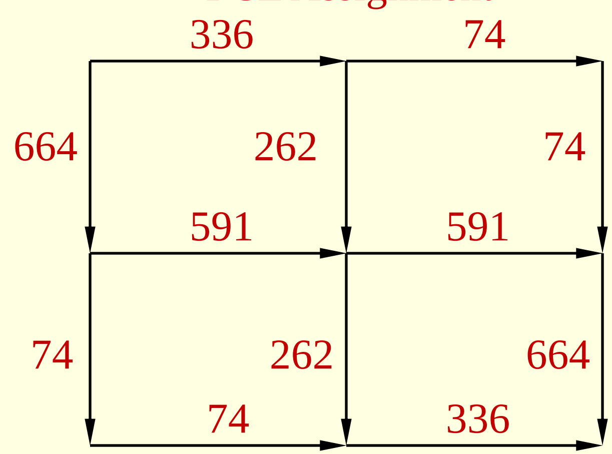
MNL Assignment



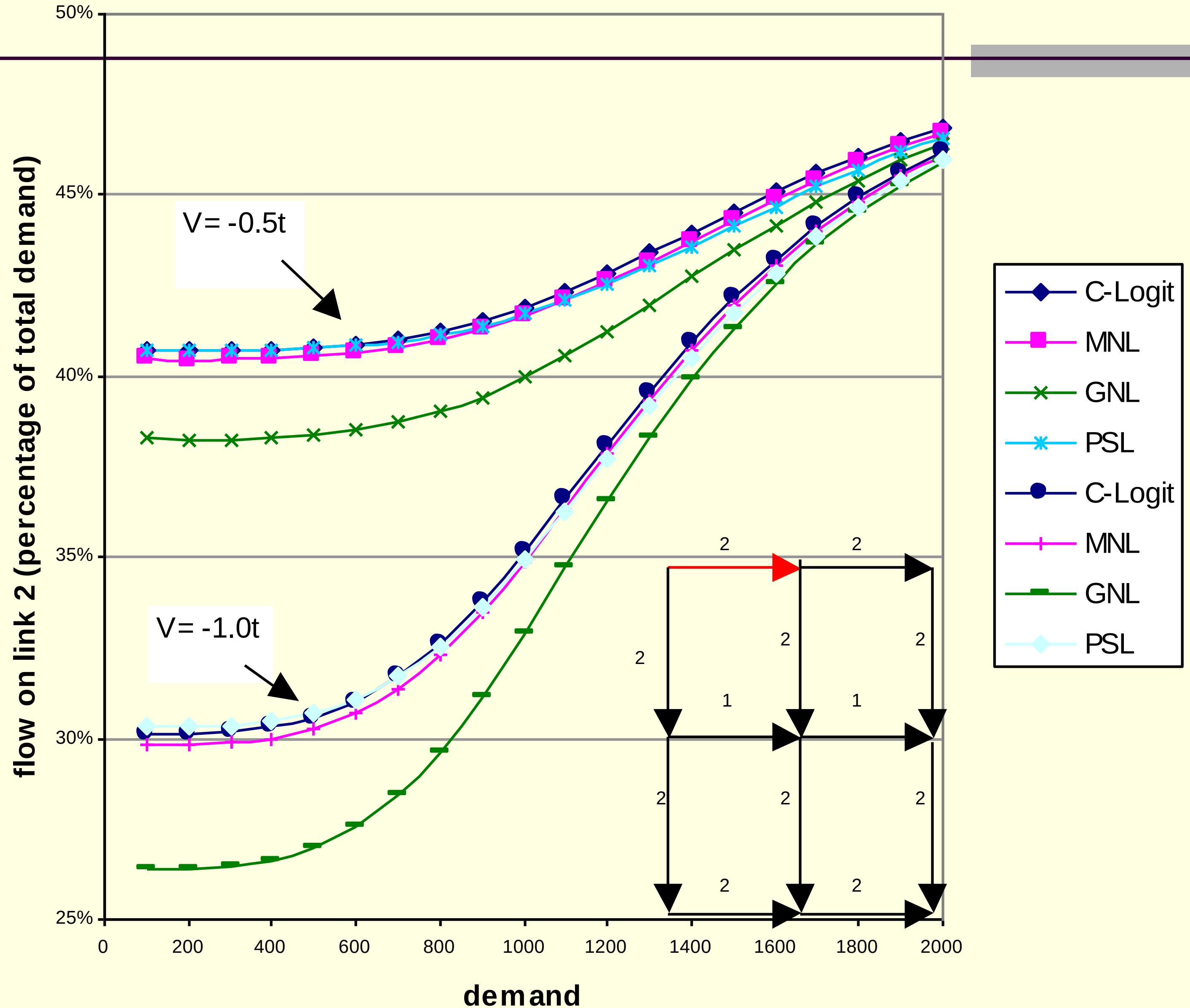
CNL Assignment



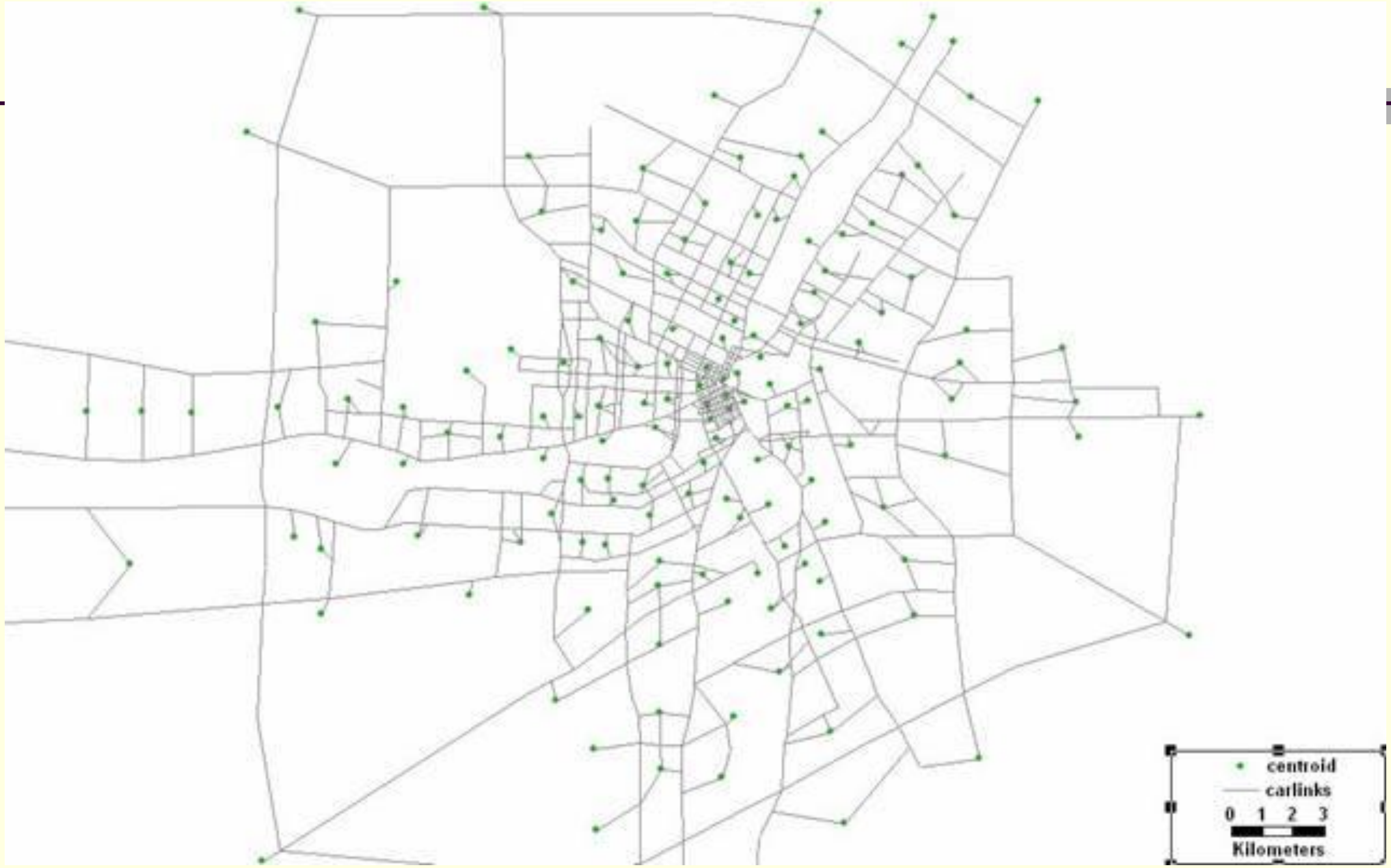
PCL Assignment



Example 2: Grid Network

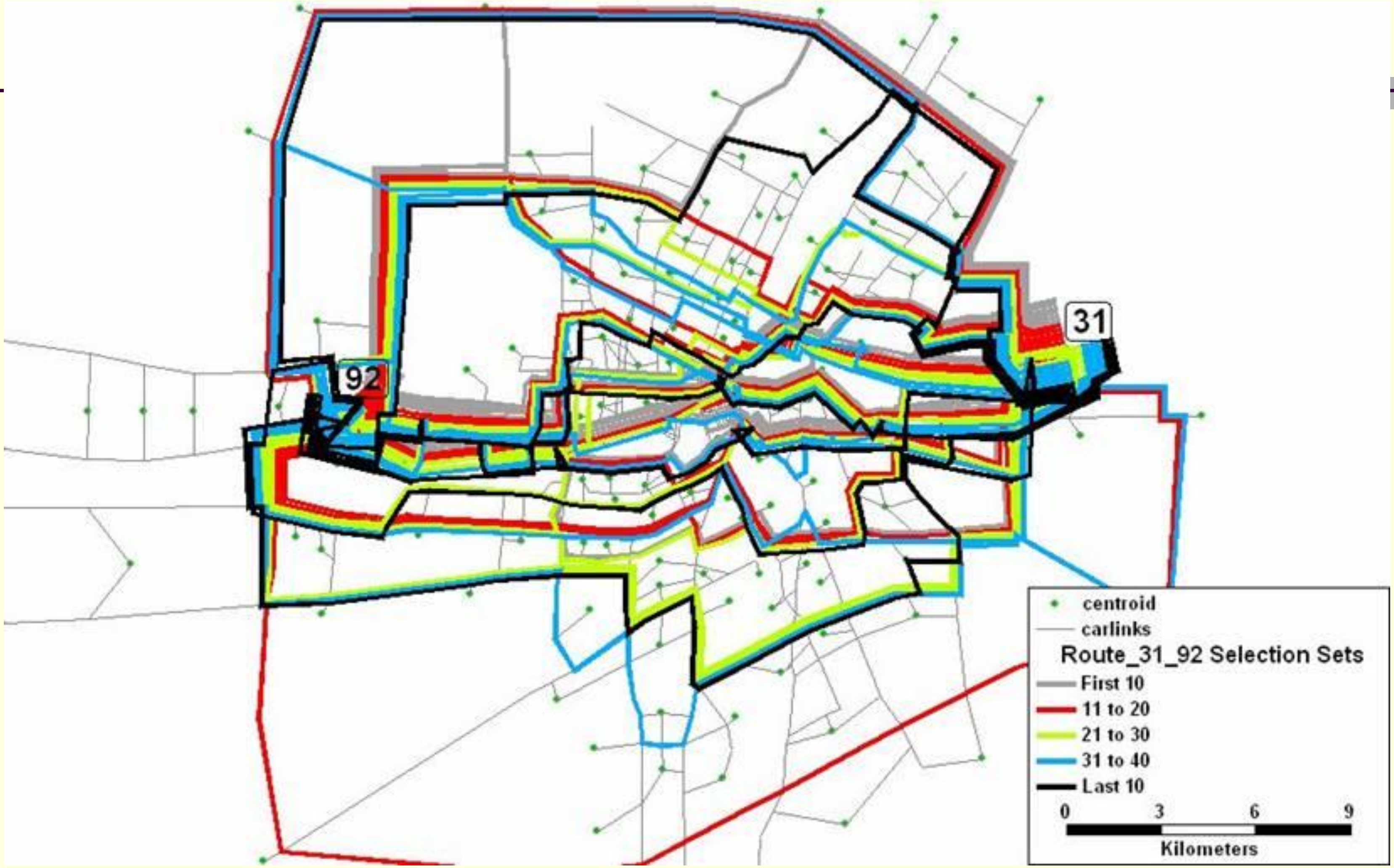


Example 3: Winnipeg Network



948 nodes, 2535 links,154 traffic area zones
4345 OD pairs with positive demand
Total demand 54459 trips.

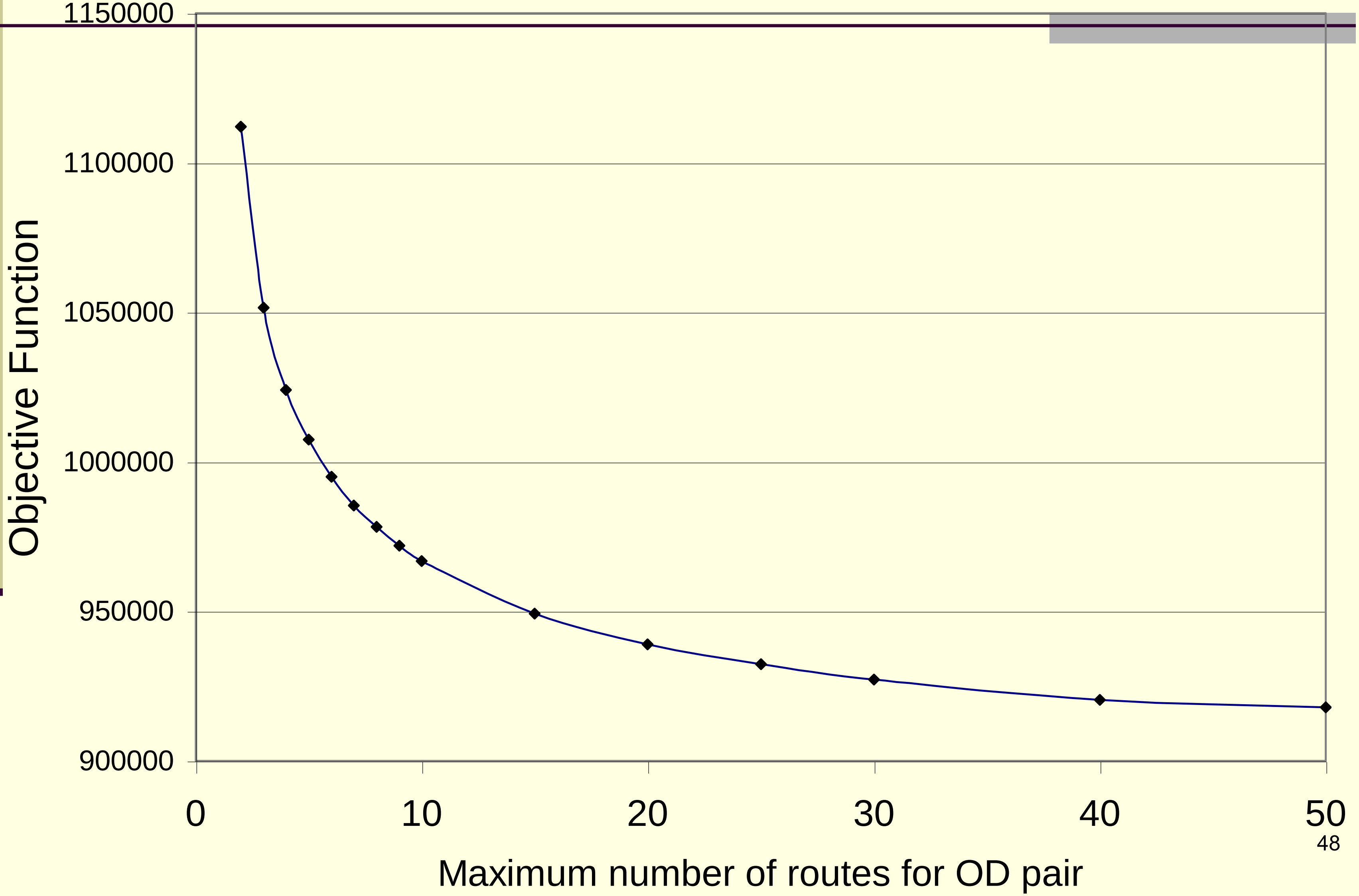
Winnipeg Network – Choice Set Generation



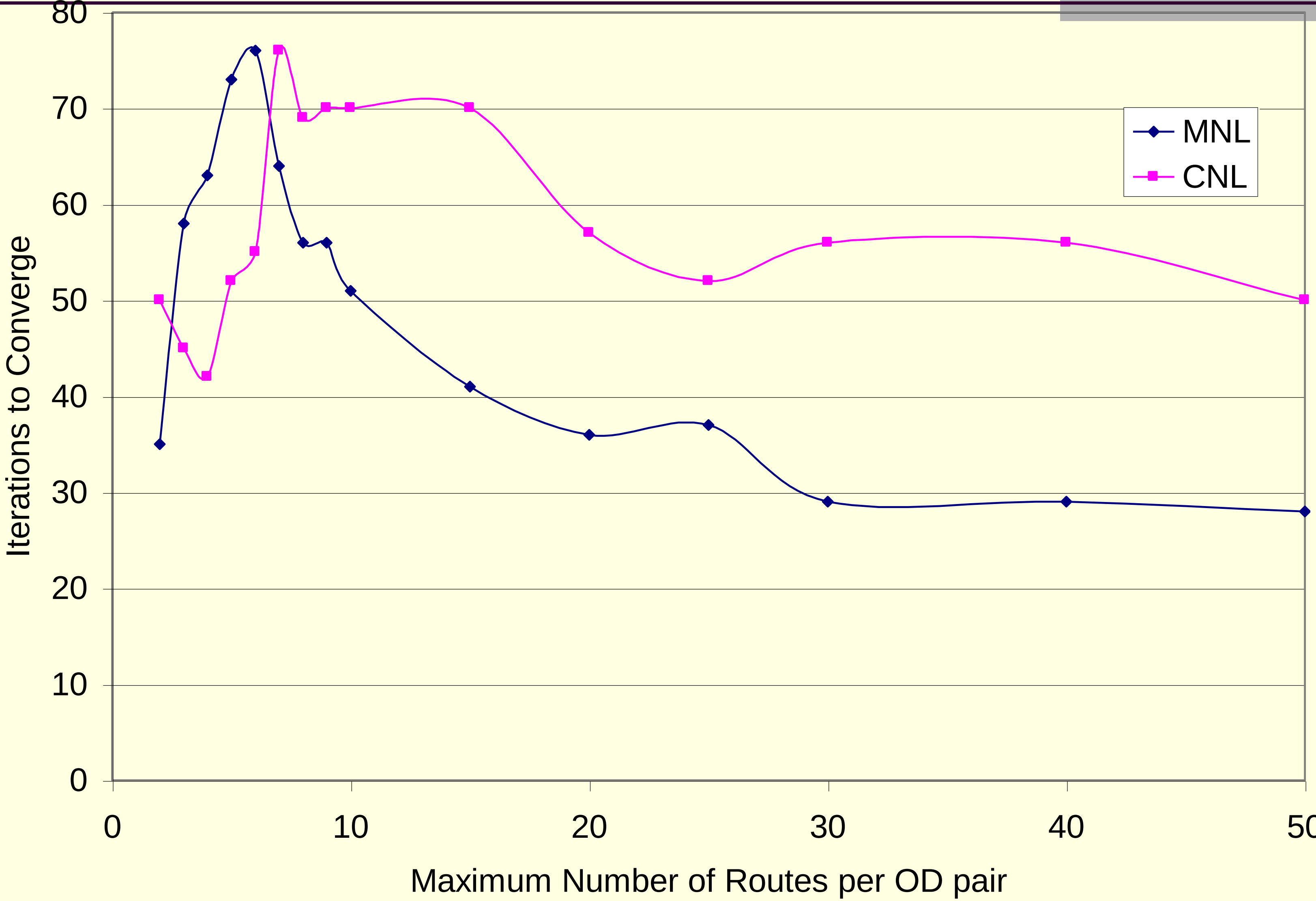
Maximum 50 routes for each OD pair

Total 174491 paths generated (40.1 paths on average)

Influence of Path-Set Size – Winnipeg



Iterations Needed to Reach Convergence



Research Directions

- Data Collection Issues
  - Passive methods
  - Smart phones
  - Map-matching
- Choice Set Generation
  - Inclusion of behavioral variables
  - Sampling correction
- Accounting for congestion
  - Path-based algorithms
  - Choice set composition