Simulation of Pedestrian Behaviour using a Discrete Choice Model Calibrated on Actual Motion Data

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Abstract. Pedestrian behavioral modeling is a topic that is receiving more and more attention in different areas of application such as panic analysis, building evacuation and surveillance systems. We propose a model based on discrete choice theory for pedestrian dynamic behavior. Our model is based on a local discretization of the space around each pedestrian, as opposed to other models which are continuous or which discretize the whole space. The model is calibrated using data from actual pedestrian movement taken from video sequences. In order to verify the quality of the calibrated model, we have developed a simulator based on it.

1 Introduction

The modeling of pedestrian dynamic and behavior is becoming an always more important component for different kind of complex systems, in different areas of application. Panic analysis, building evacuation and crowd simulation have taken into account by designers to build and optimize the use of the society's infrastructures, such as shopping malls, transport terminals, walking facilities and outdoor public spaces. On the other hand, also a relatively young discipline as computer vision starts to need pedestrian behavioral models to improve image segmentation and pedestrian tracking algorithms in automatic video surveillance applications (Johnson and Hogg (1996), Wren and Pentland (1998), Senior (2002) , Isard and Blake (1998)).

The state of the art of the pedestrian behavioral models is based on the following two main approaches: *microscopic* and *macroscopic* models. Belong to the first category all that models describing the time-space behavior of individual pedestrians, such as the *social force* model, the *Cellular Automata* model and the model proposed by Hoogendoorn (see, respectively, Helbing and Molnár (1995),Schadschneider (2002) and Hoogendoorn *et al.* (2002)). Belong to the second category all that models describing pedestrians with fluid-like properties. Examples of this approach are Henderson (1971) and D.Helbing (1992). For a deeper literature review we refer the reader to Bierlaire *et al.* (2003).

The main contribution of this paper is the specification and calibration of a discrete choice model for pedestrian behavior. To validate the proposed model we have implemented a pedestrian dynamic simulator. The use of discrete choice models for pedestrian dynamics is justified by the fact that they are completely disaggregate, being therefore well compatible with the microscopic approach. Moreover, aggregate forecasting techniques allow the computation of macroscopic measurements keeping a microscopic approach.

Finally, we have noticed that few models presented in the literature have been calibrated and validated on real data. Data collection for pedestrian dynamics is indeed particularly difficult. For these reasons, we have decided to collect manually the necessary data set. The calibrated model is integrated in an automatic pedestrian tracking system (see G.Antonini *et al.* (2004), S.Venegas *et al.* (2004)) that we aim to use in the future as a reliable automatic source of pedestrian trajectories for the calibration of more complex behavioral models.

The paper is structured as follows: in section 2 we introduce the spatial discretization we have adopted; in section 3, 4 and 5 we describe the elements of the discrete choice model; in section 6 we describe our data set; in section 7 we report the estimation results, in section 8 we describe the dynamic pedestrian simulator. We present our concluding remarks and future works in section 9.

2 The space model

The representation of the physical space plays an important role in the definition of the behavioral model. In our approach, we use a *dynamic and individual-based* spatial discretization representing the physical space where the current pedestrian can move the next step. The basic elements that we use to define our spatial structure are illustrated in figure 1.

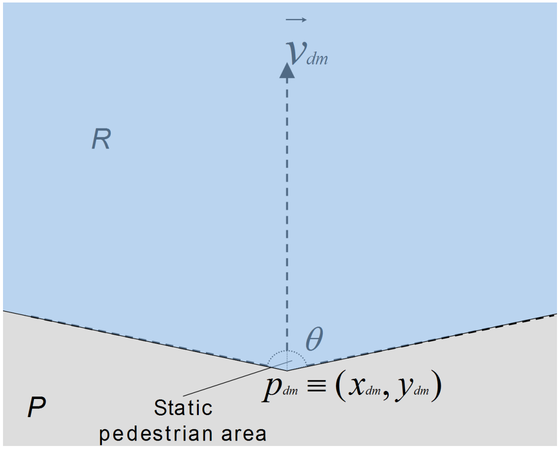

Fig. 1. The basic geometrical elements of the space structure

The decision maker current position $\mathbf{p}_{dm}$, the current speed direction $\overrightarrow{v}_{dm}$ and the visual angle θ generate our region of interest $R \subset P$ within the walking plane P.
Starting from the current speed intensity value v_{dm}, we assume that the decision maker has three different speed regimes that are available: **accelerated**, **constant speed** and **decelerated** that correspond, respectively, to 1.5 times v_{dm}, v_{dm} and 0.5 times v_{dm}. Along with the changes in speed, the decision maker can modify his/her direction in accordance with a predefined set of 11 radial directions as illustrated in figure 2.

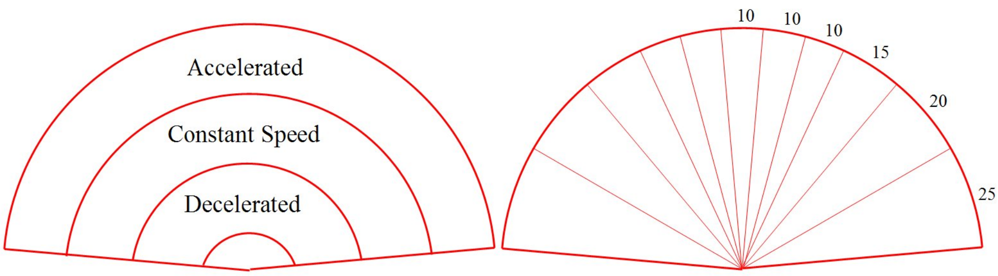

Fig. 2. Discretization of the space based on 3 speed regimes and 11 radial directions. The numbers in the figure on the right represent the angles, in degrees, of each direction.

Differently from other approaches, we propose a radial scheme that adapts to each individual. The size and orientation of our space model depend infact on the current speed vector of the decision maker.

3 The behavioral model

Each pedestrian is treated as an *agent*. This concept has been developed in artificial intelligence (see, among the others, Ferber, 1998) and widely used in traffic simulations. It provides a great deal of flexibility, as the behavior of each individual can be modeled independently, and complex interactions can be captured. We model the behavior of each agent as a sequence of specific choices related to where to put the next step. In this context, discrete choice theory represents a natural theoretical framework.

A discrete choice model is defined by four elements:

1. a *choice set*;
2. a set of *attributes* describing the alternatives;
3. a set of *socio-economic* attributes describing the decision maker;
4. a random term ϵ to capture the correlation structure between alternatives.

3.1 Choice set

The choice set $C = c_1, ..., c_N$ is naturally defined by the spatial discretization. The $s = 3$ speed regimes and the $d = 11$ radial directions create a set of $N = 33$ dynamic alternatives and a static one. We have added a static alternative for simulation pourposes but the current model does not take into account the static behavior of pedestrians. It is indeed a pure dynamical model. We assume that each cell middle point is attainable in a one-step movement by the decision maker, with an adequate change in speed intensity and direction. We have chosen a non uniform radial discretization with smaller angles around the current direction axes. This is justified by the aim to make the model more sensible to directional changes with respect to the current direction.

3.2 The attributes

The specification of the model is based on 8 variables that take into account the interactions between the decision maker and the other pedestrians in the scene as well as the dynamic aspects of the decision maker itself. We discuss here the meaning of each variable.

1. We assume that the decision maker tends to keep his/her current direction (when is phisically possible) and to go toward his/her final destination. We define the related attributes as follows:

   - *direction*: for the alternative c_j, it represents the angle (in degrees) between the direction corresponding to that alternative, represented by the radial line passing through the middle point of the cell, and the decision maker current direction.

   - *destination*: if we consider the triangle having for vertex the current pedestrian position, the final destination point (can be the last point in the trajectory or an intermediate destination) and the center of the cell c_j, the destination attribute is the angle at the current pedestrian vertex. It represents the change in direction between the alternative c_j and the destination (see fig 3).

   We want to underline the fact that we assume to know the decision maker destination. No destination choice model have been specified.
2. We model the interactions between pedestrians by means of two variables that describe how the occupation of the space and the relative movement directions of pedestrians influence the decision process. We define the related attributes as follows:

   - *occupation*:for an alternative c_j, we consider the circular sector defined by the two radial directions that delimit the alternative itself (the shadow area in fig 4). We define the occupation value for the cell c_j as follows:

$$occupation_j = \sum_{k=1}^{N} I_k \cdot \frac{1}{d_{kj}} \tag{1}$$

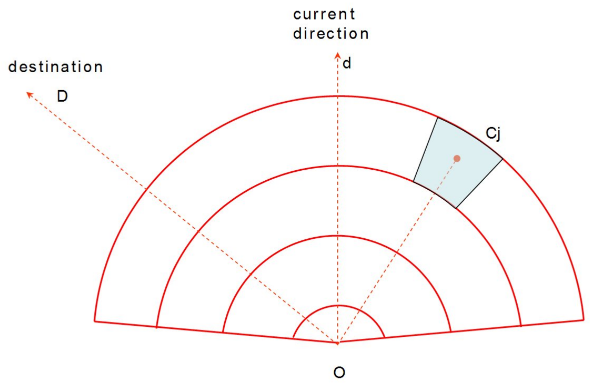

Fig. 3. Given a destination D and the current direction d, the angles defined by the *direction* and *destination* attributes are respectively $d\widehat{O}Cj$ and $D\widehat{O}Cj$.

where N is the number of pedestrians in the scene, I_k is an indicator function that is equal to one if pedestrian k is inside the circular sector and zero otherwise and d_{kj} is the distance between pedestrian k and the center of alternative j.

- *angle*: similarly to the occupation value, we define the angle attribute as follows:

$$angle_j = \sum_{k=1}^{N} I_k \cdot \frac{\alpha_{ki}}{d_{kj}} \tag{2}$$

where α_{ki} is the angle between the movement direction of pedestrian k and the movement direction of decision maker i.

In fig 4 we illustrate the definitions of these two attributes.

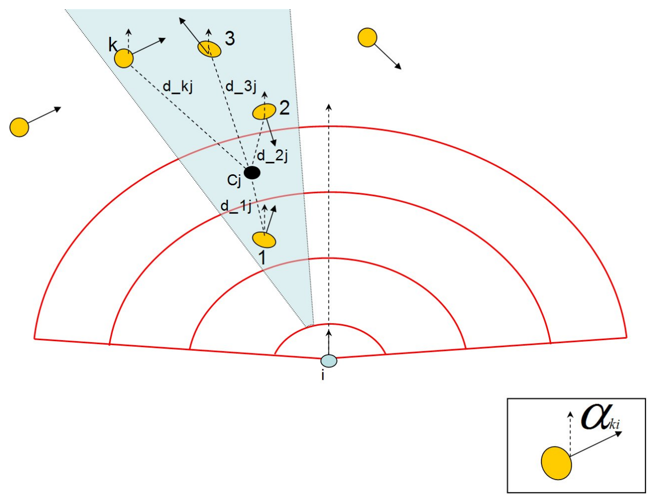

Fig. 4. An illustration of the *occupation* and *angle* attributes. They take into account the interaction between the decision maker and the other individuals.

4 Socio-economics attributes

We interpret the speed module of the decision maker along his/her trajectory as an *alternative-specific socioeconomic* attribute. It has infact two different coefficients for the accelerated and decelerated alternatives.

We introduce the elasticities of the speed module variable. We define the speed term S_{acc} for the accelerated alternatives as follows:

$$S_{acc} = \beta_{acc} \cdot v_{norm}^{\lambda_{acc}}$$

where v_{norm} represents the normalized speed module of the decision maker and β_{acc} is the alternative-specific coefficient defined above. The λ_{acc} term represents the elasticity of the speed variable. Infact we have:

$$\frac{\partial S_{acc}}{\partial v_{norm}} = \beta_{acc} \cdot \lambda_{acc} \cdot v_{norm}^{\lambda_{acc}-1} \tag{3}$$

and multiplying both sides for $\frac{v_{norm}}{S_{acc}}$ we obtain

$$\begin{aligned} \frac{v_{norm}}{S_{acc}} \cdot \frac{\partial S_{acc}}{\partial v_{norm}} &= \lambda_{acc} \cdot \frac{v_{norm}}{S_{acc}} \cdot \beta_{acc} \cdot v_{norm}^{\lambda_{acc}-1} \\ &= \lambda_{acc} \end{aligned} \tag{4}$$

Adding the elasticities we obtain a non-linear in parameters utility function. The λ_{acc} coefficient measures how responsive is the S_{acc} term to changes in the v_{norm} value. The same arguments hold for the decelerated alternatives. We report here the expression of the systematic utility function:

$$\begin{aligned} V_j = {} & \beta_{occupation} \cdot occupation_j + \beta_{direction} \cdot direction_j + \beta_{destination} \cdot destination_j \\ & + \beta_{acc} \cdot v_{norm}^{\lambda_{acc}} + \beta_{dec} \cdot v_{norm}^{\lambda_{dec}} \end{aligned} \tag{5}$$

5 The random variable

In discrete choice models the utility of each alternative is a latent variable composed by a systematic part and a random part. Different assumptions about the random term give rise to different models. In this paper we present two different model formulations: a cross nested logit model and a mixed nested logit model.

5.1 Cross nested logit formulation

This model allows flexible correlation structures in the choice set keeping a closed form solution. The general formulation of the CNL model is derived from the Generalized Extreme Value model (McFadden (1978)). The probability of choosing alternative i within the choice set C of a given choice maker is:

$$P(i|C) = \frac{y_i \frac{\partial G}{\partial y_i}(y_1, ..., y_N)}{\mu G(y_1, ..., y_N)} \tag{6}$$

basing on the following generating function:

$$G(y_1, ..., y_N) = \sum_m \left(\sum_{j \in C} \alpha_{jm} y_j^{\mu_m} \right)^{\frac{\mu}{\mu_m}} \tag{7}$$

where $\alpha_{jm} \geq 0 \ \forall j, m$; $\mu > 0$; $\mu_m > 0 \ \forall m$; $\mu \leq \mu_m \ \forall m$. We assume a correlation structure dependent on the speed and direction and we identify five nests: *accelerated*, *constant*, *decelerated*, *central* and *not central*. This correlation structure is illustrated in figure 5. We fix the degrees of membership to the different nests (α_{jm}) to the constant value 0.5.

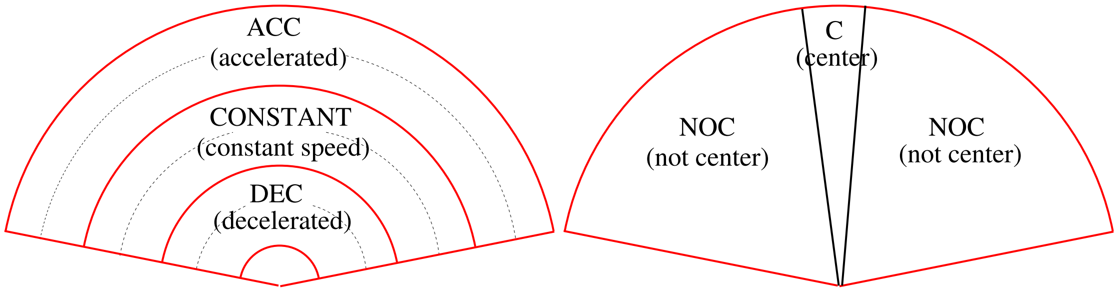

Fig. 5. left: Nesting based on speed **right:** Nesting based on direction

5.2 Mixed nested logit formulation

The assumption that the disturbances are i.i.d Gumbel distributed leads to the tractable logit models. The assumption that the disturbances are normal distributed leads to the flexible but computationally demanding probit model. The family of mixed models (logit kernel) is an hybrid between logit and probit and represents an effort to incorporate the advantages of each (Ben-Akiva and Bolduc (1996),J.L.Walker (2001)). In our model we specify an error components formulation, where the correlation between alternatives still depends on speed and direction. The Gumbel term refers to the speed related nests (accelerated, constant and decelerated), while 11 error components capture the correlation between alternatives along the 11 radial directions, one component for each direction. We show this structure in figure 6. The utility function as perceived by the individual n will have the following vector form:

$$U_n = V_n + \xi_k + \nu_s \tag{8}$$

where $n = 1, ..., N$, $k = \{n \; mod \; d : k = 1, ..., d = 11; \; n = 1, ..., N = 33\}$ and $s = \{acc, const, dec\}$ for accelerated, constant speed and decelerated nests. The ξ_k is normal distributed with zero mean and unknown variance σ_k while the ν_s are the Gumbel terms. If the ξ_k are known, the model corresponds to a MNL formulation:

$$\Lambda(i|\xi_k) = \frac{e^{\mu(V_{in}+\xi_k)}}{\sum_{j \in C} e^{\mu(V_{jn}+\xi_k)}} \tag{9}$$

where $\Lambda(i|\xi_k)$ is the probability that the choice is i conditional in ξ_k. Since the ξ_k are unknown, the unconditional choice probability is given by:

$$P(i) = \int_{\xi} \Lambda(i|\xi) n(\xi, I_d) d\xi \tag{10}$$

where $n(\xi, I_d)$ is the joint density function of ξ (a product of d standard univariate normals).

6 Data

Many different pedestrian models have been formulated in literature, using several different approaches (Schreckenberg and Sharma (2002)). As already said in the introduction of the paper, few of these models have been calibrated and validated on real data. We have addressed the problem using digital video sequences of real scenarios. In figure 7 we show a frame from the used sequence with a set of tracked pedestrians. Knowing the camera parameters (height, angle respect to the camera axis and the focal distance) we store the position of each pedestrian in the scene, at each observation, projecting it from the image plane on the top-view plane. The top-view plan is a reconstruction of the position of each region in the real scene, obtained by a calibrated camera. In the case of pedestrian trajectory, this reconstruction gives the position of each pedestrian on the top-view plan of the scene and not its position projected on the image plan. Our data set is made up of 36 pedestrian trajectories.

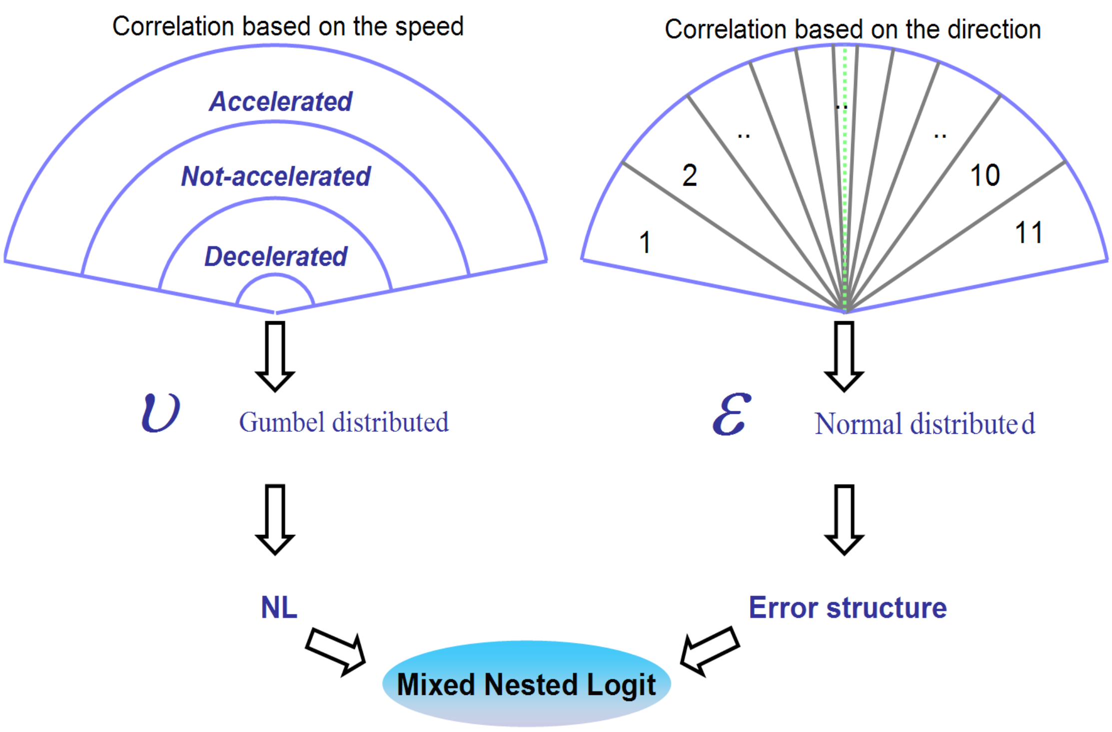

Fig. 6. Correlation structure in the Mixed Nested Logit formulation

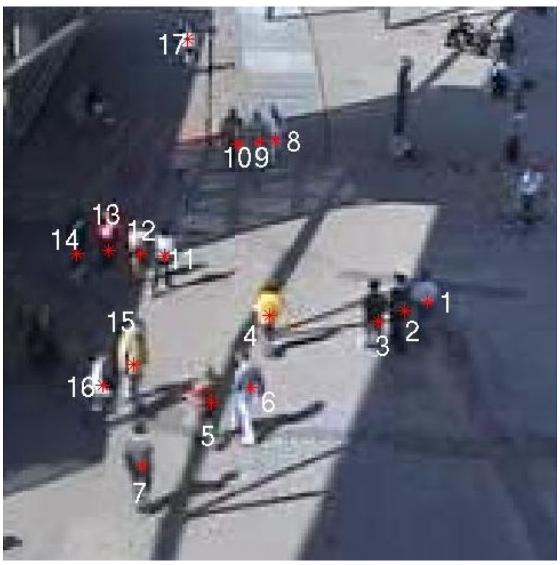

Fig. 7. A frame from the test video sequence

7 Estimation results

All the models have been estimated using the Biogeme package (Bierlaire (2003)). We report the results for the two models in table 1 and 2.

Variable number	Variable name	Coefficient estimate	Asymptotic standard error	t test 0	t test 1
Utility parameters:					
1	$\beta_{occupation}$	-1.4685811e-01	+4.9796722e-02	-2.9491522e+00	
2	$\beta_{direction}$	-2.8000636e-02	+3.0120329e-03	-9.2962585e+00	
3	$\beta_{destination}$	-2.8000636e-02	+3.0120329e-03	-9.2962585e+00	
4	β_{acc}	-3.1927386e+01	+7.7964798e+00	-4.0951028e+00	
5	β_{dec}	-5.1566031e-01	+8.0317120e-02	-6.4203038e+00	
6	λ_{acc}	+1.8938746e+00	+1.5790968e-01	+1.1993404e+01	
7	λ_{dec}	-8.5610599e-01	+9.5022575e-02	-9.0095010e+00	
Model parameters:					
8	$\mu_{accelerated}$	+2.1343940e+00	+5.7973719e-01	+3.6816579e+00	+1.9567384e+00
9	μ_{const}	+2.5955476e+00	+4.2728762e-01	+6.0744741e+00	+3.7341302e+00
10	$\mu_{not\_central}$	+1.2875363e+00	+1.3287501e-01	+9.6898299e+00	+2.1639604e+00

Summary statistics
Sample size = 1410
Number of estimated parameters = 10
Init log-likelihood = -4929.66
Final log-likelihood = -3406.57
Likelihood ratio test = 3047.02
Rho-square = 0.309023

Table 1. CNL: Estimation of utility and model parameters

Variable number	Variable name	Coefficient estimate	Asymptotic standard error	*t* test 0	*t* test 1
Utility parameters:					
1	$\beta_{occupation}$	-1.5051876e-01	+5.6931773e-02	-2.6438445e+00	
2	$\beta_{direction}$	-5.2491495e-02	+9.6579148e-03	-5.4350753e+00	
3	$\beta_{destination}$	-4.0572846e-02	+5.0527673e-03	-8.0298268e+00	
4	β_{acc}	-3.0922166e+01	+7.1640206e+00	-4.3163145e+00	
5	β_{dec}	-6.5567518e-01	+1.1817957e-01	-5.5481262e+00	
6	λ_{acc}	+1.7525719e+00	+1.7028307e-01	+1.0292108e+01	
7	λ_{dec}	-7.9586430e-01	+9.3527313e-02	-8.5094319e+00	
8	σ_1	+1.8573870e+00	+3.8892378e-01	+4.7757097e+00	
9	σ_2	-1.5691682e+00	+5.5359137e-01	-2.8345244e+00	
10	σ_3	-1.0134361e+00	+4.8586314e-01	-2.0858468e+00	
11	σ_7	+6.6238055e-01	+1.8646290e-01	+3.5523450e+00	
12	σ_8	+5.9938734e-01	+2.6174407e-01	+2.2899749e+00	
13	σ_9	+1.0150646e+00	+2.6239843e-01	+3.8684095e+00	
14	σ_{10}	+2.6667886e+00	+7.4026154e-01	+3.6024952e+00	
15	σ_{11}	+2.5289053e+00	+4.9287960e-01	+5.1308784e+00	
Model parameters:					
16	μ_{const}	+1.4235036e+00	+1.7582124e-01	+8.0963116e+00	+2.4087167e+00

Summary statistics
Number of Halton draws = 150
Sample size = 1410
Number of estimated parameters = 16
Init log-likelihood = -4930.08
Final log-likelihood = -3384.94
Likelihood ratio test = 3090.28
Rho-square = 0.313411

Table 2. Mixed NL: Estimation of utility and model parameters

The signs of the estimated coefficients follow our expectations. Infact, the negative signs of the direction and destination variable's coefficients reflect the tendency of an individual to keep his/her current direction together with the tendency to move, if it is possible, toward destination. The negative sign of the occupation coefficient reflects the fact that pedestrians will tend to prefer nearby spatial zones less crowded by other pedestrians, as it is logical to expect. The speed related coefficients show that acceleration and deceleration are two distinct behavioral patterns. The negative sign of their coefficients reflect the intuitive fact that, when it is possible, an individual will tend to keep his/her current speed value. Finally, the two elasticities parameters show that the tendency to accelerate reduces with higher speed values and the tendency to decelerate reduces with lower speed values.

In order to verify the quality of the calibrated model, we have developed a simulator based on it [1].

8 Simulation

There are essentially two approaches to simulation: *time-based* and *event-based*. In the time-based approach, the simulation proceeds in fixed time steps and all actors of the simulation are updated at each of these steps. In the event-based approach, events (e.g. collisions) are generated and inserted into a priority queue and are then executed in increasing time order. For now, we have chosen a time-based approach because the model is simpler, but we might move to an event-based approach later if the evolution of our model requires each footstep to be controlled precisely. We currently use a time step of $\Delta t = 0.9s$ in our simulations.

We provide here a brief description of the design of our simulator.

- **Initialization**
  The input to our simulator is a time-dependent origin-destination matrix, where each cell correspond to an origin o, a destination d and a time interval ΔT, exactly like the OD matrices used for transportation applications. The cells contain the number of individuals departing from o, targeting d during the time interval ΔT.
  From the time-dependent OD matrix, we create a population of pedestrians. Each pedestrian is associated with a list of characteristics (height, desired speed, age, etc.) The exact list of characteristics will obviously be determined by the behavioral models that will be used. This approach is consistent with the concept of demand simulation proposed by Antoniou *et al.* (1997) and Bierlaire *et al.* (2000). Also, we associate an itinerary with each pedestrian. An itinerary is defined as a sequence of intermediate targets, such that target k in the itinerary is visible from the position of target $k-1$, consistently with the network presentation presented in Bierlaire *et al.* (2003).

- **Moving decisions**
  First, new pedestrians are loaded in the system, with an initial speed corresponding to their desired speed, and an initial direction corresponding to the next target in their itinerary.
  Then, at each time step (Δt), the utility value of each possible move is calculated for each pedestrian. These values are then transformed into probabilities consistent with the discrete choice model and each pedestrian's move is randomly selected according to these probabilities.
  Then, the speed and direction of all individuals in the system are updated to reflect the chosen move, using the model described previously in the paper.
  Then the position of all individuals in the system are updated using the formula $x_{i+1} = x_i + \Delta t\, v_i$, where x is the position, i the time step and v the speed.

Figure 8 shows a pedestrian as depicted by our simulator. Here the choice set is shown with a coloring based on the choice's probability, from blue = lowest probability to red = highest probability. In this example, the probability of accelerating is low (outer cells) and the choice with highest probability is the one straight ahead keeping the speed constant.

[1] The videos generated by the simulator can be found at http://ltswww.epfl.ch/ltsftp/antonini/

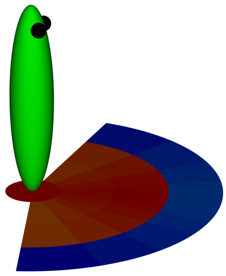

Fig. 8. Pedestrian with choice set

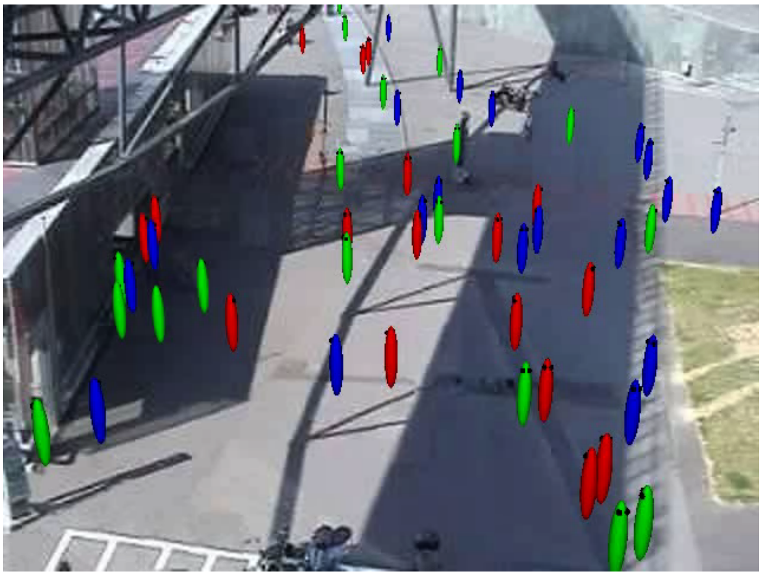

Fig. 9. Example simulated sequence

Figure 9 shows an example of a simulated situation generated by our simulator. Pedestrians are generated with an origin and a destination at doors and at some selected points on the border of the picture, and are left to evolve according to the behavioral model.
Figure 10 shows the same situation with each pedestrian's choice set.
Figure 11 shows the same situation seen from above.

9 Conclusion and future research

In this paper we have shown how to apply discrete choice models for pedestrian dynamics. The alternatives in the choice set show a strong spatial intercorrelation. The cross nested logit and mixed nested logit formulations attempt to capture these interdependencies in the choice set keeping the computational advantages of the logit kernel formulation. In the future works we aim to increase the complexity of the model, extending it to high density scenarios and add an explicit model for obstacles.

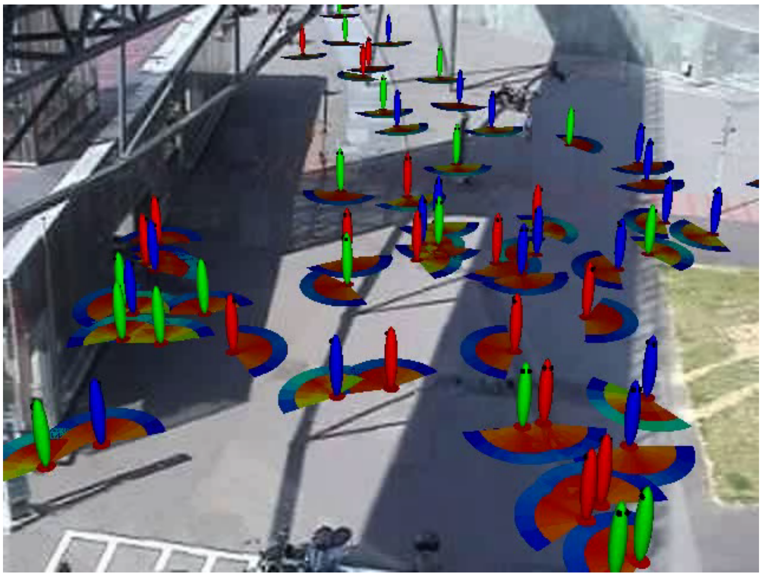

Fig. 10. Example simulated sequence showing choice sets

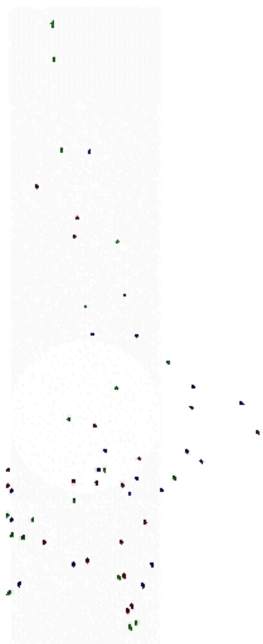

Fig. 11. Example simulated sequence: top view

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