New Insight into Track Modulus of ballasted Track

Dipl. Ing. ETH Michael Kohler,
Institut für Verkehrsplanung, Transporttechnik, Strassen- und Eisenbahnbau ETH Zürich

Conference paper STRC 2002
Session XXX

STRC | 2nd Swiss Transport Research Conference
Monte Verità / Ascona, March 20-22, 2002

New Insight into Track Modulus of ballasted Track

Dipl. Ing. ETH Michael Kohler
Institut für Verkehrsplanung, Transporttechnik, Strassen- und Eisenbahnbau
ETH Zürich
Zürich

Phone: 01-6332418
Fax: 01-6331057
e-Mail: kohler@IVT.baug.ethz.ch

Abstract

In order to analyse the stress of the superstructure the theory of Winkler is often used, which considers the rail as an infinitely long beam continuously supported by an elastic foundation. It is also known as the theory of Zimmermann or Talbot in the English-speaking world. Provided that there is a rough estimate of the Track Modulus, the attainable results are in most cases fairly good. Either the track modulus has to be determined by measurements on site or it has to be estimated by comparing with other measurements. The first approach is rather expensive and impracticable. The second approach suffers from the wide variety of parameters. Most of the known tests refer to standard-gauge and an axleload of about 20 t. Hence, it was impossible to give well-founded figures on the track modulus for axleloads below 20 t and for narrow-gauge, which makes up about 15 % of the world wide networks. In order to close this gap of knowledge, another measuring campaign was at need. Two different methods believed to be equivalent were used and revealed new insights into the theory of track modulus.

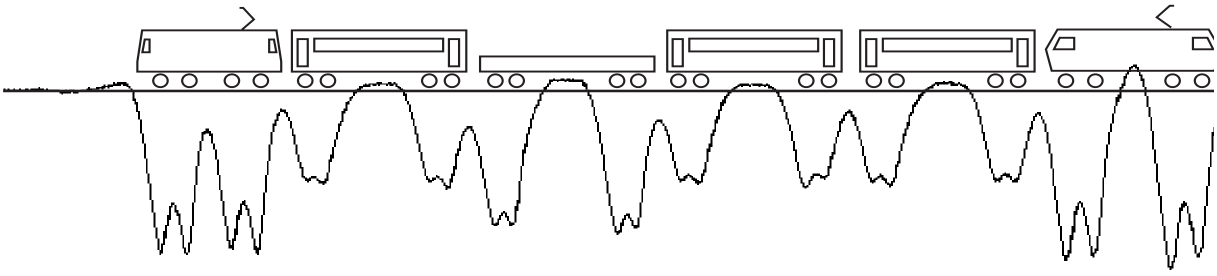
Keywords

Track Modulus – Superstructure –Railway Engineering – 2nd Swiss Transport Research Conference – STRC 2002 – Monte Verità

Introduction

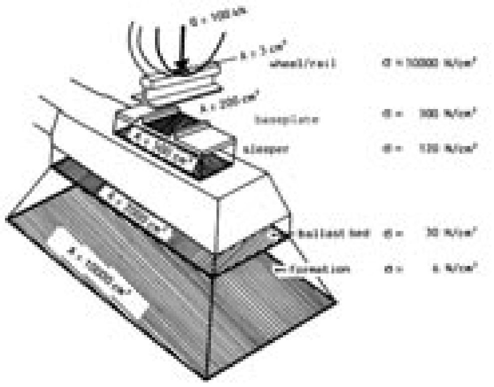
If a train runs over the track, then the rail bends under each axle. This is part of the suspension and influences the travelling comfort considerably. In order to achieve a good performance, it is imperative to analyse the behaviour of the track structure carefully.

Figure 1 Deflection of track



The main purpose of the track structure is to reduce the loads to a level, which is admissible for the subgrade. Each element distributes the loads little a more.

Figure 2 Load distribution

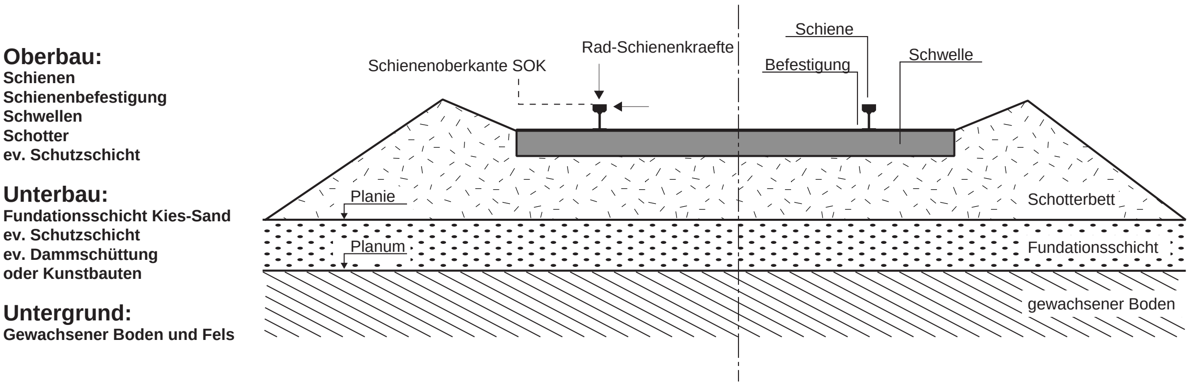


Source: Esveld (1989)

The a ballasted track is divided into superstructure, subgrade or foundation and soil. The superstructure consists of rails , sleepers and ballast. The distance between the sleepers is about

600 mm and the depth of ballast is about 300 to 400 mm. The track quality and riding comfort is given by the alignment, condition of each element.

Figure 3 Track structure

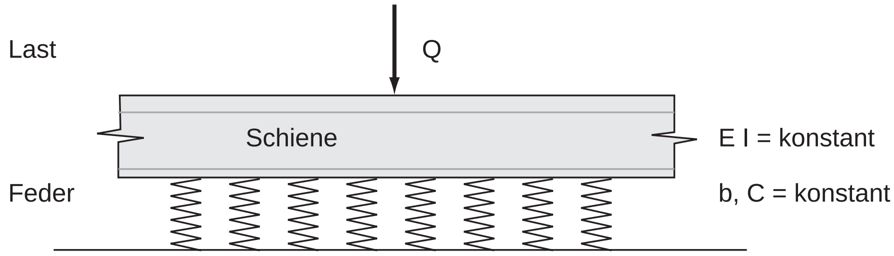


The track is an overdetermined system and the distribution of load depends on the elasticity of all components, even the subgrade. In order to analyse the stress of the superstructure the question arises, how many sleepers are involved in the load distribution. In a first approach it can be assumed that one sleeper carries approximately 40 % of the vertical and approximately 70 % of the lateral forces. This paper looks at vertical forces only.

Theory

In order to achieve a higher accuracy, the theory of Winkler and Zimmermann is frequently used. It allows a fairly good estimation of the essential strains. It considers the rail as an infinitely long beam continuously supported by an elastic foundation.

Figure 4 Beam on elastic foundation



The beam on elastic foundation can be described by a differential equation of 4th order. On condition that the deflection is proportional to the load, the following equation applies:

$$\frac{d^4 y}{dx^4} = -\frac{bC}{EI} y$$

y : Deflection
EI : Stiffness of Rail
b : Form Factor characterising Size of Sleeper
C : Track Modulus

In this case exists a simple solution for the equation. With the abbreviation

$$\beta = \sqrt[4]{\frac{bC}{4EI}}$$

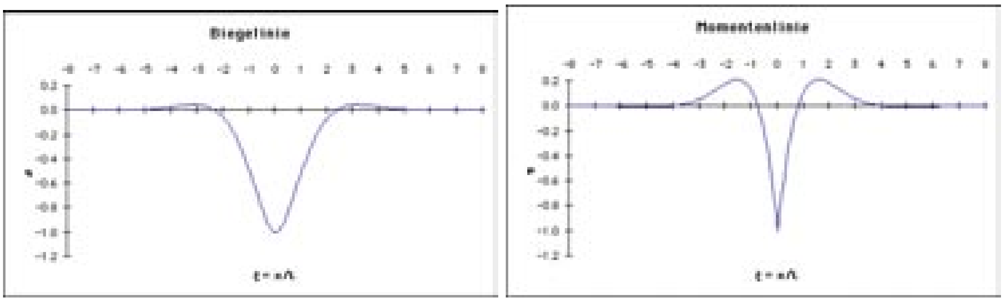
the equation of the deflection curve becomes

$$y = \frac{Q\beta}{2bC} e^{-\beta x} \left(\cos \beta x + \sin \beta x \right) \qquad (1)$$

and the bending moment

$$M = -EI \frac{d^2 y}{dx^2} = -\frac{Q}{4\beta} e^{-\beta x} \left(\cos \beta x - \sin \beta x \right). \qquad (2)$$

Figure 5 Deflection an Bending Moment



The Factor C is called track modulus. It can be understood in the transferred sense as spring constant of all items involved. It depends on many parameters such as rail profile, depth of ballast layer and size of sleeper.

Provided that there is a rough estimate of the Track Modulus C, the attainable results are in most cases fairly good. Many measurements were carried out with the goal to determine the Track modulus. Nowadays the following values are used:

Table 1 Track Modulus

Foundation	[N/mm^3]
Very soft	0.02
Soft	0.05
Good	0.10 – 0.15
Hard	0.30 – 0.60

Unfortunately these values refer to standard gauge only, with axle loads of 20 t. but There are a lot of railways with narrow gauge, with clearly smaller contact area between sleeper and ballast. Furthermore simple considerations indicate that the track modulus is dependent on the axle load.

Both questions the general usability or validity of the given known values for narrow gauge. Besides it is well-known that the track becomes harder in the winter.

Goal

In order to close this gap of knowledge, another measuring campaign was started. The principal purpose was to prove the dependency of the track modulus on the axle load and to find out the difference to the standard gauge.

The Track Modulus can be determined with the equation (1) or (2), as the deflection respectively the strain in the rail is measured. On the basis of a single load the equations become

$$\boxed{C(y) = \frac{1}{4b(EI)^{1/3}}\left(\frac{Q}{y}\right)^{4/3}}$$

$$\boxed{C(\sigma) = \frac{4EI}{b}\left(\frac{Q}{4\sigma W}\right)^{4}}\ .$$

Normally the two methods were used independently. It was decided for the new campaign to use both methods in parallel in order to enable for the first time a direct comparison.

According to the theory it is to be expected that both methods would produce the same results, at least in the qualitative way.

Measurement

The deflection was measured by an opto-electronic system. A LED (light emitting diode) attached to the rail head, marked the Position. A camera standing a few meters away from the track recorded the movements with an accuracy of about 0.01 mm. The system proved to be very reliable, although it was very sensitive to vibrations.

The strain was measured with strain gauges applied in the centre of the rail foot. The accuracy is much higher than ever needed. It posed no problem at all.

A special train was composed with axle loads of 4, 8, 12 and 16 t. It was run with 20, 60 and 90 km/h. Eight sites were chosen with different sleeper types and different soils. Each site consisted of 6 sleepers. The measurements were executed once in the summer and repeated in the winter under the same conditions.

The train had to run 54 times on each site, which was possible only during the break of operation at night. More than 80'000 measured values had to be analysed.

The zero-position of the measuring signals was adjusted automatically before each run on the basis of the values within the range between 20 and 25 m before the first axle. Thus possible temperature influences were eliminated on the DMS measurement and possible displacements of the cameras were corrected.

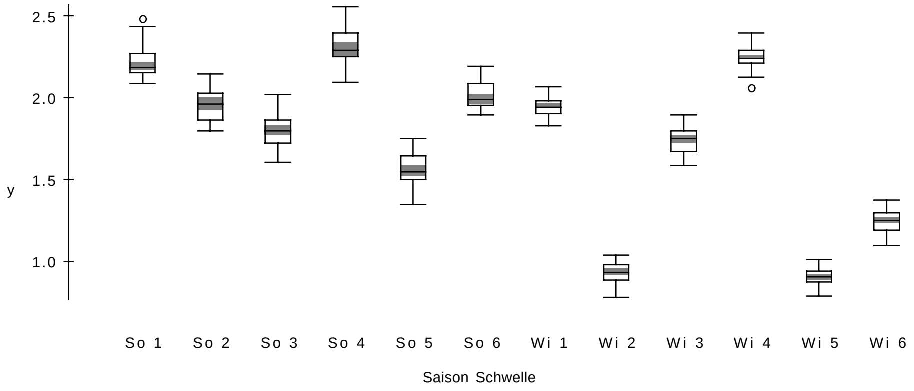
The scannrate was adapted in dependency of the speed, so that the distance between the measured values always amounts to 25 mm.

Unsatisfactory recordings were separated on the basis of predefined quality criteria. About 20 % had to be excluded for many different reasons.

One criteria is the difference of the zero-positions in front of and behind the train. In the case of the deflection this difference averaged about 0.05 mm or 3 %.

The whole discussion about accuracy has to be looked at in the context of the individual behaviour of the sleepers. The average values of the sleepers vary strongly. And the scatter caused by a single sleeper is much smaller than the overall scatter on a measuring site.

Figure 6 Deflection Boxplots (6 sleepers in summer and winter)



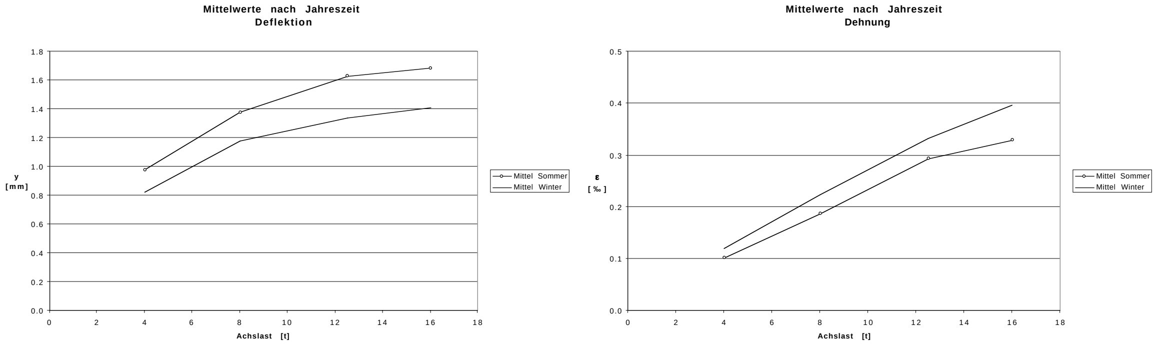
Analysis

At first some parameters were checked to make sure that they have no significant influence. For example it can be proven that the axle loads hardly differ or that the driving direction can be neglected.

Initially it was intended to likewise rule out the influence of speed. But the results for the strain reveal an unexpected dependency. The strain increases with the speed – however only in summer. Since the speed is limited to 90 km/h, no general statement can be made.

On the other hand the assumed dependency can partly be confirmed for the axle load. Concerning the deflection the track becomes harder with increasing axle load. Whereas for the strains this phenomenon is not obvious.

Figure 7 Deflection/Strain – Axle load



The two figures above show also that the two measured variables react differently to the influence of frost. While the deflections become smaller in winter as expected, the strains become larger. This is a clear contradiction to the theory.

Calculation of track modulus based on the strains is absolutely unreliable for numerical reasons. A very small change in strain causes a immense change in track modulus. It is useless to compare these two methods. Hence the track modulus should only be determined on the basis of deflection measurements.

The new figures for the track modulus can be summarised as follows:

Table 2 Track Modulus [N/mm^3]

Axle load [t]	4	8	12	16
soft	0.01	0.03	0.28	0.06
good	0.08	0.10	0.05	0.22
hard	0.15	0.17	0.16	0.39

This table represents a clear progress compared to the well-known values. In particular it becomes evident that for narrow gauge different values can be used.

References

Esveld, C. (1989) Modern Railway Track.

Zimmermann, H. (1888) Die Berechnung des Eisenbahn-Oberbaues, Berlin Verlag von Ernst & Korn (Wilhelm Ernst).